

# 纵向磁场激励下简支输流微梁的动力学行为研究\*

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**摘要:** 受磁场驱动的微机电系统在工作中存在着力、磁、流-固耦合等非线性特征,其力学行为非常复杂,并将影响系统运行的安全性与可靠性.该文采用非局部 Euler 梁模型研究磁场激励下简支输流微梁(一种微机电系统)的动力学行为,通过动力系统分支理论和谐波平衡法来考察系统的稳定性和幅频特性曲线.结果表明,可以采用改变磁场强度、流速和阻尼的三重方式调节微机电系统的频率.研究中还发现,小尺度效应和磁场强度可以影响临界流速,阻尼的存在可以改变临界流速的个数和系统的分岔类型.

**关键词:** 输流微梁; 纵向磁场; 流固耦合; 稳定性; 振动频率

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## 引言

微梁作为微机电系统工程、微流控装置等许多高科技领域中的核心部件,其应用范围非常广泛,涉及微型谐振器、微驱动器、微传感器和微米输送系统<sup>[1-2]</sup>.几乎所有涉及微型设备工作效率改善的问题都要求对其动力学行为的深入了解<sup>[3]</sup>.为此,国内外学者做了大量的研究工作,其中 Rinaldi 等在经典连续介质结构环境下,分析了尺寸、边界条件、耗散、材料、流速对输流微型管道、微型谐振器的稳定性、阻尼、频率的数值结果的影响<sup>[4]</sup>.Setoodeh 等基于应变梯度理论,研究了功能梯度输流微管道的非线性动力学行为<sup>[5]</sup>.受磁场驱动的微机电系统,在工作中存在着微构件的机械力与磁场力相互耦合及流固耦合等非线性特征,其力学行为非常复杂,并且涉及小尺度效应问题,影响系统运行的安全性与可靠性<sup>[6-9]</sup>.Zhen 等研究了单壁纳米管输流梁热力耦合以及非局部弹性振动的问题<sup>[10]</sup>.据笔者所知,目前对于磁场激励下输流微梁动力学特性的研究还处于初级阶段,为此本文采用非局部 Euler 梁模型,通过 Galerkin 数值离散方法、动力系统分支理论和谐波平衡法来研究纵向磁场激励下输流微梁的动力学行为,考察了不同磁场强度、流速和阻尼对系统振动频率的影响.

## 1 流固耦合控制方程

如图 1 所示,输流微梁的长度为  $L$ ,横截面积为  $A$ ,抗弯刚度为  $EI$ ,单位长度质量为  $m$ ,通

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过该管道的非黏性流体的单位长度质量为  $M$ , 流体在微梁中的流速为  $U_f$ , 黏性阻尼系数为  $C$ , 假定管道及流体的重力、表面张力以及外部压力(例如气压的影响)可以忽略不计, 并且微梁只在  $XZ$  平面内振动. 基于 Euler 梁模型有下面的位移方程:

$$\begin{cases} \bar{U}(X, Z, T) = U(X, T) - Z \frac{\partial W}{\partial X}, \\ \bar{W}(X, Z, T) = W(X, T), \end{cases} \quad (1)$$

其中,  $X$  是轴向坐标,  $T$  是时间,  $U$  和  $W$  代表微梁中面沿  $X$  轴和  $Z$  轴的位移,  $\bar{U}$  和  $\bar{W}$  分别是管沿着  $X$  轴和  $Z$  轴方向的总位移.

在纵向磁场的作用下简支微梁发生横向位移和轴向位移, 产生的应变如下:

$$\begin{cases} \varepsilon_{XX} = \frac{\partial U}{\partial X} + \frac{1}{2} \left( \frac{\partial W}{\partial X} \right)^2 - Z \frac{\partial^2 W}{\partial X^2}, \\ \varepsilon_{ZZ} = \varepsilon_{XZ} = 0, \end{cases} \quad (2)$$

其中  $\varepsilon_{XX}$  是轴向应变.

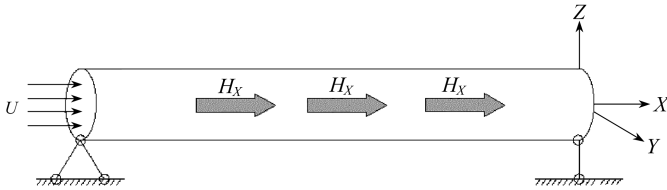


图 1 简支输流微梁在纵向磁场作用下的模型图

Fig. 1 The model for a simply supported micro beam conveying fluid in a longitudinal magnetic field

对于管道输送流体时所做的横向振动, 管道与内部流体的动能表示为

$$\begin{aligned} T_1 = & \frac{m}{2} \int_0^L \left[ \left( \frac{\partial U}{\partial T} - Z \frac{\partial^2 W}{\partial X \partial T} \right)^2 + \left( \frac{\partial W}{\partial T} \right)^2 \right] dX + \\ & \frac{M}{2} \int_0^L \left[ \left( \frac{\partial U}{\partial T} + U_f \cos \theta \right)^2 + \left( \frac{\partial W}{\partial T} - U_f \sin \theta \right)^2 + Z^2 \left( \frac{\partial^2 W}{\partial X \partial T} \right)^2 \right] dX, \end{aligned} \quad (3)$$

其中  $\theta = -\partial W / \partial X$ .

