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一维六方压电准晶双材料界面共线裂纹问题*

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摘要： 利用复变函数理论中的解析延拓、奇性主部分析和推广的 Liouville 定理，求解了一维六方压电准晶双材料在集中载荷作用下界面共线裂纹反平面弹性问题.导出了含有一条和两条有限长界面裂纹的封闭解，同时给出了裂纹尖端场强度因子(包含声子场和相位子场应力强度因子和电位移强度因子)的表达式.数值算例分析了外荷载与耦合系数之比对裂纹尖端场强度因子变化规律的影响.从数值结果中可以看出,当裂纹长度增加时,裂纹尖端场强度因子随之增加；应力强度因子随双材料耦合系数之比的增大而增大，电位移强度因子几乎不变；不同载荷作用下,裂纹尖端场强度因子随着裂纹长度改变时的变化趋势也不尽相同.研究结果可为压电准晶双材料的设计和制备提供一定的理论参考.

关　键　词： 一维六方压电准晶双材料；　界面共线裂纹；　复变函数方法；　场强度因子
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Research on Interfacial Collinear Cracks Between 1D Hexagonal Piezoelectric Quasicrystal Bimaterials

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Abstract: By means of the analytic continuation, the singularity principal part analysis and the extended Liouville theorem in the complex function theory, the anti-plane elastic problem of interfacial collinear cracks between 1D hexagonal piezoelectric quasicrystal bimaterials under concentrated loads, was addressed. The closed solutions for biomaterial interface containing 1 and 2 finite-length cracks under concentrated loads were derived. At the same time, the crack tip field intensity factors (including the phonon field, the phason field stress intensity factors and the electric displacement intensity factor) were given. The effects of the ratio of the external load to the coupling coefficient on the intensity factor variation of the crack tip field were analyzed by numerical examples. The numerical results show that, the intensity factor of the crack tip field increases with the crack length and with the ratio of coupling coefficients, while the electric displacement intensity factor keeps almost unchanged. The field intensity factor of the crack tip varies with the crack length in different styles under different loads. The research results provide a theoretical reference for the design and preparation of piezoelec-

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tric quasicrystals.

Key words: 1D hexagonal piezoelectric quasicrystal bimaterial; interfacial collinear crack; complex function method; field strength factor

0 引 言

1982 年发现的准晶具有宏观对称性和结构上的完全有序性[1].常温下准晶呈脆性，其对裂纹、孔洞、位错等缺陷非常敏感[2].为了使准晶材料被充分利用，研究裂纹、位错等缺陷具有一定的意义.在考虑表/界面效应下，苏梦雨等[3]分析了纳米尺度下一维六方准晶中含有椭圆形裂纹孔的Ⅲ型断裂问题；Cheng 等[4]研究了一维六方准晶矩形板在有限维度下的断裂问题；利用复变函数方法和 Stroh 公式，张炳彩等[5]研究了一维六方准晶中孔边不对称共线界面裂纹问题.借助边值问题，Su 等[6]研究了一维六方准晶中纳米椭圆孔边裂纹的Ⅲ型断裂问题.利用广义 Stroh 公式，Radi 等[7]求解了二维十次对称准晶直线裂纹问题.通过构造广义保角映射，高媛媛等[8]研究了一维正交准晶中具有穿透性的椭圆孔口的平面弹性问题.

压电准晶相较普通的压电材料而言具有更广阔的应用前景.基于保角映射技术，在部分渗透边界条件下，Yang 等[9]讨论了一维六方压电准晶中带两个非对称裂纹椭圆孔的断裂问题.Hu 等[10]研究了不同准晶材料之间的界面裂纹问题.运用 Fourier 积分变化，Zhou 和 Li[11-12]分别研究了有限厚度一维六方压电准晶板中币形裂纹和 Yoffe 型运动裂纹问题.利用 Schwarz-Christoffel 变换，刘兴伟等[13]研究了一维六方压电准晶中正 n 边形孔边裂纹问题.

由两种材质不同的准晶材料沿界面结合在一起，称为“准晶双材料”.目前，关于准晶双材料界面断裂问题的研究多采用积分方程方法，得到的是数值解，外载荷多为均匀分布.集中载荷作用下压电准晶双材料界面多裂纹问题的研究还未有文献报道.基于此，本文利用复变函数方法求解了集中载荷(力和电荷)作用下压电准晶双材料界面共线裂纹反平面问题，得到了形式简洁的裂纹尖端场强度因子(包含声子场、相位子场应力强度因子和电位移强度因子)的解析解.解析解严谨性更强,且在工程计算中更加便捷，同时解析解对于理解物理现象和本质具有重要的作用和意义.

