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具任意次非线性项的 Camassa-Holm 方程的双孤子新解*

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摘要: 给出辅助方程、函数变换与变量分离解相结合的方法,构造了具任意次非线性项的 Camassa-Holm 方程的双孤子和双周期新解.首先,通过两个辅助方程、函数变换与变量分离解,将具任意次非线性项的 Camassa-Holm 方程的求解问题转化为非线性代数方程的求解问题.然后,借助符号计算系统 Mathematica 求出该方程组的解,并用辅助方程的相关结论,构造了双周期解和双孤子新解.

关　键　词: 函数变换; 变量分离解; 具任意次非线性项的 Camassa-Holm 方程; 双孤子新解

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引　　言

1993 年,Camassa 和 Holm 用物理学原理导出了完全可积的 C-H 方程

$$u_t + 2ku_x + 3uu_x - 2u_x u_{xx} - u_{xxt} - uu_{xxx} = 0, \tag{1}$$

并首次获得了尖峰孤立子解[1],其中 k 是常数, $u(x,t)$ 表示水波在 x 方向的流速.

KdV 方程、BBM 方程和 C-H 方程是研究浅水波运动的重要数学模型.KdV 方程和 BBM 方程存在光滑孤立子解,但 C-H 方程存在尖峰孤子解.浅水波运动存在间断现象,该现象是水波理论的重要研究内容之一.

文献[2]用试探函数法,获得了具任意次非线性项的 Camassa-Holm 方程

$$u_t + ku_x + \alpha u_{xxt} + \beta(u^{p+1})_x + \gamma u_x(u^p)_{xx} + \delta u(u^p)_{xxx} = 0 \tag{2}$$

的三角函数型紧孤立子解和双曲函数型孤立子解,其中 $k,\alpha,\beta,\gamma,\delta$ 和 p 均为任意常数.

当 $\alpha = -1, \beta = 3/2, \gamma = -2, \delta = -1, p = 1$ 时,具任意次非线性项的 Camassa-Holm 方程(2)转化为 Camassa-Holm 方程(1).通常,具任意次非线性项的 Camassa-Holm 方程是不可积系统.获得该方程的尖峰孤子解、紧孤子解、双周期解和双孤子解,对于解释浅水波运动的多种性质具有重要的参考价值.

在非线性发展方程的求解领域,用辅助方程法[3-10],构造单孤子解、类孤子解等新解.本文给出辅助方程、函数变换与变量分离解相结合的方法,构造了具任意次非线性项的 Camassa-

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Holm 方程的新解.首先,通过两个辅助方程、函数变换与变量分离解,将具任意次非线性项的 Camassa-Holm 方程的求解问题转化为非线性代数方程的求解问题.其次,借助符号计算系统 Mathematica 求出该方程组的解,并用辅助方程的相关结论,构造了双周期解和双孤子解.

1 一种辅助方程及其解

下面给出一种辅助方程的几种解:

$$(z'(\xi))^2=\left(\frac{dz(\xi)}{d\xi}\right)^2=a+bz^2(\xi), \tag{3}$$

这里 a,b 是常数.

经计算获得了辅助方程(3)的如下解:

$$z(\xi)=\pm\sqrt{\frac{a}{b}}\sinh(\sqrt{b}\xi)\qquad(a>0,\ b>0), \tag{4}$$

$$z(\xi)=\pm\sqrt{-\frac{a}{b}}\cosh(\sqrt{b}\xi)\qquad(a<0,\ b>0), \tag{5}$$

$$z(\xi)=\frac{1}{2\sqrt{b}}[(-1+a)\cosh(\sqrt{b}\xi)\ \pm(1+a)\sinh(\sqrt{b}\xi)]\qquad(b>0), \tag{6}$$

