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带有变化分布时滞的复值神经网络 Lagrange 稳定性*

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摘要: 研究了带有变化分布时滞的复值神经网络 Lagrange 稳定性问题.通过构造合适的 Lyapunov-Krasovskii 泛函, 并使用矩阵不等式技巧,建立了网络全局指数 Lagrange 稳定性的判定条件.提供的判据是复值线性矩阵不等式, 能够使用 MATLAB 软件的 YALMIP 工具箱快速计算.

关 键 词: 复值神经网络; 变化分布时滞; Lagrange 稳定性; 复值线性矩阵不等式

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引 言

自 1982 年美国加州理工学院生物物理学家 Hopfield 建立了神经网络的数学模型以来[1],许多神经网络模型被陆续提出, 它们的动力学行为被研究.神经网络现已广泛应用于图像处理、信号处理、模式识别、联想记忆、优化计算、保密通讯等诸多领域.应用神经网络解决实际问题时, 首要任务是要考虑模型的稳定性[2].然而, 在用硬件实现神经网络的过程中, 由于放大器转换速度的限制, 传递延迟是不可避免的[3],时滞的出现不仅会降低网络的传递速度, 而且会导致本来稳定的网络变得不稳定, 并有可能引起震荡甚至出现混沌现象[4].因此,将时滞引入神经网络, 建立时滞神经网络模型并研究其稳定性具有重要的理论意义和实用价值[5].近年来, 各类时滞神经网络的稳定性得到了大量的研究, 许多稳定性结果相继被提出[2-9].

以上提到的神经网络是实值神经网络,即网络的神经元状态、输出、权值和激活函数都取实数值.虽然实值神经网络已在诸多领域得到了应用, 但也有其局限性[10], 如在电子信息工程领域, 人们就需要处理复数数据.因此, 复值神经网络应运而生[11].复值神经网络的神经元状态、输出、权值和激活函数都为复值, 能够直接处理复值数据.正因如此, 复值神经网络稳定性研究受到人们的特别关注, 并先后获得了一些 Lyapunov 全局稳定性结论[12-21].

众所周知, 当神经网络用于优化时, 要求网络有唯一平衡点, 该平衡点对应于待求解的目标, 而且随着时间的增加, 要求网络的所有状态趋近于这个平衡点.从动力系统的观点来看, 神经网络 Lyapunov 全局稳定性是单稳定性, 这就意味着神经网络具有唯一平衡点,并且

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所有轨线都趋于该平衡点[1].当神经网络用于图像处理时，希望网络的平衡点尽可能多，这样就可以将处理后的结果存储于这些平衡点上，而且网络的状态在一段时间后也要趋近于某个平衡点，此时的神经网络不再是全局稳定的.因此 Lagrange 稳定性被提出，它是 Lyapunov 全局稳定性的拓展[22].近年来，实值神经网络的 Lagrange 稳定性问题得到了较多的研究[22-25].复值神经网络的 Lagrange 稳定性问题也开始受到人们的关注.最近，文献[26]研究了带有离散变化时滞的复值神经网络 Lagrange 稳定性问题，在激活函数分解为实部函数和虚部函数的情形下，通过将复值神经网络分解为两个实值动力系统，建立了网络的 Lagrange 稳定性条件.本文继续了这方面的工作，研究带有变化分布时滞的复值神经网络 Lagrange 稳定性问题，建立网络 Lagrange 稳定性判据.不同于文献[26]的研究方法，本文没有将复值神经网络分解为两个实值动力系统来进行研究.