管道的势能表示为

$$\Pi = \frac{1}{2} \int_0^L \int_A \sigma_{XX} \varepsilon_{XX} dA dx, \quad (4)$$

其中  $\sigma_{XX}$  为正应力. 将方程(2)代入方程(4), 得到  $\Pi$  的表达式:

$$\Pi = \Pi_1 + \frac{1}{2} \int_0^L N_X \left[ \frac{\partial U}{\partial X} + \frac{1}{2} \left( \frac{\partial W}{\partial X} \right)^2 \right] dX, \quad (5)$$

其中  $\Pi_1 = \int_V (u_1 + u_2 + u_3) dV$  (具体的表达式见文献[11]),  $N_X = \int_A \sigma_{XX} dA$ .

外力所做的功可以表示为

$$W_1 = \int_0^L \left( f_z - C \frac{\partial W}{\partial T} \right) W dX, \quad (6)$$

其中  $f_z$  为纵向磁场作用下的 Lorentz 力:

$$f_z = \eta H_x^2 \frac{\partial^2 W}{\partial X^2}. \quad (7)$$

对于一维 Euler 梁模型,考虑非局部效应的本构关系<sup>[12-14]</sup>可以表示为

$$\sigma_{XX} - (e_0 a)^2 \frac{\partial^2 \sigma_{XX}}{\partial X^2} = E \varepsilon_{XX}, \quad (8)$$

式中  $E$  为弹性模量,  $e_0 a$  为小尺度参数.

基于 Hamilton 变分原理,磁场激励下输流微梁的振动方程可以写为

$$\int_{t_0}^{t_1} (\delta \Pi - \delta T_1 - \delta W_1) dt = 0, \quad (9)$$

把方程(3)、(5)~(8)代入方程(9)可得系统无量纲运动微分方程:

$$\begin{aligned} - \sum_{n=1}^{\infty} (2n-3) \tau^{2(n-1)} \frac{\partial^{2(n+1)} w}{\partial x^{2(n+1)}} + \frac{\partial^2 w}{\partial t^2} + c \frac{\partial w}{\partial t} + 2u\sqrt{\beta} \frac{\partial^2 w}{\partial x \partial t} + \\ \left( u^2 - \psi - \frac{1}{2} \vartheta \int_0^1 \left( \frac{\partial w}{\partial x} \right)^2 dx \right) \frac{\partial^2 w}{\partial x^2} = 0, \end{aligned} \quad (10)$$

其中

$$\begin{cases} x = \frac{X}{L}, w = \frac{W}{L}, t = \frac{T}{L^2} \left( \frac{EI}{M+m} \right)^{1/2}, u = \left( \frac{M}{EI} \right)^{1/2} U_i L, c = \frac{CL^2}{\sqrt{EI(M+m)}}, \\ \tau = \frac{e_0 a}{L}, \beta = \frac{M}{M+m}, \vartheta = \frac{AL^2}{I}, \psi = \eta AH_x^2 \frac{L^2}{EI}. \end{cases} \quad (11)$$

在研究非局部效应的影响时,只需取无穷阶微分方程中级数的前两个非局部项 ( $n=1,2$ ) 即可满足精度要求<sup>[10]</sup>,于是运动微分方程(11)可转化为

$$\begin{aligned} - \tau^2 \frac{\partial^6 w}{\partial x^6} + \frac{\partial^4 w}{\partial x^4} + \frac{\partial^2 w}{\partial t^2} + c \frac{\partial w}{\partial t} + 2u\sqrt{\beta} \frac{\partial^2 w}{\partial x \partial t} + \\ \left( u^2 - \psi - \frac{1}{2} \vartheta \int_0^1 \left( \frac{\partial w}{\partial x} \right)^2 dx \right) \frac{\partial^2 w}{\partial x^2} = 0, \end{aligned} \quad (12)$$

采用 Galerkin 数值离散方法,Euler 梁的变形可以表示为

$$w = \sum_{i=1}^n q_i(t) \phi_i(x), \quad (13)$$

其中  $\phi_i(x)$  表示梁的特征模态,  $q_i(t)$  为关于时间  $t$  的广义坐标.将方程(13)代入方程(12)中,两端同乘  $\phi_i$ ,然后在  $[0,1]$  区域内积分,可得非线性耦合常微分方程组,用矩阵表示为

$$\begin{aligned} \mathbf{M} \ddot{\mathbf{q}}(t) + (c\mathbf{M} + 2u\sqrt{\beta}\mathbf{B}) \dot{\mathbf{q}}(t) + (-\tau^2 \mathbf{F} + \mathbf{E} + u^2 \mathbf{D} - \psi \mathbf{D}) \mathbf{q}(t) + \\ \frac{1}{2} \vartheta \mathbf{q}^T(t) \mathbf{S} \mathbf{q}(t) \mathbf{S} \mathbf{q}(t) = \mathbf{0}, \end{aligned} \quad (14)$$