$$z(\xi)=\pm\sqrt{-\frac{a}{b}}\sin(\sqrt{-b}\xi)\qquad(a>0,\ b<0), \tag{7}$$

$$z(\xi)=\pm\sqrt{-\frac{a}{b}}\cos(\sqrt{-b}\xi)\qquad(a>0,\ b<0), \tag{8}$$

$$z(\xi)=\begin{cases}\sqrt{-\frac{a}{b}}\sin(\sqrt{-b}\xi), \\ \qquad 2p\pi+\frac{\pi}{2}\leqslant\sqrt{-b}\xi\leqslant 2p\pi+\frac{5\pi}{2}\ (a>0,\ b<0,\ p\in\mathbf{Z}), \\ \sqrt{-\frac{a}{b}},\qquad \text{other},\end{cases} \tag{9}$$

$$z(\xi)=\begin{cases}\sqrt{-\frac{a}{b}},\qquad \sqrt{-b}\xi<2p\pi+\frac{\pi}{2}\ (a>0,\ b<0,\ p\in\mathbf{Z}), \\ \sqrt{-\frac{a}{b}}\sin(\sqrt{-b}\xi), \\ \qquad 2p\pi+\frac{\pi}{2}\leqslant\sqrt{-b}\xi\leqslant 2p\pi+\frac{3\pi}{2}\ (a>0,\ b<0,\ p\in\mathbf{Z}), \\ -\sqrt{-\frac{a}{b}},\qquad \sqrt{-b}\xi>2p\pi+\frac{3\pi}{2}\ (a>0,\ b<0,\ p\in\mathbf{Z}),\end{cases} \tag{10}$$

$$z(\xi)=\begin{cases}\sqrt{-\frac{a}{b}}\cos(\sqrt{-b}\xi), \\ \qquad 2p\pi-\pi\leqslant\sqrt{-b}\xi\leqslant 2p\pi+\pi\ (a>0,\ b<0,\ p\in\mathbf{Z}), \\ -\sqrt{-\frac{a}{b}},\qquad \text{other},\end{cases} \tag{11}$$

$$z(\xi)=\begin{cases}-\sqrt{-\dfrac{a}{b}}, & \sqrt{-b}\xi < 2p\pi-\pi\ (a>0,\ b<0,\ p\in\mathbf{Z}),\\ \sqrt{-\dfrac{a}{b}}\cos(\sqrt{-b}\xi), & \\ \quad 2p\pi-\pi\leqslant\sqrt{-b}\xi\leqslant 2p\pi\ (a>0,\ b<0,\ p\in\mathbf{Z}), & \\ \sqrt{-\dfrac{a}{b}}, & \sqrt{-b}\xi>2p\pi\ (a>0,\ b<0,\ p\in\mathbf{Z}).\end{cases}\tag{12}$$

$$z(\xi)=\pm\sqrt{a}\xi\qquad(b=0),\tag{13}$$

$$z(\xi)=\pm\exp(|\sqrt{b}\xi|)\qquad(a=0).\tag{14}$$

2 具任意次非线性项的 Camassa-Holm 方程的新解

2.1 方法及其应用

将下列函数变换

$$u(x,t)=v^{1/p}(x,t)\tag{15}$$

代入方程(2)后得到

$$\begin{aligned}&-pv(x,t)[(-1+p)\alpha v_t(x,t)v_{xx}(x,t)+\\&\quad v_x(x,t)[2(-1+p)\alpha v_{xt}(x,t)-p\gamma v(x,t)v_{xx}(x,t)]]+\\&\quad p^2v^2(x,t)[v_t(x,t)+[k+(1+p)\beta v(x,t)]v_x(x,t)+\alpha v_{xxt}(x,t)+\\&\quad p\delta v(x,t)v_{xxx}(x,t)]+(1-3p+2p^2)\alpha v_t(x,t)v_x^2(x,t)=0.\end{aligned}\tag{16}$$

假设方程(16)存在如下变量分离解：

$$v(x,t)=\frac{g_1P'(\xi)+g_2Q'(\eta)}{f_1P(\xi)+f_2Q(\eta)}=\frac{g_1P'(\theta x+\omega t)+g_2Q'(\vartheta x+\varpi t)}{f_1P(\theta x+\omega t)+f_2Q(\vartheta x+\varpi t)},\tag{17}$$