1 一维六方压电准晶基本公式

取 z_1 轴为一维六方压电准晶准周期方向，xOy 面为其周期平面，建立直角坐标系.此时，一维六方压电准晶反平面弹性平衡方程为[14]

$$\nabla^2 u_{z_1}=0,\ \nabla^2 w_{z_1}=0,\ \nabla^2 v=0. \tag{1}$$

其通解为[15]

$$u_{z_1}=\operatorname{Re}\phi(z),\ w_{z_1}=\operatorname{Re}\psi(z),\ v=\operatorname{Re}\eta(z). \tag{2}$$

应力分量表示为[14]

$$\begin{cases}\sigma_{xz_1}-\mathrm{i}\sigma_{yz_1}=C_{44}\phi'(z)+R_3\psi'(z)+e_{15}^1\eta'(z),\\ H_{xz_1}-\mathrm{i}H_{yz_1}=K_2\psi'(z)+R_3\phi'(z)+e_{15}^2\eta'(z),\\ D_x-\mathrm{i}D_y=e_{15}^1\phi'(z)+e_{15}^2\psi'(z)-\epsilon_{11}\eta'(z).\end{cases} \tag{3}$$

式(1)—(3)中 z 表示坐标轴 z_1 上的分量，“′”表示对 z 求导数，这里 $z=x+\mathrm{i}y$；C_{44}，K_2 和 R_3 分别表示声子场、相位子场以及声子场-相位子场系数的弹性常数；e_{15}^1，e_{15}^2 表示压电常数，ϵ_{11} 表示介电常数；u_{z_1} 和 w_{z_1} 分别为声子场和相位子场位移，v 表示电势；σ_{xz_1}，σ_{yz_1}，H_{xz_1}，H_{yz_1} 表示声子场和相位子场剪应力，D_x，D_y 表示电位移.

2 一维六方压电准晶双材料反平面弹性问题

如图 1 所示，上下平面分别为弹性常数不同的一维六方压电准晶材料，记上半平面为 S^+，下半平面为 S^-.上下半平面的量分别用上标 1 和 2 标记.设反平面集中力 P(假设作用在声子场的力为 P_1，作用在相位子场的力为 P_2) 和电荷 Q 集中作用在 S^+ 内任意点 z_0 处.这里沿实轴的区间 $L=L_1+L_2+\cdots+L_n$，L_j 上两种一

维六方压电准晶材料完全黏合, 应力和位移保持连续, 即

$$\sigma_{yz_1}^{(1)+}=\sigma_{yz_1}^{(2)-},\ H_{yz_1}^{(1)+}=H_{yz_1}^{(2)-},\ D_y^{(1)+}=D_y^{(2)-},\qquad t\in L,\tag{4}$$

$$u_{z_1}^{(1)+}=u_{z_1}^{(2)-},\ w_{z_1}^{(1)+}=w_{z_1}^{(2)-},\ v^{(1)+}=v^{(2)-},\qquad t\in L;\tag{5}$$

沿界面其余部分 $L'=L'_1+L'_2+\cdots+L'_n$ 互相裂开, 其中 L'_i 表示第 i 条有限长裂纹.假设在裂纹上无面力作用, 裂纹端点依次是 $a_1,b_1;a_2,b_2;\cdots;a_n,b_n$,

$$\sigma_{yz_1}^{(1)+}(t)=\sigma_{yz_1}^{(2)-}(t)=0,\ H_{yz_1}^{(1)+}(t)=H_{yz_1}^{(2)-}(t)=0,\ D_y^{(1)+}(t)=D_y^{(2)-}(t)=0,\qquad t\in L'.\tag{6}$$

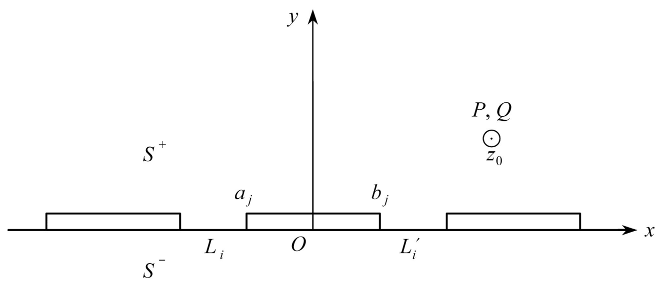

图 1　集中载荷作用于含共线界面裂纹的一维六方压电准晶双材料

Fig. 1　A 1D hexagonal piezoelectric quasicrystal bimaterial with collinear interfacial cracks under a concentrated load