其中  $\mathbf{q}(t) = \{q_j(t)\}$ ,  $(\cdot)^T$  表示矩阵的转置,

$$\mathbf{M} = [m_{ij}], \mathbf{B} = [b_{ij}], \mathbf{F} = [f_{ij}], \mathbf{E} = [e_{ij}], \mathbf{D} = [d_{ij}], \mathbf{S} = [s_{ij}],$$

表达式分别为

$$\begin{cases} m_{ij} = \delta_{ij} \int_0^1 \phi_i \phi_j dx, b_{ij} = \int_0^1 \phi_i \phi_j' dx, f_{ij} = \delta_{ij} \int_0^1 \phi_i \phi_j^{(6)} dx, e_{ij} = \delta_{ij} \int_0^1 \phi_i \phi_j''' dx, \\ d_{ij} = \delta_{ij} \int_0^1 \phi_i \phi_j'' dx, s_{ij} = \delta_{ij} \int_0^1 \phi_i' \phi_j' dx, \delta_{ij} = \begin{cases} 1, & i=j, \\ 0, & i \neq j. \end{cases} \end{cases} \quad (15)$$

采用方程(13)的前两项离散方程时,即可获得简支输流微梁的振动方程组:

$$\begin{cases} m_{11}\ddot{q}_1(t) + (cm_{11} + 2u\sqrt{\beta}b_{11})\dot{q}_1(t) + 2u\sqrt{\beta}b_{12}\dot{q}_2(t) + \\ \quad (-\tau^2f_{11} + e_{11} + u^2d_{11} - \psi d_{11})q_1(t) + \frac{1}{2}\vartheta s_{11}q_1(t)(s_{11}q_1^2(t) + s_{22}q_2^2(t)) = 0, \\ m_{22}\ddot{q}_2(t) + (cm_{22} + 2u\sqrt{\beta}b_{22})\dot{q}_2(t) + 2u\sqrt{\beta}b_{21}\dot{q}_1(t) + \\ \quad (-\tau^2f_{22} + e_{22} + u^2d_{22} - \psi d_{22})q_2(t) + \frac{1}{2}\vartheta s_{22}q_2(t)(s_{11}q_1^2(t) + s_{22}q_2^2(t)) = 0. \end{cases} \quad (16)$$

令  $\dot{q}_1(t) = p_1(t)$ ,  $\dot{q}_2(t) = p_2(t)$ . 于是, 方程组(16)可化为4个一次微分方程, 并写成矩阵形式, 可得

$$\begin{bmatrix} \dot{q}_1(t) \\ \dot{p}_1(t) \\ \dot{q}_2(t) \\ \dot{p}_2(t) \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ H_1 & H_2 & 0 & H_3 \\ 0 & 0 & 0 & 1 \\ 0 & H_4 & H_5 & H_6 \end{bmatrix} \begin{bmatrix} q_1(t) \\ p_1(t) \\ q_2(t) \\ p_2(t) \end{bmatrix} - \begin{bmatrix} 0 \\ H_7 \\ 0 \\ H_8 \end{bmatrix}, \quad (17)$$

其中

$$\begin{cases} H_1 = -\frac{-\tau^2f_{11} + e_{11} + u^2d_{11} - \psi d_{11}}{m_{11}}, H_2 = -\left(c + \frac{2u\sqrt{\beta}}{m_{11}}b_{11}\right), \\ H_3 = -\frac{2u\sqrt{\beta}b_{12}}{m_{11}}, H_4 = -\frac{2u\sqrt{\beta}b_{21}}{m_{22}}, \\ H_5 = -\frac{-\tau^2f_{22} + e_{22} + u^2d_{22} - \psi d_{22}}{m_{22}}, H_6 = -\left(c + \frac{2u\sqrt{\beta}}{m_{22}}b_{22}\right), \\ H_7 = \frac{1}{2m_{11}}\vartheta s_{11}q_1(t)(s_{11}q_1^2(t) + s_{22}q_2^2(t)), \\ H_8 = \frac{1}{2m_{22}}\vartheta s_{22}q_2(t)(s_{11}q_1^2(t) + s_{22}q_2^2(t)). \end{cases} \quad (18)$$

线性系统系数矩阵的特征方程可以写作

$$\det(\lambda \mathbf{I} - \mathbf{B}) = 0, \quad (19)$$

其中  $\mathbf{I}$  是单位矩阵,  $\mathbf{B}$  是方程(17)右端的系数矩阵;  $\lambda$  是系统的特征值, 通常特征值是复数形式, 其实部代表系统阻尼, 虚部代表振动频率.