这里的 $P(\xi)$ 和 $Q(\eta)$ 由下列辅助方程来确定

$$(P'(\xi))^2=\left(\frac{\mathrm{d}P(\xi)}{\mathrm{d}\xi}\right)^2=a_1+b_1P^2(\xi),\tag{18}$$

$$(Q'(\eta))^2=\left(\frac{\mathrm{d}Q(\eta)}{\mathrm{d}\eta}\right)^2=a_2+b_2Q^2(\eta).\tag{19}$$

这里 $a_i,b_i(i=1,2)$ 是辅助方程(18)、(19)的系数；$f_i,g_j(i,j=1,2),\theta,\vartheta,\varpi,\omega$ 是待定常数，而且 $\theta\neq\vartheta,\omega\neq\varpi$.

将解(17)和辅助方程(18)、(19)一起代入方程(16)，并令 $P^p(\xi)Q^q(\eta)P'(\xi)(p=0,1,2;q=0,1,2,\cdots,5)$，$P^l(\xi)Q^k(\eta)(l=k=0,1,2,\cdots,5)$，$P^i(\xi)Q^j(\eta)Q'(\eta)(i=0,1,2,\cdots,5;j=0,1,2,3,4)$，$P^m(\xi)Q^n(\eta)Q'(\eta)P'(\xi)(m=n=0,1,2,3,4)$ 的系数为 0 后，得到一个非线性代数分方程组.用符号计算系统 Mathematica 求出该方程组的如下几组解：

$$b_2=\frac{b_1f_2^2g_1^2}{f_1^2g_2^2},\ \theta=-\frac{f_2g_1\vartheta}{f_1g_2},\ a_1=\frac{a_2g_2^2}{g_1^2},\ \omega=-\frac{f_2g_1\varpi}{f_1g_2};\tag{20}$$

$$b_2=\frac{b_1f_2^2g_1^2}{f_1^2g_2^2},\ \theta=-\frac{f_2g_1\vartheta}{f_1g_2},\ a_1=\frac{a_2g_2^2}{g_1^2},\ \gamma=-3p\delta,\ \omega=-\frac{f_2g_1\varpi}{f_1g_2};\tag{21}$$

$$b_2=\frac{b_1f_2^2g_1^2}{f_1^2g_2^2},\ \alpha=0,\ \theta=-\frac{f_2g_1\vartheta}{f_1g_2},\ a_1=\frac{a_2g_2^2}{g_1^2},\ \omega=-\frac{f_2g_1\varpi}{f_1g_2};\tag{22}$$

$$\begin{cases} b_2 = \dfrac{b_1 f_2^2 g_1^2}{f_1^2 g_2^2},\ \alpha = 0,\ \theta = \dfrac{f_2 g_1 \vartheta}{g_2 f_1},\ \beta = -\dfrac{4 b_1 p f_2^2 g_1^2 \delta \vartheta^2}{f_1^2 g_2^2 (1+p)}, \\ \gamma = -3p\delta,\ \omega = -\dfrac{k f_2 g_1 \vartheta}{f_1 g_2},\ \varpi = -k\vartheta; \end{cases} \tag{23}$$

$$\begin{cases} b_2 = \dfrac{b_1 f_2^2 g_1^2}{f_1^2 g_2^2},\ \alpha = 0,\ a_1 = \dfrac{a_2 g_2^2}{g_1^2},\ \beta = -\dfrac{b_1 p \delta (f_1 g_2 \theta + f_2 g_1 \vartheta)^2}{f_1^2 g_2^2 (1+p)}, \\ \gamma = -3p\delta,\ \omega = -\dfrac{f_1 g_2 k\theta + (k\vartheta + \varpi) f_2 g_1}{f_1 g_2}; \end{cases} \tag{24}$$

$$a_1 = a_2 = 0,\ b_1 = \frac{b_2 f_1^2 g_2^2}{f_2^2 g_1^2},\ \vartheta = -\frac{f_1 g_2}{f_2 g_1}\theta,\ \varpi = -\frac{f_1 g_2 \omega}{f_2 g_1}. \tag{25}$$