1 预 备 知 识

本文考虑如下带有变化分布时滞的复值神经网络模型：

$$\dot{\boldsymbol{z}}(t) = -\boldsymbol{C}\boldsymbol{z}(t) + \boldsymbol{A}\boldsymbol{f}(\boldsymbol{z}(t)) + \boldsymbol{B}\int_{t-\sigma(t)}^{t}\boldsymbol{f}(\boldsymbol{z}(s))\mathrm{d}s + \boldsymbol{J}, \qquad t \geqslant 0, \tag{1}$$

其中 $\boldsymbol{z}(t) = (z_1(t), z_2(t), \cdots, z_n(t))^{\mathrm{T}} \in \mathbb{C}^n$ 表示 t 时刻 n 个神经元的状态向量；$\sigma(t)$ 是变化分布时滞并且满足 $0 \leqslant \sigma(t) \leqslant \sigma$，$\sigma$ 是正常数；$\boldsymbol{C} = \mathrm{diag}(c_1, c_2, \cdots, c_n) \in \mathbb{R}^{n\times n}$，$c_j > 0\ (j = 1, 2, \cdots, n)$ 表示自反馈连接权矩阵；$\boldsymbol{f}(\boldsymbol{z}(t)) = (f_1(z_1(t)), f_2(z_2(t)), \cdots, f_n(z_n(t)))^{\mathrm{T}} \in \mathbb{C}^n$ 表示神经元激活函数；$\boldsymbol{A}$ 和 $\boldsymbol{B}$ 分别表示连接权矩阵和分布时滞连接权矩阵；$\boldsymbol{J} \in \mathbb{C}^n$ 表示外部输入常向量.

模型(1)的初始条件为

$$\boldsymbol{z}(s) = \boldsymbol{\varphi}(s), \qquad s \in [-\sigma, 0],$$

其中 $\boldsymbol{\varphi}(s)$ 在 $[-\sigma, 0]$ 内有界且连续.

本文给出如下假设：

(H) 对于任意 $i \in \{1, 2, \cdots, n\}$，存在一个正对角矩阵 $\boldsymbol{L} = \mathrm{diag}(l_1, l_2, \cdots, l_n)$ 使得 $f_i(0) = 0$ 并且对于任意的 $\alpha_1 \neq \alpha_2$，有

$$| f_i(\alpha_1) - f_i(\alpha_2) | \leqslant l_i | \alpha_1 - \alpha_2 |.$$

定义 1[22] 对于任意的正实数 $\alpha > 0$，如果存在一个常数 $K = K(\alpha) > 0$，使得 $\| \boldsymbol{z}(t, \varphi) \| < K$ 对任意的初始条件 $\boldsymbol{\varphi} \in C_\alpha = \{\boldsymbol{\varphi} \in C([-h, 0], \mathbb{C}^n) \mid \|\boldsymbol{\varphi}\| < K, t \geqslant 0\}$ 都成立，则称复值神经网络模型(1)是一致 Lagrange 稳定的.

定义 2[22] 如果存在一个径向无界正定函数 $V(\cdot)$ 和一个非负连续函数 $K(\cdot)$，两个正常数 β 和 ε，使得对于模型(1)的任意解 $\boldsymbol{z}(t)$，当 $V(\boldsymbol{z}(t)) > \beta$ 时，有 $V(\boldsymbol{z}(t)) - \beta \leqslant K(\boldsymbol{\varphi})\mathrm{e}^{-\varepsilon t}$ 对于任意的 $t \geqslant 0$ 和 $\boldsymbol{\varphi} \in C_\alpha$ 时都成立，则称复值神经网络模型(1)是全局指数吸引的，并称 $\boldsymbol{\Omega} = \{\boldsymbol{z}(t) \in \mathbb{C}^n \mid V(\boldsymbol{z}(t)) \leqslant \beta\}$ 是模型(1)的吸引集.

定义 3[22] 如果复值神经网络模型(1)既是一致 Lagrange 稳定的，又是全局指数吸引的，则称它是 Lagrange 全局指数稳定的.

