文章编号:1000-0887(2017)05-0561-09

一类两参数非线性反应扩散方程奇摄动问题的广义解*

冯依虎[1], 刘树德[2], 莫嘉琪[2]

(1. 亳州学院 电子与信息工程系, 安徽 亳州 236800;
2. 安徽师范大学 数学计算机科学学院, 安徽 芜湖 241003)

摘要: 利用奇异摄动方法讨论了一类两参数广义奇摄动反应扩散方程问题.首先,在适当的条件下,对两个小参数进行幂级数展开,构造了问题的形式外部解.其次,在区域边界邻近,建立局部坐标系,利用多重尺度变量方法分别构造了问题解的第一、第二边界层校正项.最后,利用合成展开理论,得到了问题广义解的渐近表示式,并用泛函分析不动点原理,估计了渐近展开式的精度.该文得到问题的广义解在重叠区域内具有两个不同厚度的校正函数.它们分别对边界条件起着校正的作用,扩展了问题研究范围,同时还提供了构造这类在重叠区域上不同厚度的校正项的方法,因此具有广泛的研究前景.

关 键 词: 反应扩散; 奇异摄动; 广义解
中图分类号: O175.29 **文献标志码:** A doi: 10.21656/1000-0887.370177

引 言

非线性奇摄动微分方程问题的研究是学术界十分重视的课题[1-2].许多学者作了许多工作,如,Martínez 与 Wolanski[3],Kellogg 与 Kopteva[4],Tian 与 Zhu[5],Skrynnikov[6] 和 Samusenko[7] 等.莫嘉琪等也研究了一系列非线性奇摄动问题[8-19].本文涉及的是奇摄动问题的广义解.

考虑如下非线性反应扩散问题:

$$\frac{\partial u}{\partial t}-\varepsilon^{2m}L^m[u]-\mu^{2k}L^k[u]=f(t,\boldsymbol{x},u),$$

$$t\in[0,T];\ \boldsymbol{x}=(x_1,x_2,\cdots,x_n)\in\Omega,\tag{1}$$

$$\frac{\partial^i u}{\partial n^i}=g_i(t,x),\qquad i=0,1,\cdots,m-1;\ x\in\partial\Omega,\tag{2}$$

$$u(t,x)=h(x),\qquad t=0;\ x\in\Omega,\tag{3}$$

其中 m,k 为正整数, 且 $m>k>0$,ε, μ 为小参数, T 为足够大的正常数;Ω 为 R^n 中的凸域, $\partial\Omega$

* 收稿日期: 2016-06-06; 修订日期: 2016-10-01
基金项目: 国家自然科学基金(11202106);安徽省教育厅自然科学重点基金(KJ2015A347;KJ2017A702);安徽省高校优秀青年人才支持计划重点项目(gxyqZD2016520)
作者简介: 冯依虎(1982—),男,副教授,硕士(E-mail: fengyihubzsz@163.com);
莫嘉琪(1937—),男,教授(通讯作者. E-mail: mojiaqi@mail.ahnu.edu.cn).

为 Ω 光滑边界, $\partial/\partial \boldsymbol{n}$ 为在 $\partial\Omega$ 上的外法向导数,而

$$L_i \equiv \sum_{0\leqslant\mu,\sigma\leqslant i}(-1)^{\mu}\mathrm{D}^{\mu}(a^{\mu\sigma}(x)\mathrm{D}^{\sigma}), \qquad i=k,m,$$

$$\mathrm{D}_0=\frac{\partial}{\partial t},\ \mathrm{D}_j=\frac{\partial}{\partial x_j}, \qquad j=1,2,\cdots,n,$$

$$\mathrm{D}^{\alpha}=\mathrm{D}_1^{\alpha_1}\mathrm{D}_2^{\alpha_2}\cdots\mathrm{D}_n^{\alpha_n}, \qquad \alpha=\sum_{j=1}^{n}\alpha_j,\ x^{\alpha}=x_1^{\alpha_1}x_2^{\alpha_2}\cdots x_n^{\alpha_n},$$

系数 $a^{\mu\sigma}$ 为在 $C^{\infty}(\Omega)$ 上的实值函数, L_i 为在 $\bar{\Omega}$ 上的一致椭圆型算子; f,g_i 和 h 为连续函数, 且 $g_1(0,x)=h(x),x\in\partial\Omega$.