2.2 构造具任意次非线性项的 Camassa-Holm 方程的双孤子新解

将辅助方程(2)的解与非线性代数分方程组的解(20)~(25)代入式(26),获得具任意次非线性项的 Camassa-Holm 方程的单孤子解、双孤子解、单周期解和双周期解.

$$u(x,t) = v^{1/p}(x,t) = \left\{ \frac{g_1 P'(\theta x + \omega t) + g_2 Q'(\vartheta x + \varpi t)}{f_1 P(\theta x + \omega t) + f_2 Q(\vartheta x + \varpi t)} \right\}^{1/p}. \tag{26}$$

情形 1 单周期新解

将辅助方程(2)的解(7)~(12)两两组合,并与式(20)(或式(21)~(23))一起代入式(26),获得光滑型三角函数周期解、紧孤子型三角函数周期解以及光滑型与紧孤子型组合的三角函数周期解(这里未讨论).

情形 2 光滑单孤子解

将辅助方程(2)的解(4)~(6)两两组合,并与式(20)(或式(21)~(23))一起代入式(26),获得光滑单孤子解.

将辅助方程(2)的解(4)~(6)两两组合,并与式(20)一起代入式(26),获得下列光滑单孤子解:

$$u_1(x,t) = \left\{ \left(\pm \frac{f_2 g_1 \sqrt{b_1}}{f_1 g_2} \left(-(1+a_2) g_2 + \left[-2\sqrt{-a_2 g_2^2} + (-1+a_2) g_2 \right] \tanh\left[\frac{f_2 g_1 \sqrt{b_1}}{f_1 g_2} (\vartheta x + \varpi t) \right] \right) \right) \Big/ \left(\pm 2\sqrt{-a_2} f_2 + f_2 \left((-1+a_2) - (1+a_2) \tanh\left[\frac{f_2 g_1 \sqrt{b_1}}{f_1 g_2} (\vartheta x + \varpi t) \right] \right) \right) \right\}^{1/p}, \tag{27}$$

这里 $a_2 < 0,\ b_1 > 0$.

$$u_2(x,t) = \left\{ \left(\pm \frac{b_1 f_2 g_1}{f_1 g_2} \left(-2\sqrt{a_2} g_2 + g_2 \left[(1+a_2) - (-1+a_2) \tanh\left[\frac{f_2 g_1 \sqrt{b_1}}{f_1 g_2} (\vartheta x + \varpi t) \right] \right] \right) \right) \Big/ \left(-(-1+a_2)\sqrt{b_1} f_2 + \left[2\sqrt{a_2 b_1} f_2 + (1+a_2)\sqrt{b_1} f_2 \right] \tanh\left[\frac{f_2 g_1 \sqrt{b_1}}{f_1 g_2} (\vartheta x + \varpi t) \right] \right) \right\}^{1/p}, \tag{28}$$

这里 $a_2 > 0,\ b_1 > 0$.

$$u_3(x,t)=\left\{\pm\frac{b_1g_1}{f_1g_2\sqrt{a_2b_1}}\coth\left[\frac{f_2g_1\sqrt{b_1}}{f_1g_2}(\vartheta x+\varpi t)\right]\right\}^{1/p}\quad(a_2>0,b_1>0),\tag{29}$$

$$u_4(x,t)=\left\{\frac{\mp\sqrt{b_1}g_1}{f_2}\tanh\left[\frac{f_2g_1\sqrt{b_1}}{f_1g_2}(\vartheta x+\varpi t)\right]\right\}^{1/p}\quad(b_1>0).\tag{30}$$

情形 3 尖峰单孤子解

将辅助方程(2)的解(14)与式(25)一起代入式(26),获得尖峰单孤子解(这里未讨论).

情形 4 双周期新解

将辅助方程(2)的解(7)~(12)两两组合,并与式(24)一起代入式(26),获得光滑型三角函数双周期解、紧孤子型三角函数双周期解以及光滑型与紧孤子型组合的三角函数双周期解(这里未讨论).