3　集中载荷作用下一维六方压电准晶双材料界面共线裂纹反平面弹性问题求解

如图 1, 由式(2), 在 S^+ 内, 设

$$\begin{cases}\phi'_1(z)=\dfrac{M_1}{z-z_0}+\phi_{10}(z),\\[2ex] \psi'_1(z)=\dfrac{M_2}{z-z_0}+\psi_{10}(z),\\[2ex] \eta'_1(z)=\dfrac{M_3}{z-z_0}+\eta_{10}(z),\end{cases}\tag{7}$$

式中, M_1,M_2,M_3 的表达式见附录, $\phi_{10}(z),\psi_{10}(z)$ 和 $\eta_{10}(z)$ 在 S^+ 内全纯.

在 S^- 内无集中作用载荷, 可设

$$u_{z_1}^{(2)}=\mathrm{Re}\,\phi_2(z),\ w_{z_1}^{(2)}=\mathrm{Re}\,\psi_2(z),\ v^{(2)}=\mathrm{Re}\,\eta_2(z),\tag{8}$$

这里, $\phi_2(z),\psi_2(z)$ 和 $\eta_2(z)$ 在 S^- 内全纯.

由 Schwarz 对称原理[16], 对 $\phi'_1(z),\psi'_1(z),\eta'_1(z)$ 和 $\phi'_2(z),\psi'_2(z),\eta'_2(z)$ 做如下的延拓:

$$\begin{cases}\phi'_1(z)=\bar{\phi}'_1(z),\ \psi'_1(z)=\bar{\psi}'_1(z),\ \eta'_1(z)=\bar{\eta}'_1(z),\qquad z\in S^-,\\ \phi'_2(z)=\bar{\phi}'_2(z),\ \psi'_2(z)=\bar{\psi}'_2(z),\ \eta'_2(z)=\bar{\eta}'_2(z),\qquad z\in S^+.\end{cases}\tag{9}$$

将式(7)代入式(9)得到

$$\begin{cases}\phi'_1(z)=M_1G(z)+\phi_{10}(z),\\ \psi'_1(z)=M_2G(z)+\psi_{10}(z),\\ \eta'_1(z)=M_3G(z)+\eta_{10}(z),\end{cases}\tag{10}$$

式中, $\phi_{10}(z),\psi_{10}(z)$ 和 $\eta_{10}(z)$ 在沿 L 隔开的全平面全纯,

$$G(z)=\frac{1}{z-z_0}+\frac{1}{z-\bar{z}_0}.\tag{11}$$

在 S^- 内无外载荷作用, 故 $\phi'_2(z)$, $\psi'_2(z)$ 和 $\eta'_2(z)$ 在 S^- 内全纯, 做如式(10)的解析延拓可知, $\phi'_2(z)$, $\psi'_2(z)$ 和 $\eta'_2(z)$ 在沿 L 割开的全平面全纯.

注意到式(4)和式(6), 下式成立:

$$\bar{t}=t,\ \bar{\phi}'^{+}_1(t)=\phi'^{-}_1(t),\ \bar{\phi}'^{-}_2(t)=\phi'^{+}_2(t),\ \bar{\psi}'^{+}_1(t)=\psi'^{-}_1(t),$$
$$\bar{\psi}'^{-}_2(t)=\psi'^{+}_2(t),\ \bar{\eta}'^{+}_1(t)=\eta'^{-}_1(t),\ \bar{\eta}'^{-}_2(t)=\eta'^{+}_2(t).$$

因此有

$$\begin{cases}\sigma_{yz_1}^{(1)+}(t)=\sigma_{yz_1}^{(2)-}(t),\\ H_{yz_1}^{(1)+}(t)=H_{yz_1}^{(2)-}(t),\\ D_y^{(1)+}(t)=D_y^{(2)-}(t),\end{cases}\tag{12}$$

其中, t 属于全实轴.