运用谐波平衡法, 令  $q_1(t) = A_1 \cos(\omega t)$ ,  $q_2(t) = A_2 \cos(\omega t)$ , 将其代入输流微梁的振动方程组, 令一次谐波系数为0, 忽略超过一次的高阶谐波, 可以得到

$$\begin{cases} A_1 = \pm \left[ \frac{8}{3} \frac{m_{11}\omega^2 - (-\tau^2f_{11} + e_{11} + u^2d_{11} - \psi d_{11})}{\vartheta s_{11}^2} \right]^{1/2}, \\ A_2 = \pm \left[ \frac{8}{3} \frac{m_{22}\omega^2 - (-\tau^2f_{22} + e_{22} + u^2d_{22} - \psi d_{22})}{\vartheta s_{22}^2} \right]^{1/2}. \end{cases} \quad (20)$$

## 2 数值结果分析

纵向磁场激励下输流微梁的振动表现出多值、分岔等非线性现象, 现对式(12)所表示的系统进行数值求解, 选取系统无量纲参数  $\beta = 0.2$ ,  $\vartheta = 1$ ,  $\tau = 0.1$  和  $\tau = 0.2$ <sup>[11]</sup>,  $\psi = 12.02$  和  $\psi$

$= 48.08^{[6]}$ ,  $c = 10$  和  $c = 15$ . 通过特征值虚部与实部随流速的演化图和幅频响应曲线, 得到系统参数对微梁动力学行为的影响.

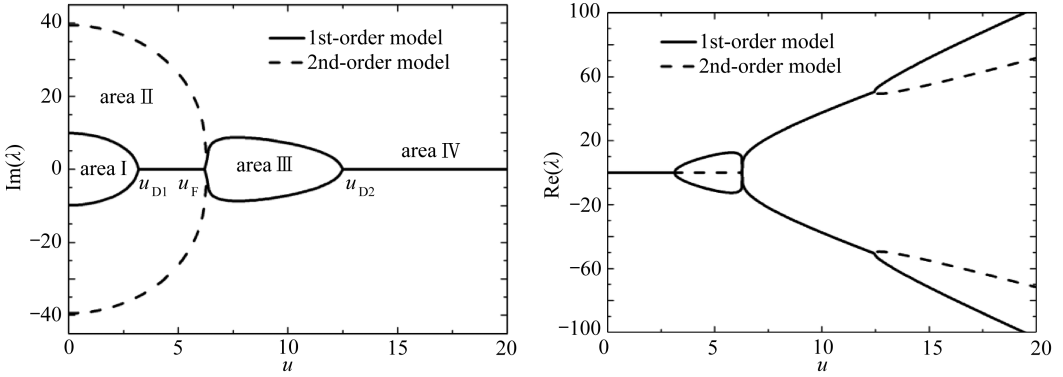


图2  $\tau = 0, c = 0, \psi = 0$  时, 特征值  $\lambda$  的虚部与实部随流速的演化图

Fig. 2 For  $\tau = 0, c = 0, \psi = 0$ , the evolution of the imaginary and real parts of eigenvalue  $\lambda$  with the flow velocity

图2为不考虑小尺度效应, 无阻尼、无纵向磁场时特征值虚部与实部随流速  $u$  的演化图, 从图中可以看出内部流体有效地影响了输流管道的振动频率和稳定性. 在 I 区 ( $0 \leq u \leq u_{D1} = \pi$ ), 4 个特征值的实部均为 0, 当流速增加到临界流速  $u_{D1}$ , 一阶模态所对应的一对特征值 ( $\lambda_{1,2}$ ) 的虚部减小为 0, 相应运动出现第一次失稳, 并通过叉形分岔进入 II 区, 与  $u_{D1}$  对应的奇点为鞍点. 随着流速的增加, 特征区域进入 II 区 ( $u_{D1} < u < u_F = 2\pi$ ), 显而易见, 由于存在正实部的特征值, 在该区域系统的解是不稳定的, 当流速穿过  $u_F$  点时, 出现的第二次分岔为 Hopf 分岔, 与  $u_F$  对应的奇点为焦点, 在  $u_F$  处, 一阶和二阶模态开始互相耦合. 在 III 区 ( $u_F < u < u_{D2} = 12.4548$ ), 特征值的实部特征为  $\lambda_{ReIII} = \{+, -, +, -\}$ , 当流速增加到  $u_{D2}$  时, 再次发生叉形分岔进入 IV 区, 与  $u_{D2}$  对应的奇点为鞍点.