情形 5 双孤子新解

将辅助方程(2)的解(4)~(6)两两组合,并与式(24)一起代入式(26),获得双孤子解:

$$\begin{cases}u_5(x,t)=\left\{\left(\pm\frac{\sqrt{b_1}f_2g_1}{f_1g_2}\left[-(1+a_2)g_2\cosh\left[\frac{\sqrt{b_1}f_2g_1}{f_1g_2}(\vartheta x+\varpi t)\right]+\right.\right.\right.\\\qquad\left.\left.(-1+a_2)g_2\sinh\left[\frac{\sqrt{b_1}f_2g_1}{f_1g_2}(\vartheta x+\varpi t)\right]\mp\Omega_1(x,t)\right]\right)\Big/\\\qquad\left((-1+a_2)f_2\cosh\left[\frac{\sqrt{b_1}f_2g_1}{f_1g_2}(\vartheta x+\varpi t)\right]-\right.\\\qquad\left.\left.(1+a_2)f_2\sinh\left[\frac{\sqrt{b_1}f_2g_1}{f_1g_2}(\vartheta x+\varpi t)\right]\pm\Omega_2(x,t)\right)\right\}^{1/p},\\\Omega_1(x,t)=2\sqrt{-a_2}g_2\sinh\left[\frac{\sqrt{b_1}}{f_1g_2}[f_1g_2(kt-x)\theta+f_2g_1(k\vartheta+\varpi)t]\right],\\\Omega_2(x,t)=2\sqrt{-a_2}f_2\cosh\left[\frac{\sqrt{b_1}}{f_1g_2}[f_1g_2(kt-x)\theta+f_2g_1(k\vartheta+\varpi)t]\right]\\\qquad\qquad(b_1>0,a_2<0).\end{cases}\tag{31}$$

$$u_6(x,t)=\left\{\left(\pm\sqrt{a_2}g_2\cosh\left[\frac{\sqrt{b_1}}{f_1g_2}[f_1g_2(kt-x)\theta+f_2g_1(k\vartheta+\varpi)t]\right]-\Omega_3(x,t)\right)\Big/\left(\pm\frac{f_1g_2}{2\sqrt{b_1}g_1}\Omega_4(x,t)\mp\frac{f_1g_2\sqrt{a_2}}{g_1\sqrt{b_1}}\sinh\left[\frac{\sqrt{b_1}}{f_1g_2}[f_1g_2(kt-x)\theta+f_2g_1(k\vartheta+\varpi)t]\right]\right)\right\}^{1/p},\tag{32a}$$

$$\Omega_3(x,t)=\frac{1}{2}g_2\left\{(1+a_2)\cosh\left[\frac{\sqrt{b_1}f_2g_1}{f_1g_2}(\vartheta x+\varpi t)\right]-(-1+a_2)\sinh\left[\frac{\sqrt{b_1}f_2g_1}{f_1g_2}(\vartheta x+\varpi t)\right]\right\},\tag{32b}$$

$$\Omega_4(x,t)=(-1+a_2)\cosh\left[\frac{\sqrt{b_1}f_2g_1}{f_1g_2}(\vartheta x+\varpi t)\right]-$$

$$(1+a_2)\sinh\left[\frac{\sqrt{b_1}f_2g_1}{f_1g_2}(\vartheta x+\varpi t)\right]\quad(b_1>0,a_2>0).\tag{32c}$$

$$u_7(x,t)=\Bigg\{\Bigg(\sqrt{a_2}g_2\cosh\left[\frac{\sqrt{b_1}f_2g_1}{f_1g_2}(\vartheta x+\varpi t)\right]\pm$$

$$\sqrt{a_2}g_2\cosh\left[\frac{\sqrt{b_1}}{f_1g_2}[f_1g_2(kt-x)\theta+f_2g_1(k\vartheta+\varpi)t]\right]\Bigg)\Bigg/$$

$$\Bigg(\frac{f_1g_2}{g_1}\sqrt{\frac{a_2}{b_1}}\Bigg(\sinh\left[\frac{\sqrt{b_1}f_2g_1}{f_1g_2}(\vartheta x+\varpi t)\right]\mp$$

$$\sinh\left[\frac{\sqrt{b_1}}{f_1g_2}[f_1g_2(kt-x)\theta+f_2g_1(k\vartheta+\varpi)t]\right]\Bigg)\Bigg)\Bigg\}^{1/p},\tag{33}$$