根据推广的 Liouville 定理[17], 并考虑式(9)和式(10)有

$$\begin{cases}C_{44}^{(1)}\phi'_1(z)+R_3^{(1)}\psi'_1(z)+e_{15}^{1\,(1)}\eta'_1(z)+C_{44}^{(2)}\phi'_2(z)+R_3^{(2)}\psi'_2(z)+e_{15}^{1\,(2)}\eta'_2(z)=\\ \qquad -\dfrac{P'}{2\pi}G(z)+\dfrac{P_3K_2^{(1)}e_{15}^{1\,(1)}-P_2e_{15}^{1\,(1)}e_{15}^{2\,(1)}-P_2R_3^{(1)}\epsilon_{11}^{(1)}-P_3R_3^{(1)}e_{15}^{2\,(1)}}{2\pi[K_2^{(1)}\epsilon_{11}^{(1)}+(e_{15}^{2\,(1)})^2]}G(z)+D_0,\\ K_2^{(1)}\psi'_1(z)+R_3^{(1)}\phi'_1(z)+e_{15}^{2\,(1)}\eta'_1(z)+K_2^{(2)}\psi'_2(z)+R_3^{(2)}\phi'_2(z)+e_{15}^{2\,(2)}\eta'_2(z)=\\ \qquad -\dfrac{P_2}{2\pi}G(z)+D_1,\\ e_{15}^{1\,(1)}\phi'_1(z)+e_{15}^{2\,(1)}\psi'_1(z)-\epsilon_{11}^{(1)}\eta'_1(z)+e_{15}^{1\,(2)}\phi'_2(z)+e_{15}^{2\,(2)}\psi'_2(z)-\epsilon_{11}^{(2)}\eta'_2(z)=\\ \qquad -\dfrac{Q}{2\pi}G(z)+D_2,\end{cases}\tag{13}$$

这里, z 属于全平面, D_0, D_1, D_2 为待定常数.

由位移连续条件, 可得

$$\begin{cases}\phi'^{+}_2(t)+\phi'^{-}_2(t)=N_1G(t)+D_3, & t\in L,\\ \psi'^{+}_2(t)+\psi'^{-}_2(t)=N_2G(t)+D_4, & t\in L,\\ \eta'^{+}_2(t)+\eta'^{-}_2(t)=N_3G(t)+D_5, & t\in L,\end{cases}\tag{14}$$

其中, N_1, N_2, N_3, D_3, D_4, D_5 的表达式见附录.

式(14)的积分形式解为

$$\begin{cases}\phi'_2(z)=\dfrac{X_0(z)}{2\pi i}\displaystyle\int_L\dfrac{N_1G(t)+D_3}{X_0^+(t)(t-z)}dt+X_0(z)P_n(z),\\ \psi'_2(z)=\dfrac{X_0(z)}{2\pi i}\displaystyle\int_L\dfrac{N_2G(t)+D_4}{X_0^+(t)(t-z)}dt+X_0(z)Q_n(z),\\ \eta'_2(z)=\dfrac{X_0(z)}{2\pi i}\displaystyle\int_L\dfrac{N_3G(t)+D_5}{X_0^+(t)(t-z)}dt+X_0(z)F_n(z),\end{cases}\tag{15}$$

其中

$$X_0(z)=\prod_{j=1}^{n}(z-a_j)^{-1/2}(z-b_j)^{-1/2},\tag{16}$$

$X_0(z)$ 是沿 L 割开的 z 平面上的单值分支, 满足

$$\lim_{|z|\to\infty}[X_0(z)\cdot z^n]=1,\qquad z\in S^+,\tag{17}$$

$$\begin{cases}P_n(z)=c_0z^n+c_1z^{n-1}+c_2z^{n-2}+\cdots+c_n,\\ Q_n(z)=d_0z^n+d_1z^{n-1}+d_2z^{n-2}+\cdots+d_n,\\ F_n(z)=f_0z^n+f_1z^{n-1}+f_2z^{n-2}+\cdots+f_n.\end{cases}\tag{18}$$