现讨论对应于古典问题(1)~(3)的广义非线性反应扩散初始边值问题:

$$(\psi,\mathrm{D}_0u)-\varepsilon^{2m}B_m[\psi,u]-\mu^{2k}B_k[\psi,u]=(\psi,f(t,x,u)),$$
$$0<t\leqslant T;\ x\in\Omega;\ \forall\psi\in C_0^{\infty}(\Omega), \tag{4}$$

$$(\psi,\bar{\mathrm{D}}_ju)=(\psi,g_j), \qquad j=0,1,\cdots,m-1;\ x\in\partial\Omega;\ \forall\psi\in C_0^{\infty}(\Omega), \tag{5}$$

$$(\psi,u)=(\psi,h), \qquad t=0;\ \forall\psi\in C_0^{\infty}(\Omega), \tag{6}$$

其中

$$B_i[\psi,u]=(\psi,L^i[u]),\ \bar{\mathrm{D}}_i=\frac{\partial^i}{\partial n^i}, \qquad i=m,k,$$

$C_0^{\infty}(\Omega)$ 为在 Ω 中的无限连续可微空间 $C^{\infty}(\Omega)$ 中具有紧致函数的子集, $B_1[\nu,u]$ 为在 Ω 中关于 u,ν 的双线性算子, $H^1(\Omega)$ 为具有有限模

$$\|\psi\|_j=\left\{\sum_{\alpha\leqslant j}\int_{\Omega}|\mathrm{D}^{\alpha}\psi(x)|^2\mathrm{d}x\right\}^{1/2}, \qquad \forall\psi\in C^1(\Omega);\ j=0,1$$

的 Hilbert 空间, (u,ν) 表示在 $H^1(\Omega)$ 上的内积.

假设:

[H1] 小参数 ε, μ 满足: $\lim\limits_{\mu\to 0}\dfrac{\varepsilon}{\mu}=0$.

[H2] 存在常数 $C_{i1},i=m,k$, 使得

$$|B_i[\nu,u]|\leqslant C_{i1}\|\nu\|_1\cdot\|u\|_1,\ |B_i[\nu,\nu]|\leqslant C_{i1}\|\nu\|^2,$$
$$i=m,k;\ \forall\nu,u\in H^1.$$

[H3] 对于 $a^{\mu\sigma}$, 在 Ω 中具有上界 C_2, 且

$$|a^{\mu\sigma}(x)-a^{\mu\sigma}(y)|\leqslant C_2(|x-y|), \qquad \forall x,y\in\Omega.$$

[H4] 存在常数 $\delta_i,i=1,2$, 使得

$$-\delta_2\leqslant\frac{\partial f}{\partial u}\leqslant-\delta_1<0, \qquad \forall x\in\bar{\Omega},\ \forall u\in H^1.$$

1 初边值问题的外部解

当 $\varepsilon=\mu=0$ 时, 非线性反应扩散初始边值问题(4)~(6)退化为广义初值问题:

$$(\psi,\mathrm{D}_0u)=(\psi,f(t,x,u)), \qquad 0<t\leqslant T;\ x\in\Omega;\ \forall\psi\in C_0^{\infty}(\Omega), \tag{7}$$

$$(\psi,u)=(\psi,h), \qquad t=0;\ x\in\Omega;\ \forall\psi\in C_0^{\infty}(\Omega). \tag{8}$$

由假设，初值问题(7)、(8)有解 $U_{00}(t,x)$.

现构造初始边值问题(4)～(6)的外部解 U.设它的渐近展开式为

$$U=\sum_{i,j=0}^{\infty}U_{ij}(t,x)\varepsilon^{i}\mu^{j}. \tag{9}$$

将式(9)代入式(4)和(6)，按 ε 和 μ 的幂展开.合并对应的同次幂 $\varepsilon^{i}\mu^{j}(i,j=0,1,\cdots)$ 的系数.由 $\varepsilon^{0}\mu^{0}$ 的系数为0可得解为 $U_{00}(t,x)$.再由 $\varepsilon^{i}\mu^{j}(i,j=0,1,\cdots,i+j\neq 0)$ 的系数为0，可依次得到

$$(\psi,\mathrm{D}_0U_{ij})=B_m[\phi,U_{i-2m,j}]+B_k[\phi,U_{i,j-2k}]+(\psi,F_{ij}),$$
$$0<t\leqslant T;\ x\in\Omega;\ \forall\psi\in C_0^{\infty}(\Omega), \tag{10}$$