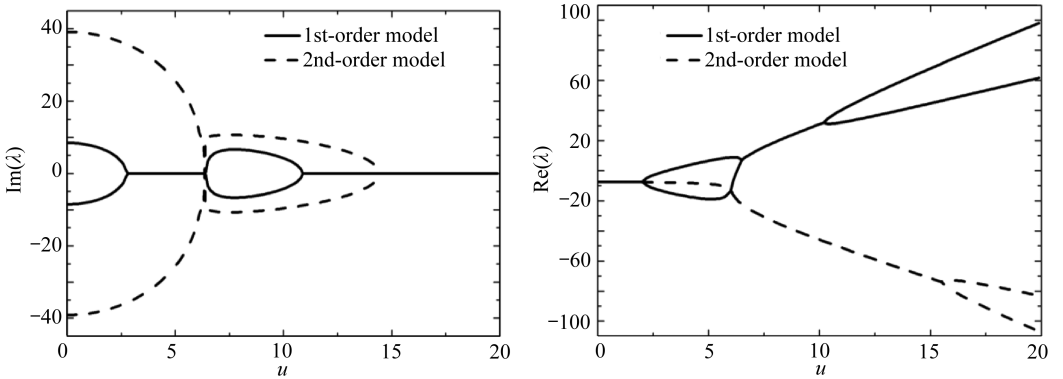


图3  $\tau = 0, c = 10, \psi = 0$  时, 特征值  $\lambda$  的虚部与实部随流速的演化图

Fig. 3 For  $\tau = 0, c = 10, \psi = 0$ , the evolution of the imaginary and real parts of eigenvalue  $\lambda$  with the flow velocity

图3为不考虑小尺度效应, 阻尼项  $c = 10$ , 无纵向磁场时特征值虚部与实部随流速  $u$  的演化图. 显而易见, 在  $u$  较小时系统实部一直小于 0, 此时振动系统是稳定的, 直到  $u = 2.7240$  时产生叉式分岔, 此后系统含有正实部的特征值, 因此系统处于不稳定状态. 当流速  $u = 6.2456, 10.8768, 14.4382$  发生第二次、第三次、第四次分岔. 图4为不考虑小尺度效应, 阻尼项  $c = 15$ ,

无纵向磁场时特征值虚部与实部随流速  $u$  的演化图.从图 2~4 可以看出,阻尼可以改变临界流速的个数,并且使系统的耦合模态颤振消失,这与文献[15]的结论一致.

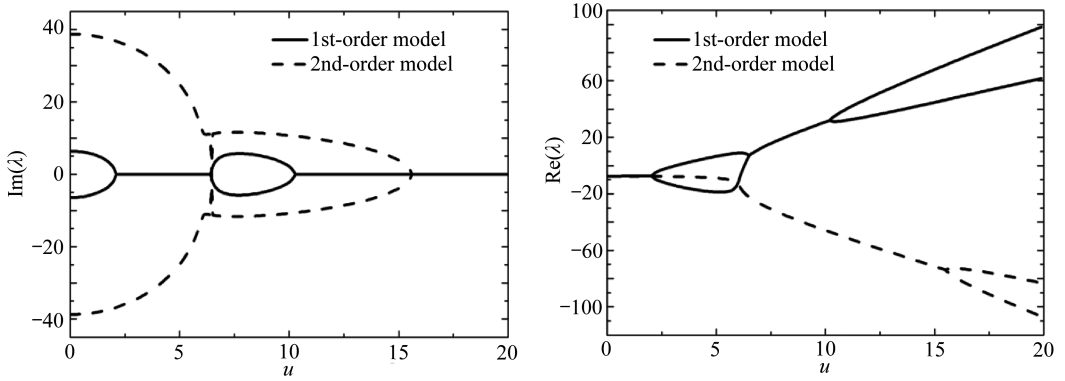


图 4  $\tau = 0, c = 15, \psi = 0$  时,特征值  $\lambda$  的虚部与实部随流速的演化图

Fig. 4 For  $\tau = 0, c = 15, \psi = 0$ , the evolution of the imaginary and real parts of eigenvalue  $\lambda$  with the flow velocity

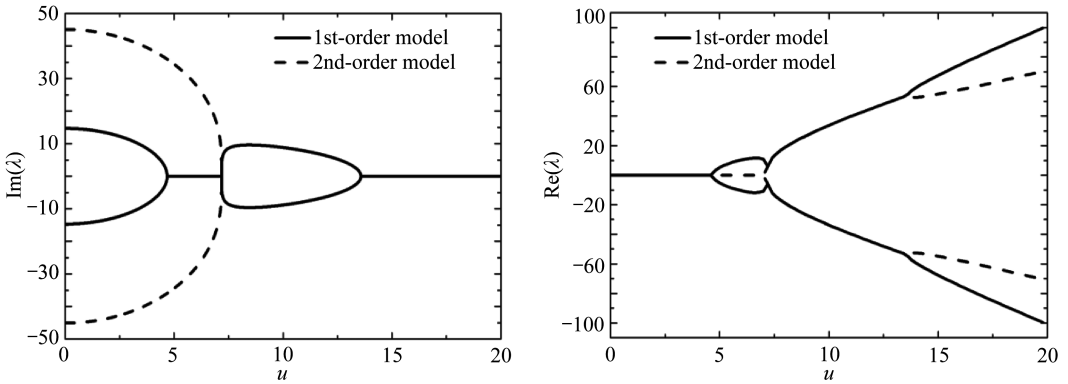


图 5  $\tau = 0, c = 0, \psi = 12.02$  时,特征值  $\lambda$  的虚部与实部随流速的演化图

Fig. 5 For  $\tau = 0, c = 0, \psi = 12.02$ , the evolution of the imaginary and real parts of eigenvalue  $\lambda$  with the flow velocity