计算式(15)中的积分后, 得到

$$\begin{cases}\phi_2'(z)=X_0(z)\left\{P_n(z)-\dfrac{1}{2}\left[G_{z_0}^{\phi}(z)+G_{\bar{z}_0}^{\phi}(z)+G_{\infty}^{\phi}(z)\right]\right\}+\dfrac{N_1}{2}G(z)+\dfrac{D_3}{2},\\ \psi_2'(z)=X_0(z)\left\{Q_n(z)-\dfrac{1}{2}\left[G_{z_0}^{\psi}(z)+G_{\bar{z}_0}^{\psi}(z)+G_{\infty}^{\psi}(z)\right]\right\}+\dfrac{N_2}{2}G(z)+\dfrac{D_4}{2},\\ \eta_2'(z)=X_0(z)\left\{F_n(z)-\dfrac{1}{2}\left[G_{z_0}^{\eta}(z)+G_{\bar{z}_0}^{\eta}(z)+G_{\infty}^{\eta}(z)\right]\right\}+\dfrac{N_3}{2}G(z)+\dfrac{D_5}{2},\end{cases}\tag{19}$$

其中，$G_{z_0}^{\phi}(z),G_{\bar{z}_0}^{\phi}(z),G_{\infty}^{\phi}(z)$ 分别是式(15)中被积函数 $(N_1G(z)+D_3)/X_0(z)$ 在 $z=z_0,\bar{z}_0,\infty$ 处的奇性主部；同理，$G_{z_0}^{\psi}(z),G_{\bar{z}_0}^{\psi}(z),G_{\infty}^{\psi}(z)$($G_{z_0}^{\eta}(z),G_{\bar{z}_0}^{\eta}(z),G_{\infty}^{\eta}(z)$) 分别是 $(N_2G(z)+D_4)/X_0(z)$($(N_3G(z)+D_5)/X_0(z)$) 在 $z=z_0,\bar{z}_0,\infty$ 处的奇性主部，可由 Laurent 级数展开式获得.

在 $\phi_2'(z),\psi_2'(z)$ 和 $\eta_2'(z)$ 中，有 $c_0,c_1,\cdots,c_n,d_0,d_1,\cdots,d_n,f_0,f_1,\cdots,f_n,D_3,D_4$ 和 D_5 共 $3(n+2)$ 个常数需要确定，这就需要 $3(n+2)$ 个方程.根据无穷远处的受力情况，可得到 6 个常数；其余 $3n$ 个方程由位移单值条件导出.式(13)仅利用了 L 两边(S^+ 与 S^-) 声子场和相位子场位移的导数相等.另外还需补充在 n 个裂纹端点处位移相等的条件以满足 L 上两边位移相等.由各裂纹两端位移相等，有

$$\begin{cases}\mathrm{Re}\displaystyle\int_{a_j}^{b_j}\phi_1'^{+}(t)\,\mathrm{d}t=\mathrm{Re}\int_{a_j}^{b_j}\phi_2'^{-}(t)\,\mathrm{d}t, & t\in L',\\ \mathrm{Re}\displaystyle\int_{a_j}^{b_j}\psi_1'^{+}(t)\,\mathrm{d}t=\mathrm{Re}\int_{a_j}^{b_j}\psi_1'^{-}(t)\,\mathrm{d}t, & t\in L',\\ \mathrm{Re}\displaystyle\int_{a_j}^{b_j}\eta_1'^{+}(t)\,\mathrm{d}t=\mathrm{Re}\int_{a_j}^{b_j}\eta_2'^{-}(t)\,\mathrm{d}t, & t\in L'.\end{cases}\tag{20}$$

在 L' 上，$C_{44}^{(1)}\phi_1'(z)+R_3^{(1)}\psi_1'(z)+e_{15}^{1\,(1)}\eta_1'(z)$，$C_{44}^{(2)}\phi_1'(z)+R_3^{(2)}\psi_1'(z)+e_{15}^{1\,(2)}\eta_1'(z)$，$K_2^{(1)}\psi_1'(z)+R_3^{(1)}\phi_1'(z)+e_{15}^{2\,(1)}\eta_1'(z)$，$K_2^{(2)}\psi_1'(z)+R_3^{(2)}\phi_1'(z)+e_{15}^{2\,(2)}\eta_1'(z)$，$e_{15}^{1\,(1)}\phi_1'(z)+e_{15}^{2\,(1)}\psi_1'(z)-\epsilon_{11}^{(1)}\eta_1'(z)$，$e_{15}^{1\,(2)}\phi_1'(z)+e_{15}^{2\,(2)}\psi_1'(z)-\epsilon_{11}^{(2)}\eta_1'(z)$ 的虚部为 0.