$$(\psi,U_{ij})=0,\qquad t=0;\ \forall\psi\in C_0^{\infty}(\Omega), \tag{11}$$

其中 F_{ij} 为关于 $U_{rs}(r,s=0,1,\cdots;r+s<i+j)$ 逐次已知的函数，上述和下文带有负下标的项均设为0.于是由初值问题(10)、(11)可依次得到解 $U_{ij}(t,x)(i,j=0,1,\cdots;i+j\neq 0)$.将 $U_{ij}(t,x)(i,j=0,1,\cdots)$ 代入式(9)，便得到非线性反应扩散初始边值问题(4)～(6)的外部解.但是外部解未必满足边界条件(5)，因此尚需构造满足边界条件的边界层校正项.

2 构造第一边界层校正项

首先在边界 $\partial\Omega$ 的邻域中构造一组局部坐标系 (ρ,ϕ).定义在该邻域中的每一点的 ρ（$0\leqslant\rho\leqslant\rho_0$，$\rho_0$ 为足够小的正常数，使得邻域中的每一点到边界的法线互不相交）为该点到边界 $\partial\Omega$ 的距离，而 ϕ 为通过该点的法线到边界 $\partial\Omega$ 的交点的 ϕ 的坐标相同.这里的 $\phi=(\phi_1,\phi_2,\cdots,\phi_n)$ 为 $n-1$ 维流形 $\partial\Omega$ 上的一个非奇坐标系.

在 $\partial\Omega$ 的邻域 $0\leqslant\rho\leqslant\rho_0$ 中，有

$$\frac{\partial u}{\partial t}-\varepsilon^{2m}\bar{B}_m[\psi,u]-\mu^{2k}\bar{B}_k[\psi,u]=(\psi,f),\qquad\forall\psi\in C_0^{\infty}(0\leqslant\rho\leqslant\rho_0), \tag{12}$$

$$\left(\psi,\frac{\partial^{l}u}{\partial\rho^{l}}\right)=(\psi,-g_l),\qquad\rho=0;\ l=0,1,\cdots,m-1;\ \forall\psi\in C_0^{\infty}(0\leqslant\rho\leqslant\rho_0), \tag{13}$$

$$(\psi,u)=(\psi,h),\qquad t=0;\ (\rho,\phi)\in\Omega;\ \forall\psi\in C_0^{\infty}(0\leqslant\rho\leqslant\rho_0), \tag{14}$$

其中

$$\bar{B}_l[\psi,u]\equiv(\psi,\bar{L}_l[u]),\qquad l=k,m,$$
$$\bar{L}_l\equiv\sum_{1\leqslant|\nu|,|\sigma|\leqslant l}(-1)^{|\nu|}\bar{\mathrm{D}}^{\nu}(\bar{a}_m^{\nu\sigma}(x)\bar{\mathrm{D}}^{\sigma})+\sum_{1\leqslant|\nu|\leqslant l}\bar{\mathrm{D}}^{\nu}\bar{B}_m^{\nu},\qquad l=k,m,$$

而

$$\bar{\mathrm{D}}_n=\frac{\partial}{\partial\rho},\ \bar{\mathrm{D}}_j=\frac{\partial}{\partial\phi_j},\qquad j=1,2,\cdots,n-1,$$

$$\bar{\mathrm{D}}^{\alpha}=\bar{\mathrm{D}}_1^{\alpha_1}\bar{\mathrm{D}}_2^{\alpha_2}\cdots\bar{\mathrm{D}}_n^{\alpha_n},\qquad\alpha=\sum_{j=1}^{n}\alpha_j,$$

$$\bar{a}_m^{nn}=\sum_{i,j=1}^{n}a_m^{ij}\frac{\partial\rho}{\partial x_i}\frac{\partial\rho}{\partial x_j}>0,\ \bar{a}_k^{nn}=\sum_{i,j=1}^{n}a_k^{ij}\frac{\partial\rho}{\partial x_i}\frac{\partial\rho}{\partial x_j}>0,$$

而 $\bar{a}_m^{ij}, \bar{a}_k^{ij}, \bar{B}_m^j, \bar{B}_k^j$ 的结构式从略.