引理 1[18] 对于任意的 $\boldsymbol{a}, \boldsymbol{b} \in \mathbb{C}^n$，如果 $\boldsymbol{P} \in \mathbb{C}^{n\times n}$ 是一个正定 Hermite 矩阵，则 $\boldsymbol{a}^*\boldsymbol{b} + \boldsymbol{b}^*\boldsymbol{a} \leqslant \boldsymbol{a}^*\boldsymbol{P}\boldsymbol{a} + \boldsymbol{b}^*\boldsymbol{P}^{-1}\boldsymbol{b}$.

引理 2[10] 对于任意的常数矩阵 $\boldsymbol{W} \in \mathbb{C}^{m\times m}$，$\boldsymbol{W} > 0$（正定）和向量函数 $\boldsymbol{\omega}: [a, b] \to \mathbb{C}^m$，$a < b$，则

$$\left(\int_a^b \boldsymbol{\omega}(s)\mathrm{d}s\right)^* \boldsymbol{W}\left(\int_a^b \boldsymbol{\omega}(s)\mathrm{d}s\right) \leqslant (b - a)\int_a^b \boldsymbol{\omega}^*(s)\boldsymbol{W}\boldsymbol{\omega}(s)\mathrm{d}s.$$

引理 3[22] 设 $V(t) \in C([0, +\infty), \mathbb{R})$，如果存在正常数 α 和 β 使得

$$D^+V(t) \leqslant -\alpha V(t) + \beta, \qquad t \geqslant 0$$

成立，则有

$$V(t) - \frac{\beta}{\alpha} \leqslant \left(V(0) - \frac{\beta}{\alpha}\right)\mathrm{e}^{-\alpha t}, \qquad t \geqslant 0.$$

2 主要结果

定理 1 在假设(H)条件下，如果存在 3 个正定 Hermite 矩阵 $\boldsymbol{P}_1$，$\boldsymbol{P}_2$ 和 $\boldsymbol{W}$，使得复值线性矩阵不等式

$$\boldsymbol{\Pi} = \begin{pmatrix} \boldsymbol{\Pi}_{11} & \boldsymbol{P}_1\boldsymbol{A} & \boldsymbol{P}_1\boldsymbol{B} & \boldsymbol{P}_1 \\ \boldsymbol{A}^*\boldsymbol{P}_1 & \sigma\boldsymbol{P}_2 - \boldsymbol{R} & \boldsymbol{0} & \boldsymbol{0} \\ \boldsymbol{B}^*\boldsymbol{P}_1 & \boldsymbol{0} & -\dfrac{\mathrm{e}^{-\alpha\sigma}}{\sigma}\boldsymbol{P}_2 & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & -\boldsymbol{W} \end{pmatrix} < 0 \tag{2}$$

(负定)成立，其中 $\boldsymbol{\Pi}_{11} = \alpha\boldsymbol{P}_1 - \boldsymbol{P}_1\boldsymbol{C} - \boldsymbol{C}\boldsymbol{P}_1 + \boldsymbol{LRL}$，则复值神经网络(1)是 Lagrange 全局指数稳定的,并且复值神经网络(1)的吸引集为

$$\boldsymbol{\Omega} = \left\{\boldsymbol{z}(t) \in \mathbb{C}^n : \|\boldsymbol{z}(t)\| \leqslant \frac{\boldsymbol{J}^*\boldsymbol{WJ}}{\alpha\lambda_{\min}(\boldsymbol{P}_1)}\right\}. \tag{3}$$

证明 构造 Lyapunov-Krasovskii 泛函

$$V(t) = V_1(t) + V_2(t), \tag{4}$$

其中

$$V_1(t) = \boldsymbol{z}^*(t)\boldsymbol{P}_1\boldsymbol{z}(t), \tag{5}$$

$$V_2(t) = \int_{-\sigma}^{0}\int_{t+\xi}^{t}\mathrm{e}^{\alpha(s-t)}\boldsymbol{f}^*(\boldsymbol{z}(s))\boldsymbol{P}_2\boldsymbol{f}(\boldsymbol{z}(s))\,\mathrm{d}s\,\mathrm{d}\xi. \tag{6}$$