考虑式(13)，得

$$\begin{cases}\displaystyle\int_{L_j'}\Bigg[(C_{44}^{(1)}+C_{44}^{(2)})\phi_2'(t)+(R_3^{(1)}+R_3^{(2)})\psi_2'(t)+(e_{15}^{1\,(1)}+e_{15}^{1\,(2)})\eta_2'(t)+\frac{P'}{2\pi}G(t)-\\ \quad\dfrac{QK_2^{(1)}e_{15}^{1\,(1)}-P_2e_{15}^{1\,(1)}e_{15}^{2\,(1)}-P_2R_3^{(1)}\epsilon_{11}^{(1)}-QR_3^{(1)}e_{15}^{2\,(1)}}{2\pi[K_2^{(1)}\epsilon_{11}^{(1)}+(e_{15}^{2\,(1)})^2]}G(t)-D_0\Bigg]\mathrm{d}t=0,\\ \displaystyle\int_{L_j'}\left[(K_2^{(1)}+K_2^{(2)})\psi_2'(t)+(R_3^{(1)}+R_3^{(2)})\phi_2'+(e_{15}^{2\,(1)}+e_{15}^{2\,(2)})\eta_2'(t)+\frac{P_2}{2\pi}G(z)-D_1\right]\mathrm{d}t=0,\\ \displaystyle\int_{L_j'}\left[(e_{15}^{1\,(1)}+e_{15}^{1\,(2)})\phi_2'+(e_{15}^{2\,(1)}+e_{15}^{2\,(2)})\psi_2'(t)-(\epsilon_{11}^{(1)}+\epsilon_{11}^{(2)})\eta_2'(t)+\frac{Q}{2\pi}G(z)-D_2\right]\mathrm{d}t=0,\end{cases}\tag{21}$$

其中　$j=1,2,\cdots,n$.

4　特　　例

假设无穷远处应力为零，当 $C_{44}^{(i)}K_2^{(i)}-R_3^{(i)2}\neq 0(i=1,2)$ 时，

$$\phi_i'(\infty)=\psi_i'(\infty)=\eta_i'(\infty)=0.\tag{22}$$

将式(22)代入式(13)，得到

$$D_0=0,\ D_1=0,\ D_2=0,\ D_3=0,\ D_4=0,\ D_5=0.\tag{23}$$

由式(15)和式(22)，可得

$$c_0=d_0=f_0=0.\tag{24}$$

4.1　一条有限长界面裂纹

如图 2 所示，设界面上有一条从 a 到 b 的裂纹，在 S^+ 内任意点 z_0 处作用集中载荷.此时 $n=1$，则

$$P_n(z)=c_1,\ Q_n(z)=d_1,\ F_n(z)=f_1, \tag{25}$$

$$X_0(z)=\frac{1}{\sqrt{z^2-l^2}}, \tag{26}$$

$$G_{z_0}^{\phi}(z)=N_1\frac{\sqrt{z_0^2-l^2}}{z-z_0},\ G_{z_0}^{\psi}(z)=N_2\frac{\sqrt{z_0^2-l^2}}{z-z_0},\ G_{z_0}^{\eta}(z)=N_3\frac{\sqrt{z_0^2-l^2}}{z-z_0}, \tag{27}$$

$$G_{\bar z_0}^{\phi}(z)=N_1\frac{\sqrt{\bar z_0^2-l^2}}{z-\bar z_0},\ G_{\bar z_0}^{\psi}(z)=N_2\frac{\sqrt{\bar z_0^2-l^2}}{z-\bar z_0},\ G_{\bar z_0}^{\eta}(z)=N_3\frac{\sqrt{\bar z_0^2-l^2}}{z-\bar z_0}, \tag{28}$$

$$G_{\infty}^{\phi}(z)=2N_1,\ G_{\infty}^{\psi}(z)=2N_2,\ G_{\infty}^{\eta}(z)=2N_3. \tag{29}$$

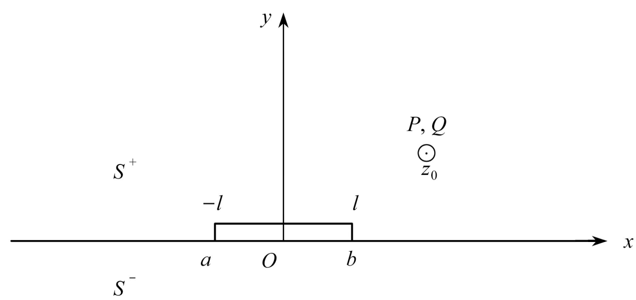

图 2　集中载荷作用在含一条界面裂纹的压电准晶双材料

Fig. 2　A piezoelectric quasicrystal bimaterial with an interfacial crack subjected to a concentrated load