现在在 $\partial\Omega$ 的邻域 $0 \leqslant \rho \leqslant \rho_0$ 中构造非线性反应扩散初始边值问题(12)~(14)解的第一边界层校正项 V.做伸长变量变换[1-2]:

$$\xi = \frac{\rho}{\mu}. \tag{15}$$

设

$$u \sim \sum_{i,j=0}^{\infty} U_{ij}(t,x)\varepsilon^i\mu^j + V, \tag{16}$$

其中

$$V = \sum_{i,j=0}^{\infty} \nu_{ij}(t,\xi,\phi)\sigma^i\mu^j, \qquad 0 < \sigma = \frac{\varepsilon}{\mu},\ \mu \ll 1. \tag{17}$$

将式(15)~(17)代入反应扩散初始边值问题(12)~(14),按 σ 和 μ 的幂展开,合并同次幂 $\sigma^i\mu^j(i,j=0,1,\cdots)$ 的系数.由 $\sigma^i\mu^j, i,j=0,1,\cdots$ 的系数为 0,可依次得到

$$\frac{\partial\nu_{00}}{\partial t} - (\tilde{\mathrm{D}}_n\psi, \bar{a}_k^{nn}\tilde{\mathrm{D}}_n\nu_{00}) = (\psi, f_u(0,\phi,U_{00})\nu_{00}),$$
$$\forall\psi \in C_0^\infty(0 \leqslant \mu\xi \leqslant \rho_0), \tag{18}$$

$$\left(\psi, \frac{\partial^l\nu_{00}}{\partial\xi^l}\right) = \begin{cases}(\psi, g_0 - U_{00}), & l = 0,\\ -U_{00}, & l = 1,2,\cdots,k-1,\end{cases}\ \xi = 0,\ \forall\psi \in C_0^\infty(0 \leqslant \mu\xi \leqslant \rho_0), \tag{19}$$

$$(\psi, \nu_{00}) = (\psi, h), \qquad t = 0,\ \forall\psi \in C_0^\infty(0 \leqslant \mu\xi \leqslant \rho_0), \tag{20}$$

$$\frac{\partial\nu_{ij}}{\partial t} - (\tilde{\mathrm{D}}_n\psi, \bar{a}_k^{nn}\tilde{\mathrm{D}}_n\nu_{ij}) = (\psi, \bar{F}_{ij}),$$
$$i,j = 0,1,\cdots;\ i+j \neq 0;\ \forall\psi \in C_0^\infty(0 \leqslant \mu\xi \leqslant \rho_0), \tag{21}$$

$$\left(\psi, \frac{\partial^l\nu_{ij}}{\partial\xi^l}\right) = \begin{cases}(\psi, g_l - U_{ij}), & l = j \leqslant k-1,\\ -U_{ij}, & l = 1,2,\cdots,k-1;\ l \neq j,\end{cases}$$
$$\xi = 0,\ \forall\psi \in C_0^\infty(0 \leqslant \mu\xi \leqslant \rho_0), \tag{22}$$

$$(\psi, \nu_{ij}) = 0, \qquad t = 0,\ \forall\psi \in C_0^\infty(0 \leqslant \mu\xi \leqslant \rho_0), \tag{23}$$

其中 $\bar{F}_{ij}(i,j=0,1,\cdots,i+j\neq 0)$ 为逐次已知的函数,且

$$\tilde{\mathrm{D}}_n = \frac{\partial}{\partial\xi},\ \tilde{\mathrm{D}}_j = \frac{\partial}{\partial\phi_j}, \qquad j = 1,2,\cdots,n-1,$$

$$\tilde{\mathrm{D}}^\alpha = \tilde{\mathrm{D}}_1^{\alpha_1}\tilde{\mathrm{D}}_2^{\alpha_2}\cdots\tilde{\mathrm{D}}_n^{\alpha_n}, \qquad \alpha = \sum_{j=1}^{n}\alpha_j.$$

由反应扩散初始边值问题(18)~(20)和(21)~(23),能依次得到 $\nu_{ij}(i,j=0,1,\cdots)$.于是由式(17),便得到了在边界 $\partial\Omega$ 邻域的第一边界层校正函数 V,由假设和非线性反应扩散初始边值问题解的性质知,ν_{ij} 具有性态:

$$\nu_{ij} = O\left(\exp\left(-k_{ij}\frac{\rho}{\mu}\right)\right), \qquad i,j = 0,1,\cdots;\ 0 \leqslant \rho \leqslant \rho_0;\ 0 < \mu \ll 1, \tag{24}$$

其中 $k_{ij}(i, j = 0,1,\cdots)$ 为正常数.