则 $V_1(t)$ 沿着模型(1) 对 t 求导，并应用引理 1，得

$$\begin{aligned}
\dot{V}_1(t) = {} & \boldsymbol{z}^*(t)\boldsymbol{P}_1\dot{\boldsymbol{z}}(t) + \dot{\boldsymbol{z}}^*(t)\boldsymbol{P}_1\boldsymbol{z}(t) = \\
& -\alpha V_1(t) + \alpha\boldsymbol{z}^*(t)\boldsymbol{P}_1\boldsymbol{z}(t) + \boldsymbol{z}^*(t)\boldsymbol{P}_1\dot{\boldsymbol{z}}(t) + \dot{\boldsymbol{z}}^*(t)\boldsymbol{P}_1\boldsymbol{z}(t) = \\
& -\alpha V_1(t) + \alpha\boldsymbol{z}^*(t)\boldsymbol{P}_1\boldsymbol{z}(t) + \\
& \boldsymbol{z}^*(t)\boldsymbol{P}_1\left(-\boldsymbol{Cz}(t) + \boldsymbol{Af}(\boldsymbol{z}(t)) + \boldsymbol{B}\int_{t-\sigma(t)}^{t}\boldsymbol{f}(\boldsymbol{z}(s))\,\mathrm{d}s + \boldsymbol{J}\right) + \\
& \left(-\boldsymbol{Cz}(t) + \boldsymbol{Af}(\boldsymbol{z}(t)) + \boldsymbol{B}\int_{t-\sigma(t)}^{t}\boldsymbol{f}(\boldsymbol{z}(s))\,\mathrm{d}s + \boldsymbol{J}\right)^*\boldsymbol{P}_1\boldsymbol{z}^*(t) \leqslant \\
& -\alpha V_1(t) + \boldsymbol{z}^*(t)(\alpha\boldsymbol{P}_1 - \boldsymbol{P}_1\boldsymbol{C} - \boldsymbol{C}\boldsymbol{P}_1 + \boldsymbol{P}_1\boldsymbol{W}^{-1}\boldsymbol{P}_1)\boldsymbol{z}(t) + \\
& \boldsymbol{z}^*(t)\boldsymbol{P}_1\boldsymbol{Af}(\boldsymbol{z}(t)) + \boldsymbol{z}^*(t)\boldsymbol{P}_1\boldsymbol{B}\int_{t-\sigma(t)}^{t}\boldsymbol{f}(\boldsymbol{z}(s))\,\mathrm{d}s + \\
& \boldsymbol{f}^*(\boldsymbol{z}(t))\boldsymbol{A}^*\boldsymbol{P}_1\boldsymbol{z}(t) + \left(\int_{t-\sigma(t)}^{t}\boldsymbol{f}(\boldsymbol{z}(s))\,\mathrm{d}s\right)^*\boldsymbol{B}^*\boldsymbol{P}_1\boldsymbol{z}(t) + \boldsymbol{J}^*\boldsymbol{WJ}.
\end{aligned} \tag{7}$$

计算 $V_2(t)$ 对 t 的导数，注意到 $0 < \sigma(t) \leqslant \sigma$，并应用引理 2，有

$$\begin{aligned}
\dot{V}_2(t) = {} & -\alpha\int_{-\sigma}^{0}\int_{t+\xi}^{t}\mathrm{e}^{\alpha(s-t)}\boldsymbol{f}^*(\boldsymbol{z}(s))\boldsymbol{P}_2\boldsymbol{f}(\boldsymbol{z}(s))\,\mathrm{d}s\,\mathrm{d}\xi + \\
& \sigma\boldsymbol{f}^*(\boldsymbol{z}(t))\boldsymbol{P}_2\boldsymbol{f}(\boldsymbol{z}(t)) - \int_{t-\sigma}^{t}\mathrm{e}^{\alpha(s-t)}\boldsymbol{f}^*(\boldsymbol{z}(s))\boldsymbol{P}_2\boldsymbol{f}(\boldsymbol{z}(s))\,\mathrm{d}s =
\end{aligned}$$