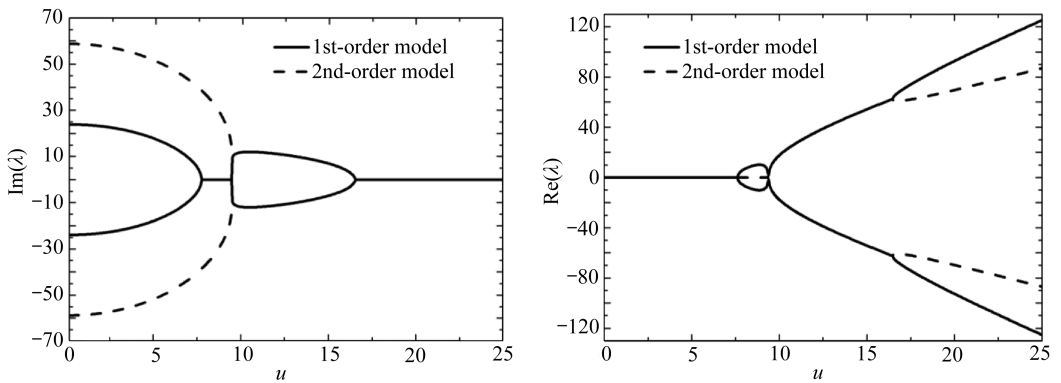


图 6  $\tau = 0, c = 0, \psi = 48.08$  时,特征值  $\lambda$  的虚部与实部随流速的演化图

Fig. 6 For  $\tau = 0, c = 0, \psi = 48.08$ , the evolution of the imaginary and real parts of eigenvalue  $\lambda$  with the flow velocity

图 5 为不考虑小尺度效应、无阻尼,纵向磁场  $\psi = 12.02$  时特征值虚部与实部随流速  $u$  的

演化图,图 6 为不考虑小尺度效应、无阻尼,纵向磁场  $\psi = 48.08$  时特征值虚部与实部随流速  $u$  的演化图。对比图 2、图 5、图 6 可知纵向磁场的存在可以增大输流管道的临界流速以及振动频率,但磁场不改变特征值随流速变化的分岔类型。

图 7 为考虑小尺度效应  $\tau = 0.1$ , 无阻尼、无纵向磁场情况下特征值虚部与实部随流速  $u$  的演化图。图 8 为考虑小尺度效应  $\tau = 0.2$ , 无阻尼、无纵向磁场情况下特征值虚部与实部随流速  $u$  的演化图。

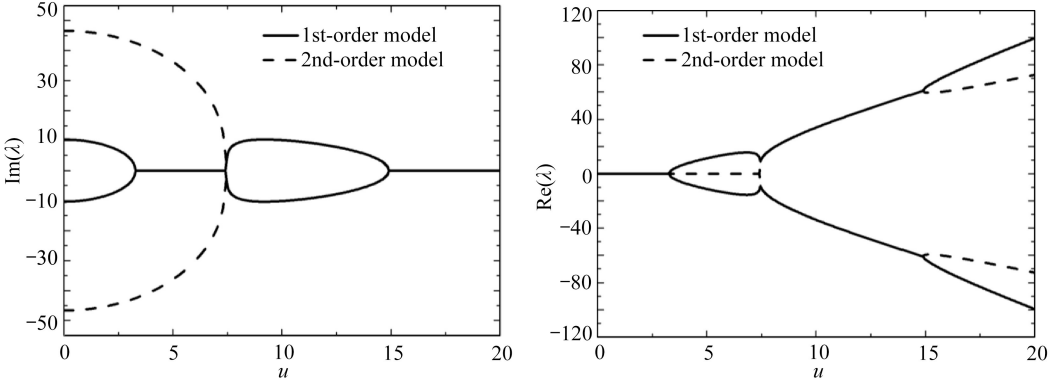


图 7  $\tau = 0.1, c = 0, \psi = 0$  时,特征值  $\lambda$  的虚部与实部随流速的演化图

Fig. 7 For  $\tau = 0.1, c = 0, \psi = 0$ , the evolution of the imaginary and real parts of eigenvalue  $\lambda$  with the flow velocity