3 构造第二边界层校正项

现在在 $\partial\Omega$ 的邻域 $0 \leqslant \rho \leqslant \rho_0$ 中构造非线性反应扩散初始边值问题(12)~(14)解的第二边界层校正项 W.做伸长变量变换[1-2]:

$$\eta = \frac{\rho}{\varepsilon}. \tag{25}$$

设

$$u \sim \sum_{i,j=0}^{\infty} U_{ij}(t,x)\varepsilon^i\mu^j + W, \tag{26}$$

其中

$$W = \sum_{i,j=0}^{\infty} w_{ij}(t,\eta,\phi)\varepsilon^i\sigma^j, \qquad 0 < \varepsilon,\ \sigma = \frac{\varepsilon}{\mu} \ll 1. \tag{27}$$

将式(25)~(27)代入反应扩散初始边值问题(12)~(14), 按 ε 和 σ 的幂展开, 合并同次幂 $\varepsilon^i\sigma^j(i, j = 0,1,\cdots)$ 的系数.由 $\varepsilon^i\sigma^j, i, j = 0,1,\cdots$ 的系数为 0, 可依次得到

$$\frac{\partial w_{00}}{\partial t} - (\hat{\mathrm{D}}_n\psi, \bar{a}_m^{nn}\hat{\mathrm{D}}_n w_{00}) = (\psi, f_u(0,\phi,U_{00})w_{00}),$$
$$\forall\psi \in C_0^\infty(0 \leqslant \varepsilon\eta \leqslant \rho_0), \tag{28}$$

$$\left(\psi, \frac{\partial^l w_{00}}{\partial\eta^l}\right) = \begin{cases}(\psi, g_0 - U_{00}), & l = 0,\\ -U_{00}, & l = 1,2,\cdots,m-1,\end{cases}$$
$$\eta = 0,\ \forall\psi \in C_0^\infty(0 \leqslant \varepsilon\eta \leqslant \rho_0), \tag{29}$$

$$(\psi, w_{00}) = (\psi, h), \qquad t = 0,\ \forall\psi \in C_0^\infty(0 \leqslant \varepsilon\eta \leqslant \rho_0), \tag{30}$$

$$\frac{\partial w_{ij}}{\partial t} - (\hat{\mathrm{D}}_n\psi, \bar{a}_m^{nn}\hat{\mathrm{D}}_n w_{ij}) = (\psi, \tilde{F}_{ij}),$$
$$i, j = 0,1,\cdots;\ i + j \neq 0;\ \forall\psi \in C_0^\infty(0 \leqslant \varepsilon\eta \leqslant \rho_0), \tag{31}$$

$$\left(\psi, \frac{\partial^l w_{ij}}{\partial\eta^l}\right) = \begin{cases}(\psi, g_l - U_{ij}), & l = j \leqslant k-1,\\ -U_{ij}, & l = k, k+1,\cdots,m-1;\ l \neq j,\end{cases}$$
$$\eta = 0,\ \forall\psi \in C_0^\infty(0 \leqslant \varepsilon\eta \leqslant \rho_0), \tag{32}$$

$$(\psi, w_{ij}) = 0, \qquad t = 0,\ \forall\psi \in C_0^\infty(0 \leqslant \varepsilon\eta \leqslant \rho_0), \tag{33}$$

其中 $\tilde{F}_{ij}(i, j = 0,1,\cdots, i + j \neq 0)$ 为逐次已知的函数, 且

$$\hat{\mathrm{D}}_n = \frac{\partial}{\partial\eta},\ \hat{\mathrm{D}}_j = \tilde{\mathrm{D}}_j = \frac{\partial}{\partial\phi_j}, \qquad j = 1,2,\cdots,n-1,$$

$$\hat{\mathrm{D}}^\alpha = \hat{\mathrm{D}}_1^{\alpha_1}\hat{\mathrm{D}}_2^{\alpha_2}\cdots\hat{\mathrm{D}}_n^{\alpha_n}, \qquad \alpha = \sum_{j=1}^{n}\alpha_j.$$

由反应扩散初始边值问题(28)~(30)和(31)~(33), 能依次得到 $w_{ij}(i, j = 0,1,\cdots)$.于是由式(27), 便得到了在边界 $\partial\Omega$ 邻域的第二边界层校正函数 W, 由假设和非线性反应扩散初始边值问题解的性质知, w_{ij} 具有性态:

$$w_{ij} = O\left(\exp\left(-\bar{k}_{ij}\frac{\rho}{\varepsilon}\right)\right), \qquad i,j = 0,1,\cdots;\ 0 \leqslant \rho \leqslant \rho_0;\ 0 < \varepsilon \ll 1, \tag{34}$$

其中 $\bar{k}_{ij}(i,j = 0,1,\cdots)$ 为正常数.