这里 $a_2>0,\ b_1>0$.

$$u_8(x,t)=\Bigg\{\Bigg(\pm\sqrt{-a_2b_1}g_2\Bigg(\cosh\left[\frac{\sqrt{b_1}}{f_1g_2}[f_1g_2(kt-x)\theta+f_2g_1(k\vartheta+\varpi)t]\right]+$$

$$\sinh\left[\frac{\sqrt{b_1}f_2g_1}{f_1g_2}(\vartheta x+\varpi t)\right]\Bigg)\Bigg)\Bigg/\Bigg(\frac{\sqrt{-a_2}f_1g_2}{g_1}\Bigg(\pm\cosh\left[\frac{\sqrt{b_1}f_2g_1}{f_1g_2}(\vartheta x+\varpi t)\right]\mp$$

$$\sinh\left[\frac{\sqrt{b_1}}{f_1g_2}[f_1g_2(kt-x)\theta+f_2g_1(k\vartheta+\varpi)t]\right]\Bigg)\Bigg)\Bigg\}^{1/p},\tag{34}$$

这里 $a_2<0,\ b_1>0$.

$$u_9(x,t)=\Bigg\{\Bigg(\sqrt{-a_2}g_2\Bigg(\sinh\left[\frac{\sqrt{b_1}f_2g_1}{f_1g_2}(\vartheta x+\varpi t)\right]\mp$$

$$\sinh\left[\frac{\sqrt{b_1}}{f_1g_2}[f_1g_2(kt-x)\theta+f_2g_1(k\vartheta+\varpi)t]\right]\Bigg)\Bigg)\Bigg/$$

$$\Bigg(\frac{f_1g_2}{g_1}\sqrt{\frac{-a_2}{b_1}}\Bigg(\cosh\left[\frac{\sqrt{b_1}f_2g_1}{f_1g_2}(\vartheta x+\varpi t)\right]\pm$$

$$\cosh\left[\frac{\sqrt{b_1}}{f_1g_2}[f_1g_2(kt-x)\theta+f_2g_1(k\vartheta+\varpi)t]\right]\Bigg)\Bigg)\Bigg\}^{1/p},\tag{35}$$

这里 $a_2<0,\ b_1>0$.

3 结 论

文献[2]用试探函数法,获得了具任意次非线性项的 Camassa-Holm 方程的双曲函数单孤子解和三角函数型紧孤子解.本文给出辅助方程、函数变换与变量分离解相结合的方法,获得了几种新解:

1) 新周期解.这里包括了三角函数紧孤子解、三角函数光滑周期解、光滑和紧孤子相组合的周期解和双周期解.

2) 新孤子解.这里包括了双曲函数光滑单孤子解、尖峰单孤子解和双孤子解.

在方程(2)中取 $p=1,\alpha=-1,\beta=3/2,\gamma=-2,\delta=-1$，获得 Camassa-Holm 方程(1)的三角函数紧孤子解、三角函数光滑周期解、光滑和紧孤子相组合的周期解、光滑单孤子解和尖峰单孤子解.

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New 2-Soliton Solutions to the Arbitrary Order Nonlinear Camassa-Holm Equation

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Abstract: The method combining the auxiliary equation, the function transformation and the variable separation solutions was proposed to construct the new 2-soliton and 2-period solutions to the arbitrary order nonlinear Camassa-Holm equation. Step 1, with 2 auxiliary equations, the function transformation and the variable separation solutions, the problem of solving the arbitrary order nonlinear Camassa-Holm equation was transformed to the problem of solving the nonlinear algebraic equations. Step 2, by means of symbolic computation system Mathematica, the solutions to the algebraic equations were obtained, and with the help of the relative conclusions on the auxiliary equation, the new 2-soliton and 2-period solutions were constructed.

Key words: function transformation; variable separation solution; arbitrary order nonlinear Camassa-Holm equation; new 2-soliton solution

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