下面确定常数 c_1 和 d_1，由式(21)和式(23)可得

$$\begin{cases}\displaystyle\int_{-l}^{l}\Bigg[(C_{44}^{(1)}+C_{44}^{(2)})\phi_2'(z)+(R_3^{(1)}+R_3^{(2)})\psi_2'(z)+(e_{15}^{1\ (1)}+e_{15}^{1\ (2)})\eta_2'(z)+\frac{P'}{2\pi}G(z)-\\ \qquad\displaystyle\frac{QK_2^{(1)}e_{15}^{1\ (1)}-P_2e_{15}^{1\ (1)}e_{15}^{2\ (1)}-P_2R_3^{(1)}\epsilon_{11}^{(1)}-QR_3^{(1)}e_{15}^{2\ (1)}}{2\pi[K_2^{(1)}\epsilon_{11}^{(1)}+(e_{15}^{2\ (1)})^2]}G(z)\Bigg]\mathrm{d}t=0,\\ \displaystyle\int_{-l}^{l}\Bigg[(K_2^{(1)}+K_2^{(2)})\psi_2'(z)+(R_3^{(1)}+R_3^{(2)})\phi_2'(z)+(e_{15}^{2\ (1)}+e_{15}^{2\ (2)})\eta_2'(z)+\frac{P_2}{2\pi}G(z)\Bigg]\mathrm{d}t=0,\\ \displaystyle\int_{-l}^{l}\Bigg[(e_{15}^{1\ (1)}+e_{15}^{1\ (2)})\phi_2'(z)+(e_{15}^{2\ (1)}+e_{15}^{2\ (2)})\psi_2'(z)-(\epsilon_{11}^{(1)}+\epsilon_{11}^{(2)})\eta_2'(z)+\frac{Q}{2\pi}G(z)\Bigg]\mathrm{d}t=0.\end{cases} \tag{30}$$

考虑围道积分，由留数定理得

$$c_1=N_1,\ d_1=N_2,\ f_1=N_3. \tag{31}$$

由式(13)，同时考虑到 $D_0=D_1=0$，当 $C_{44}^{(i)}K_2^{(i)}-R_3^{(i)\,2}\neq 0$ 时，有

$$\begin{cases}\phi_1'(z)=M_1G(z)-B_1\phi_2'(z)-E_1\psi_2'(z)-G_1\eta_2'(z),\\ \psi_1'(z)=M_2G(z)+B_2\phi_2'(z)+E_2\psi_2'(z)+G_2\eta_2'(z),\\ \eta_1'(z)=M_3G(z)+B_3\phi_2'(z)+E_3\psi_2'(z)+G_3\eta_2'(z),\end{cases} \tag{32}$$

其中，$B_1, B_2, B_3, E_1, E_2, E_3, G_1, G_2, G_3$ 的表达式见附录.

4.2　两条有限长界面裂纹

如图 3 所示，设界面上有两条等长的裂纹，裂纹端点依次为 $-b$，$-a$，a，b，在 S^+ 内任意点 z_0 处作用集中载荷.当 $n=2$ 时，

$$P_n(z)=c_1z+c_2,\ Q_n(z)=d_1z+d_2,\ F_n(z)=f_1z+f_2, \tag{33}$$

$$X_0(z)=\frac{1}{\sqrt{(z^2-a^2)(z^2-b^2)}}, \tag{34}$$

$$\begin{cases} G_{z_0}^{\phi}(z) = N_1 \dfrac{\sqrt{(z_0^2 - a^2)(z_0^2 - b^2)}}{z - z_0},\ G_{z_0}^{\psi}(z) = N_2 \dfrac{\sqrt{(z_0^2 - a^2)(z_0^2 - b^2)}}{z - z_0}, \\ G_{z_0}^{\eta}(z) = N_3 \dfrac{\sqrt{(z_0^2 - a^2)(z_0^2 - b^2)}}{z - z_0}, \end{cases} \tag{35}$$

$$\begin{cases} G_{\bar z_0}^{\phi}(z) = N_1 \dfrac{\sqrt{(\bar z_0^2 - a^2)(\bar z_0^2 - b^2)}}{z - \bar z_0},\ G_{\bar z_0}^{\psi}(z) = N_2 \dfrac{\sqrt{(\bar z_0^2 - a^2)(\bar z_0^2 - b^2)}}{z - \bar z_0}, \\ G_{\bar z_0}^{\eta}(z) = N_3 \dfrac{\sqrt{(\bar z_0^2 - a^2)(\bar z_0^2 - b^2)}}{z - \bar z_0}, \end{cases} \tag{36}$$