由式(17)、(27)，便得到广义非线性反应扩散初始边值问题(12)～(14)解的边界层校正函数 Z 的合成渐近展开式：

$$Z = \zeta(\rho)\sum_{i,j=0}^{\infty}[\nu_{ij}(t,\xi,\phi)\sigma^i\mu^j + w_{ij}(t,\eta,\phi)\varepsilon^i\sigma^j], \qquad 0 < \varepsilon, \mu, \sigma \ll 1,$$

其中 $\zeta(\rho)$ 为充分光滑的函数，且

$$\zeta(\rho) = \begin{cases} 1, & 0 \leqslant \rho \leqslant \dfrac{1}{3}\rho_0, \\ 0, & \rho \geqslant \dfrac{2}{\rho_0}. \end{cases}$$

由式(15)、(25)、(24)、(34)和假设[H1]还可看出，第二边界层的厚度比第一边界层的厚度更薄.

由式(9)、(17)、(27)，便得到广义非线性反应扩散初始边值问题(4)～(6)的形式渐近解的表达式

$$u = \sum_{i,j=0}^{\infty}[U_{ij}(t,x)\varepsilon^i\mu^j + \zeta(\rho)(\nu_{ij}(t,\xi,\phi)\sigma^i\mu^j + w_{ij}(t,\eta,\phi)\varepsilon^i\sigma^j)],$$
$$0 < \varepsilon, \mu, \sigma \ll 1. \tag{35}$$

4 余项估计

以下的讨论仅考虑 $\zeta(\rho) = 1$ 时的情形(其他的情形讨论雷同).设

$$u = \sum_{i,j=0}^{M}[U_{ij}\varepsilon^i\mu^j + \nu_{ij}\sigma^i\mu^j + w_{ij}\varepsilon^i\sigma^j] + Z_M, \tag{36}$$

其中余项 $Z_M \in H^1([0,T]\times\Omega)$，考虑广义非线性反应扩散初始边值问题(4)～(6)和式(36)，以及对于充分小的 ε，μ 和式(24)、(34)，有

$$\begin{aligned}
&(\psi,\mathrm{D}_0Z_M) - \varepsilon^{2m}B_m[\psi,Z_M] - \mu^{2k}B_k[\psi,Z_M] - (\psi,f(t,x,Z_M)) = \\
&\quad (\psi,\mathrm{D}_0U_{00}) - (\psi,f(t,x,U_{00})) + \\
&\quad \sum_{i=1}^{m}[(\psi,\mathrm{D}_0U_{ij}) - B_m[\phi,U_{i-2m,j}] - B_k[\phi,U_{i,j-2k}] - (\psi,F_{ij})]\varepsilon^i\mu^j + \\
&\quad \frac{\partial\nu_{00}}{\partial t} - (\tilde{\mathrm{D}}_n\psi,\bar{a}_k^{nn}\tilde{\mathrm{D}}_n\nu_{00}) - (\psi,f_u(0,\phi,U_{00})\nu_{00}) + \\
&\quad \sum_{i=0}^{m}\left[\frac{\partial\nu_{ij}}{\partial t} - (\tilde{\mathrm{D}}_n\psi,\bar{a}_k^{nn}\tilde{\mathrm{D}}_n\nu_{ij}) - (\psi,\bar{F}_{ij})\right]\sigma^i\mu^j + \\
&\quad \frac{\partial w_{00}}{\partial t} - (\hat{\mathrm{D}}_n\psi,\bar{a}_m^{nn}\hat{\mathrm{D}}_nw_{00}) - (\psi,f_u(0,\phi,U_{00})w_{00}) + \\
&\quad \sum_{i=0}^{m}\left[\frac{\partial w_{ij}}{\partial t} - (\hat{\mathrm{D}}_n\psi,\bar{a}_m^{nn}\hat{\mathrm{D}}_nw_{ij}) - (\psi,\tilde{F}_{ij})\right]\varepsilon^i\sigma^j + O(\lambda) = O(\lambda),
\end{aligned}$$