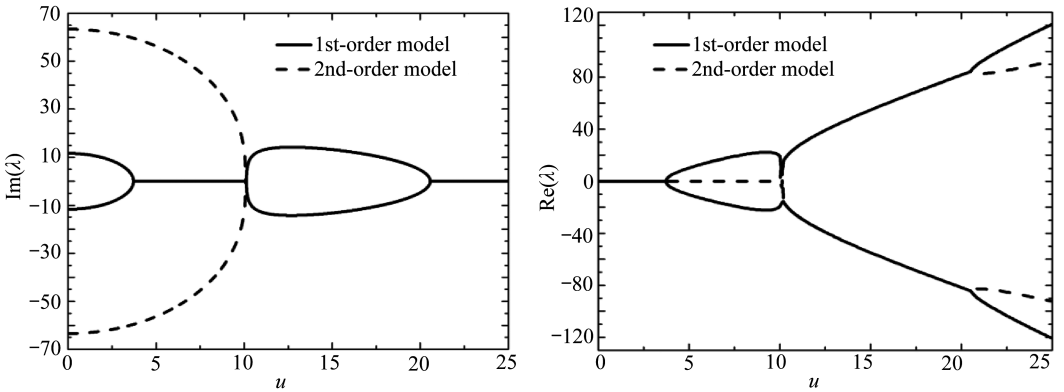
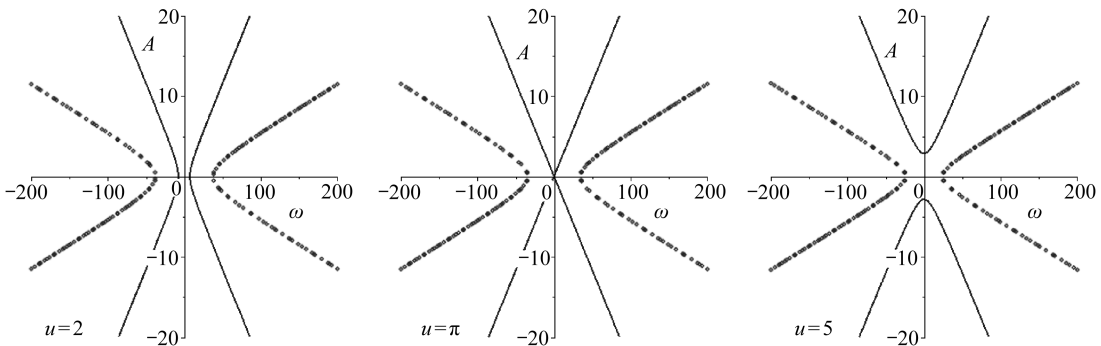


图 8  $\tau = 0.2, c = 0, \psi = 0$  时,特征值  $\lambda$  的虚部与实部随流速的演化图

Fig. 8 For  $\tau = 0.2, c = 0, \psi = 0$ , the evolution of the imaginary and real parts of eigenvalue  $\lambda$  with the flow velocity



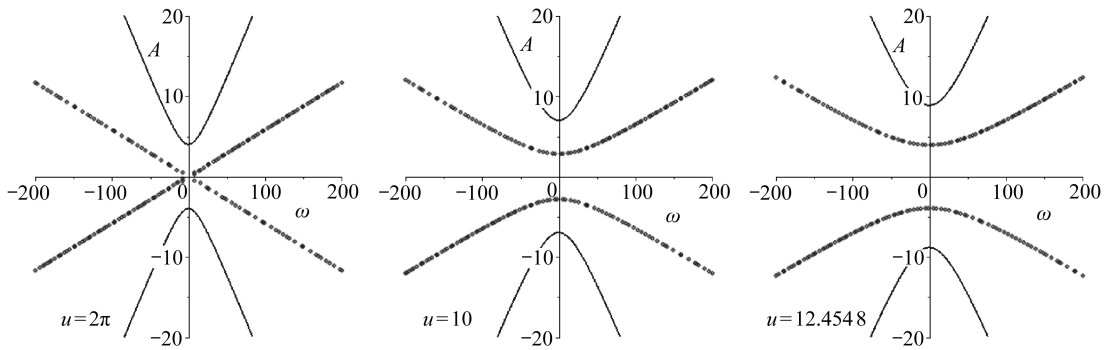


图 9  $\tau = 0, c = 0, \psi = 0$  时, 系统在不同流速下的幅频特性曲线(实线  $A_1$ , 点线  $A_2$ )

Fig. 9 For  $\tau = 0, c = 0, \psi = 0$ , amplitude-frequency characteristic curves of the system under different flow velocities (the solid lines indicate  $A_1$ , the dotted lines indicate  $A_2$ )