$$-\alpha V_2(t)+\sigma\boldsymbol{f}^*(z(t))\boldsymbol{P}_2\boldsymbol{f}(z(t))-\int_{t-\sigma}^{t}\mathrm{e}^{\alpha(s-t)}\boldsymbol{f}^*(z(s))\boldsymbol{P}_2\boldsymbol{f}(z(s))\mathrm{d}s\leqslant$$

$$-\alpha V_2(t)+\sigma\boldsymbol{f}^*(z(t))\boldsymbol{P}_2\boldsymbol{f}(z(t))-\frac{\mathrm{e}^{-\alpha\sigma}}{\sigma}\int_{t-\sigma(t)}^{t}\boldsymbol{f}^*(z(s))\boldsymbol{P}_2\boldsymbol{f}(z(s))\mathrm{d}s\leqslant$$

$$-\alpha V_2(t)+\sigma\boldsymbol{f}^*(z(t))\boldsymbol{P}_2\boldsymbol{f}(z(t))-$$
$$\frac{\mathrm{e}^{-\alpha\sigma}}{\sigma}\left(\int_{t-\sigma(t)}^{t}\boldsymbol{f}(z(s))\mathrm{d}s\right)^*\boldsymbol{P}_2\left(\int_{t-\sigma(t)}^{t}\boldsymbol{f}(z(s))\mathrm{d}s\right).\tag{8}$$

由式(7)和(8)得

$$\dot V(t)\leqslant-\alpha V(t)+\boldsymbol{z}^*(t)(\alpha\boldsymbol{P}_1-\boldsymbol{P}_1\boldsymbol{C}-\boldsymbol{C}\boldsymbol{P}_1+\boldsymbol{P}_1\boldsymbol{W}^{-1}\boldsymbol{P}_1)\boldsymbol{z}(t)+$$
$$\boldsymbol{z}^*(t)\boldsymbol{P}_1\boldsymbol{A}\boldsymbol{f}(\boldsymbol{z}(t))+\boldsymbol{z}^*(t)\boldsymbol{P}_1\boldsymbol{B}\int_{t-\sigma(t)}^{t}\boldsymbol{f}(\boldsymbol{z}(s))\mathrm{d}s+$$
$$\boldsymbol{f}^*(\boldsymbol{z}(t))\boldsymbol{A}^*\boldsymbol{P}_1\boldsymbol{z}(t)+\left(\int_{t-\sigma(t)}^{t}\boldsymbol{f}(\boldsymbol{z}(s))\mathrm{d}s\right)^*\boldsymbol{B}^*\boldsymbol{P}_1\boldsymbol{z}(t)+\boldsymbol{J}^*\boldsymbol{W}\boldsymbol{J}+$$
$$\sigma\boldsymbol{f}^*(\boldsymbol{z}(t))\boldsymbol{P}_2\boldsymbol{f}(\boldsymbol{z}(t))-\frac{\mathrm{e}^{-\alpha\sigma}}{\sigma}\left(\int_{t-\sigma(t)}^{t}\boldsymbol{f}(\boldsymbol{z}(s))\mathrm{d}s\right)^*\boldsymbol{P}_2\left(\int_{t-\sigma(t)}^{t}\boldsymbol{f}(\boldsymbol{z}(s))\mathrm{d}s\right).\tag{9}$$

使用假设(H)能够得到

$$0\leqslant\boldsymbol{z}^*(t)\boldsymbol{LRL}\boldsymbol{z}(t)-\boldsymbol{f}^*(\boldsymbol{z}(t))\boldsymbol{R}\boldsymbol{f}(\boldsymbol{z}(t)).\tag{10}$$