$$\lambda = \max(\varepsilon^{M+1}, \mu^{M+1}, \sigma^{M+1}), \ \forall \psi \in C_0^\infty(\Omega), \tag{37}$$

$$(\psi, \bar{D}_j Z_M) - (\psi, g_j) = 0, \qquad j = 0,1,\cdots,m-1;\ x \in \partial\Omega;\ \forall \psi \in C_0^\infty(\Omega), \tag{38}$$

$$(\psi, Z_M) - (\psi, h) = 0, \qquad t = 0;\ \forall \psi \in C_0^\infty(\Omega). \tag{39}$$

故由不动点定理[1-2]，可得

$$\| Z_M \|_0 = O(\lambda), \qquad 0 < \lambda = \max\left(\varepsilon^{M+1}, \mu^{M+1}, \sigma^{M+1} = \left(\frac{\varepsilon}{\mu}\right)^{M+1}\right) \ll 1.$$

于是有如下定理：

定理 在假设[H1]~[H3]下，对于充分小的 ε，μ，广义非线性反应扩散初始边值问题(2)~(4)对 $\forall (t,x) \in ([0,T] \times \Omega)$ 在 $H^1(\Omega)$ 意义下存在唯一的广义解 $u(t,x)$，并成立

$$\| u(t,u) - \sum_{i,j=0}^{M} [U_{ij}\varepsilon^i \mu^j + \nu_{ij}\sigma^i \mu^j + w_{ij}\varepsilon^i \sigma^j] \|_0 = O(\lambda),$$

$$0 < \lambda = \max\left(\varepsilon^{M+1}, \mu^{M+1}, \sigma^{M+1} = \left(\frac{\varepsilon}{\mu}\right)^{M+1}\right) \ll 1.$$

5 结　　语

本文讨论了两参数奇摄动反应扩散方程初始边值问题，用多重尺度方法分别在区域边界附近构造了两个边界层校正项，并用泛函分析不动点原理证明了得到的渐近解的一致有效性。本文问题的广义解在重叠的区域内具有两个不同厚度的校正项函数，它们分别对相应问题的边界条件起着校正的作用，扩展了奇摄动问题研究领域。本文还提供了对相应问题构造这类在重叠区域上不同厚度校正项的方法，扩展了对奇摄动问题的研究范围，具有广泛的应用前景。

参考文献(References)：

[1] de Jager E M, Furu J. *The Theory of Singular Perturbation*[M]. Amsterdam: North-Holland Publishing Co, 1996.

[2] Barbu L, Morosanu G. *Singularly Perturbed Boundary-Value Problems*[M]. Basel: Birkhauserm Verlag AG, 2007.

[3] Martínez S, Wolanski N. A singular perturbation problem for a quasi-linear operator satisfying the natural growth condition of Lieberman[J]. *SIAM Journal on Mathematical Analysis*, 2009, **41**(1): 318-359.

[4] Kellogg R B, Kopteva N A. Singularly perturbed semilinear reaction-diffusion problem in a polygonal domain[J]. *Journal of Differential Equations*, 2010, **248**(1): 184-208.

[5] Tian C, Zhu P. Existence and asymptotic behavior of solutions for quasilinear parabolic systems[J]. *Acta Applicandae Mathematicae*, 2012, **121**(1): 157-173.

[6] Skrynnikov Y. Solving initial value problem by matching asymptotic expansions[J]. *SIAM Journal on Applied Mathematics*, 2012, **72**(1): 405-416.

[7] Samusenko P. Asymptotic integration of degenerate singularly perturbed systems of parabolic partial differential equations[J]. *Journal of Mathematical Sciences*, 2013, **189**(5): 834-847.

[8] MO Jia-qi, HAN Xiang-lin, CHEN Song-lin. The singularly perturbed nonlocal reaction diffusion system[J]. *Acta Mathematica Scientia*, 2002, **22**B(4): 549-556.

[9] MO Jia-qi, HAN Xiang-lin. A class of singularly perturbed generalized solution for the reaction diffusion problems[J]. *Journal of Systems Science and Mathematical Sciences*, 2002, **22**(4): 447-451.

[10] MO Jia-qi, LIN Wan-tao. A class of nonlinear singularly perturbed problems for reaction diffusion equations with boundary perturbation[J]. *Acta Mathematicae Applicatae Sinica*, 2006, **22**(1): 27-32.