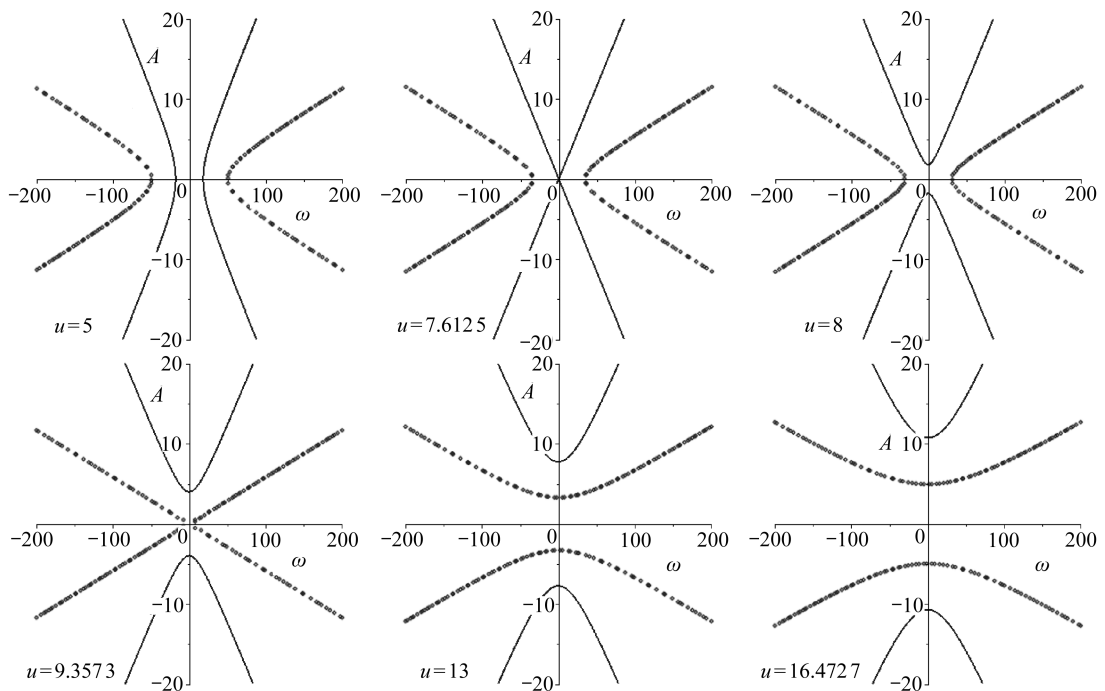


图 10  $\tau = 0, c = 0, \psi = 48.08$  时, 系统在不同流速下的幅频特性曲线(实线  $A_1$ , 点线  $A_2$ )

Fig. 10 For  $\tau = 0, c = 0, \psi = 48.08$ , amplitude-frequency characteristic curves of the system under different flow velocities (the solid lines indicate  $A_1$ , the dotted lines indicate  $A_2$ )

对比图 2、图 7、图 8 可知,小尺度效应可以增大临界流速和振动频率,但不改变特征值随流速变化的分岔类型。

由图 2~8 可知,采用改变磁场强度、流速和阻尼的三重方式来调节输流微梁的振动频率是可行的。对于一个给定的频率,它可以对应不同的流速(或者磁场),并且在临界点处特征值对载荷的变化很敏感,很小的流速变化就会导致有效的频率改变,因此可以在较大范围内调整流速以期获得需要的频率,此种调节方式对于微机电系统的优化设计具有重要意义。

下面来考察系统在不同流速下的幅频特性曲线,由图 9 与图 10 可知系统的固有频率  $\omega$  随振幅  $A$  的变化而改变。当临界流速  $u = \pi, 2\pi$  (见图 9),  $u = 7.6125, 9.3573$  (见图 10) 时,幅频



特性曲线也处于临界状态。

### 3 结 论

本文采用非局部 Euler 梁模型考察简支输流微梁在纵向磁场激励下的非线性动力学行为, 所得结果表明: 1) 小尺度效应和纵向磁场可以增大系统的临界流速和振动频率, 但不改变特征值随流速变化的分岔类型; 2) 阻尼的存在可以改变临界流速的个数和系统的分岔类型; 3) 不同流速下系统的固有频率随振幅的变化而改变; 4) 采用改变磁场强度、流速和阻尼的三重方式来调节输流微梁的振动频率是行之有效的。

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## Dynamical Behavior Analysis of Micro Beams Conveying Fluid in Longitudinal Magnetic Fields

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**Abstract:** Such nonlinear characteristics as mechanics, magnetics and fluid-structure interactions exist in the micro-electromechanical system (MEMS) driven by the magnetic field, which will affect the safety and reliability of the system. Based on the nonlocal Eulerian beam model, the dynamical behaviors of fluid-conveying micro beams (a kind of MEMS) in magnetic fields were studied. The dynamical system bifurcation theory and the harmonic balance method were used to study the stability and amplitude-frequency characteristic curves of the pinned-pinned micro beam system. The results show that, the frequency of the beam can be adjusted through changes of the magnetic field intensity, the flow velocity and the system damping. The small-scale effects tend to change the critical velocity and the existence of damping can change the number of critical velocities and the type of bifurcation.

**Key words:** micro beam conveying fluid; longitudinal magnetic field; fluid-structure interaction; stability; vibration frequency

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