由式(9)和(10)，得

$$\dot V(t)\leqslant-\alpha V(t)+\boldsymbol{z}^*(t)(\alpha\boldsymbol{P}_1-\boldsymbol{P}_1\boldsymbol{C}-\boldsymbol{C}\boldsymbol{P}_1+\boldsymbol{P}_1\boldsymbol{W}^{-1}\boldsymbol{P}_1+\boldsymbol{LRL})\boldsymbol{z}(t)+$$
$$\boldsymbol{z}^*(t)\boldsymbol{P}_1\boldsymbol{A}\boldsymbol{f}(\boldsymbol{z}(t))+\boldsymbol{z}^*(t)\boldsymbol{P}_1\boldsymbol{B}\int_{t-\sigma(t)}^{t}\boldsymbol{f}(\boldsymbol{z}(s))\mathrm{d}s+$$
$$\boldsymbol{f}^*(\boldsymbol{z}(t))\boldsymbol{A}^*\boldsymbol{P}_1\boldsymbol{z}(t)+\left(\int_{t-\sigma(t)}^{t}\boldsymbol{f}(\boldsymbol{z}(s))\mathrm{d}s\right)^*\boldsymbol{B}^*\boldsymbol{P}_1\boldsymbol{z}(t)+\boldsymbol{J}^*\boldsymbol{W}\boldsymbol{J}+$$
$$\boldsymbol{f}^*(\boldsymbol{z}(t))(\sigma\boldsymbol{P}_2-\boldsymbol{R})\boldsymbol{f}(\boldsymbol{z}(t))-$$
$$\frac{\mathrm{e}^{-\alpha\sigma}}{\sigma}\left(\int_{t-\sigma(t)}^{t}\boldsymbol{f}(\boldsymbol{z}(s))\mathrm{d}s\right)^*\boldsymbol{P}_2\left(\int_{t-\sigma(t)}^{t}\boldsymbol{f}(\boldsymbol{z}(s))\mathrm{d}s\right)=$$
$$-\alpha V(t)+\boldsymbol{\eta}^*(t)\boldsymbol{\Xi}\boldsymbol{\eta}(t)+\boldsymbol{J}^*\boldsymbol{W}\boldsymbol{J},\tag{11}$$

其中

$$\boldsymbol{\eta}(t)=\left(\boldsymbol{z}^*(t),\boldsymbol{f}^*(\boldsymbol{z}(t)),\int_{t-\sigma(t)}^{t}\boldsymbol{f}(\boldsymbol{z}(s))\mathrm{d}s\right)^*,$$

并且

$$\boldsymbol{\Xi}=\begin{pmatrix}\boldsymbol{\Xi}_{11} & \boldsymbol{P}_1\boldsymbol{A} & \boldsymbol{P}_1\boldsymbol{B}\\ \boldsymbol{A}^*\boldsymbol{P}_1 & \sigma\boldsymbol{P}_2-\boldsymbol{R} & \boldsymbol{0}\\ \boldsymbol{B}^*\boldsymbol{P}_1 & \boldsymbol{0} & -\dfrac{\mathrm{e}^{-\alpha\sigma}}{\sigma}\boldsymbol{P}_2\end{pmatrix},$$

$$\boldsymbol{\Xi}_{11}=\alpha\boldsymbol{P}_1-\boldsymbol{P}_1\boldsymbol{C}-\boldsymbol{C}\boldsymbol{P}_1+\boldsymbol{P}_1\boldsymbol{W}^{-1}\boldsymbol{P}_1+\boldsymbol{LRL}.$$

由条件(2)，运用 Schur(舒尔)定理，得到 $\boldsymbol{\Xi}<0$.因此

$$\dot V(t)\leqslant-\alpha V(t)+\boldsymbol{J}^*\boldsymbol{W}\boldsymbol{J}.\tag{12}$$

应用引理 3 得到

$$V(t)-\frac{\boldsymbol{J}^*\boldsymbol{W}\boldsymbol{J}}{\alpha}\leqslant\left(V(0)-\frac{\boldsymbol{J}^*\boldsymbol{W}\boldsymbol{J}}{\alpha}\right)\mathrm{e}^{-\alpha t},\qquad t\geqslant 0.\tag{13}$$

因此复值神经网络模型(1)是全局指数吸引的.