[11] MO Jia-qi. A class of singularly perturbed differential-difference reaction diffusion equation[J]. *Advances in Mathematics*, 2009, **38**(2): 227-231.

[12] MO Jia-qi. Homotopic mapping solving method for gain fluency of a laser pulse amplifier[J]. *Science in China*(*Ser G*): *Physics*, *Mechanics & Astronomy*, 2009, **52**(7): 1007-1070.

[13] MO Jia-qi, LIN Wan-tao. Asymptotic solution of activator inhibitor systems for nonlinear reaction diffusion equations[J]. *Journal of Systems Science and Complexity*, 2008, **20**(1): 119-128.

[14] MO Jia-qi. A singularly perturbed reaction diffusion problem for the nonlinear boundary condition with two parameters[J]. *Chinese Physics B*, 2010, **19**(1): 010203.

[15] FENG Yi-hu, MO Jia-qi. The shock asymptotic solution for nonlinear elliptic equation with two parameters[J]. *Mathematica Applicata*, 2015, **27**(3): 579-585.

[16] FENG Yi-hu, MO Jia-qi. Asymptotic solution for singularly perturbed fractional order differential equation[J]. *Journal of Mathematics*, 2016, **36**(2): 239-245.

[17] FENG Yi-hu, CHEN Xian-feng, MO Jia-qi. The generalized interior shock layer solution of a class of nonlinear singularly perturbed reaction diffusion problem[J]. *Mathematica Applicata*, 2016, **29**(1): 161-165.

[18] 冯依虎, 石兰芳, 汪维刚, 等. 一类广义非线性强阻尼扰动发展方程的行波解[J]. 应用数学和力学, 2015, **36**(3): 315-324.(FENG Yi-hu, SHI Lan-fang, WANG Wei-gang, et al. Traveling wave solution to a class of generalized nonlinear strong-damping disturbed evolution equations[J]. *Applied Mathematics and Mechanics*, 2015, **36**(3): 315-324.(in Chinese))

[19] 冯依虎, 莫嘉琪. 一类非线性非局部扰动 LGH 方程的孤子行波解[J]. 应用数学和力学, 2016, **37**(4): 426-433.(FENG Yi-hu, MO Jia-qi. Soliton travelling wave solutions to a class of nonlinear nonlocal disturbed LGH equations[J]. *Applied Mathematics and Mechanics*, 2016, **37**(4): 426-433.(in Chinese))

Generalized Solution to a Class of Singularly Perturbed Problem of Nonlinear Reaction Diffusion Equation With Two Parameters

FENG Yi-hu[1], LIU Shu-de[2], MO Jia-qi[2]

(1. *Department of Electronics and Information Engineering*, *Bozhou University*, *Bozhou*, *Anhui* 236800, *P.R.China*;

2. *School of Mathematics & Computer Science*, *Anhui Normal University*, *Wuhu*, *Anhui* 241003, *P.R.China*)

Abstract: A class of generalized singularly perturbed problems of reaction diffusion equations with two parameters were considered with the singular perturbation method. Firstly, under suitable conditions, the outer solution to the problem was found. Next, the power series of the two small parameters were developed, and the first and second boundary layer corrective terms for the solution to the problem were constructed with the multiscale variable method, respectively. Finally, based on the composite expansion method, the asymptotic expression of the generalized solution to the problem was obtained, and according to the fixed point theory for functional analysis, the precision of the asymptotic expansion was estimated. Two corrective functions with different thicknesses were obtained for the generalized solution in the overlapping area, and they take effects on the boundary conditions respectively and expand the range of study; moreover, the work provides a costruction method for this kind of corrective terms with different thicknesses in the overlapping area, thus has a wide study foreground.

Key words: reaction diffusion; singular perturbation; generalized solution

Foundation item: The National Natural Science Foundation of China(11202106)

引用本文/Cite this paper:

冯依虎, 刘树德, 莫嘉琪. 一类两参数非线性反应扩散方程奇摄动问题的广义解[J]. 应用数学和力学, 2017, **38**(5): 561-569.

FENG Yi-hu, LIU Shu-de, MO Jia-qi. Generalized solution to a class of singularly perturbed problem of nonlinear reaction diffusion equation with two parameters[J]. *Applied Mathematics and Mechanics*, 2017, **38**(5): 561-569.