由 $V(t)$ 的定义有

$$\lambda_{\min}(\boldsymbol{P}_1)\,\|\boldsymbol{z}(t)\|^2 \leqslant V(t), \tag{14}$$

并且

$$\begin{aligned}V(0) &= \boldsymbol{z}^*(0)\boldsymbol{P}_1\boldsymbol{z}(0) + \int_{-\sigma}^{0}\int_{\xi}^{0}\mathrm{e}^{\alpha s}\boldsymbol{f}^*(\boldsymbol{z}(s))\boldsymbol{P}_2\boldsymbol{f}(\boldsymbol{z}(s))\,\mathrm{d}s\mathrm{d}\xi \leqslant \\ &\lambda_{\max}(\boldsymbol{P}_1)\,\|\boldsymbol{z}(0)\|^2 + \|\boldsymbol{P}_2\|\max_{1\leqslant i\leqslant n}\{l_i^2\}\sup_{s\in[-\sigma,0]}\|\boldsymbol{\varphi}(s)\|^2 \leqslant \\ &(\lambda_{\max}(\boldsymbol{P}_1) + \|\boldsymbol{P}_2\|\max_{1\leqslant i\leqslant n}\{l_i^2\})\sup_{s\in[-\sigma,0]}\|\boldsymbol{\varphi}(s)\|^2 .\end{aligned} \tag{15}$$

因此

$$\|\boldsymbol{z}(t)\| \leqslant \left(\frac{(\lambda_{\max}(\boldsymbol{P}_1) + \|\boldsymbol{P}_2\|\max_{1\leqslant i\leqslant n}\{l_i^2\})\sup_{s\in[-\sigma,0]}\|\boldsymbol{\varphi}(s)\|^2}{\lambda_{\min}(\boldsymbol{P}_1)} + \frac{\boldsymbol{J}^*\boldsymbol{W}\boldsymbol{J}}{\alpha\lambda_{\min}(\boldsymbol{P}_1)}\right)^{1/2} . \tag{16}$$

从而复值神经网络(1)是一致 Lagrange 稳定的.故复值神经网络(1)是 Lagrange 全局指数稳定的,并且它的吸引集为

$$\boldsymbol{\Omega} = \left\{\boldsymbol{z}(t) \in \mathbb{C}^n :\ \|\boldsymbol{z}(t)\| \leqslant \frac{\boldsymbol{J}^*\boldsymbol{W}\boldsymbol{J}}{\alpha\lambda_{\min}(\boldsymbol{P}_1)}\right\} .$$

证毕.

3 结　　论

本文研究了带有变化分布时滞的复值神经网络的 Lagrange 稳定性问题.在激活函数不分解为实部函数和虚部函数的情形下,通过构造合适的复值域上的 Lyapunov-Krasovskii 泛函, 并使用复值域上的矩阵不等式技巧,获得了网络全局指数 Lagrange 稳定的一个判定条件.由于提供的判据是复值线性矩阵不等式, 能够使用 MATLAB 软件的 YALMIP 工具箱快速求解.

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Lagrangian Stability of Complex-Valued Neural Networks With Distributed Time-Varying Delays

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Abstract: The Lagrangian stability of complex-valued neural networks with distributed time-varying delays was investigated. By means of the Lyapunov-Krasovskii functional and the matrix inequality techniques, a delay-dependent sufficient condition was obtained to ensure the global exponential stability in a Lagrangian sense for the considered neural networks. The condition is expressed in the form of complex-valued linear matrix inequality, which can be checked numerically with the effective YALMIP toolbox in MATLAB.

Key words: complex-valued neural network; distributed time-varying delay; Lagrangian stability; complex-valued linear matrix inequality

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