$$G_{\infty}^{\phi}(z) = 2N_1 z,\ G_{\infty}^{\psi}(z) = 2N_2 z,\ G_{\infty}^{\eta}(z) = 2N_3 z, \tag{37}$$

$$\begin{cases} c_2 = -\dfrac{N_1 b}{2\mathrm{E}(k)}\left[\dfrac{z_0 X(z_0)\Pi(\varphi,h,k)}{b(z_0^2 - b^2)} + \dfrac{\bar z_0 X(\bar z_0)\Pi(\varphi,h,k)}{b(\bar z_0^2 - b^2)} + \pi\right], \\ d_2 = -\dfrac{N_2 b}{2\mathrm{E}(k)}\left[\dfrac{z_0 X(z_0)\Pi(\varphi,h,k)}{b(z_0^2 - b^2)} + \dfrac{\bar z_0 X(\bar z_0)\Pi(\varphi,h,k)}{b(\bar z_0^2 - b^2)} + \pi\right], \\ f_2 = -\dfrac{N_3 b}{2\mathrm{E}(k)}\left[\dfrac{z_0 X(z_0)\Pi(\varphi,h,k)}{b(z_0^2 - b^2)} + \dfrac{\bar z_0 X(\bar z_0)\Pi(\varphi,h,k)}{b(\bar z_0^2 - b^2)} + \pi\right], \end{cases} \tag{38}$$

式中 $X(z) = \sqrt{(z^2 - a^2)(z^2 - b^2)}$,$\mathrm{E}(k)$ 是第二类完全椭圆积分，$\Pi(\varphi,h,k)$ 是第三类椭圆积分，且

$$\varphi = \frac{\pi}{2},\ h = \frac{b^2 - a^2}{b^2 - z_0^2},\ k = \sqrt{1 - \frac{a^2}{b^2}},\ h' = \frac{b^2 - a^2}{b^2 - \bar z_0^2}. \tag{39}$$

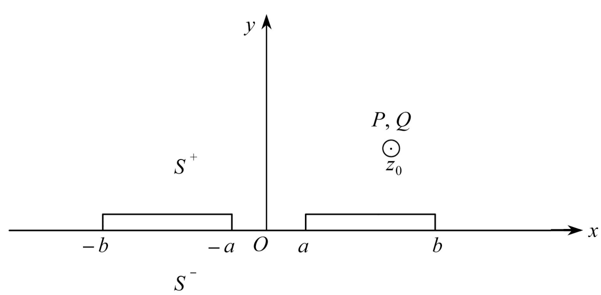

图 3　集中载荷作用在含两条界面裂纹的压电准晶双材料

Fig. 3　A piezoelectric quasicrystal bimaterial with 2 interfacial cracks subjected to a concentrated load

5　裂纹尖端场强度因子

计算双材料界面裂纹Ⅲ型场强度因子[18]：

$$\begin{cases} K_{\text{Ⅲ}}^{\sigma} = \sqrt{2\pi}\lim\limits_{z\to l}\sqrt{z-l}\,\sigma_{yz}^{(2)}(z) = -\sqrt{2\pi}\lim\limits_{z\to l}\sqrt{z-l}\,\mathrm{Im}(C_{44}^{(2)}\phi_2'(z) + R_3^{(2)}\psi_2'(z) + e_{15}^{1\,(2)}\eta_2'(z)), \\ K_{\text{Ⅲ}}^{H} = \sqrt{2\pi}\lim\limits_{z\to l}\sqrt{z-l}\,H_{yz}^{(2)}(z) = -\sqrt{2\pi}\,\mathrm{Im}\lim\limits_{z\to l}\sqrt{z-l}\,(K_2^{(2)}\psi_2'(z) + R_3^{(2)}\phi_2'(z) + e_{15}^{2\,(2)}\eta_2'(z)), \\ K_{\text{Ⅲ}}^{D} = \sqrt{2\pi}\lim\limits_{z\to l}\sqrt{z-l}\,D_{y}^{(2)}(z) = -\sqrt{2\pi}\,\mathrm{Im}\lim\limits_{z\to l}\sqrt{z-l}\,(e_{15}^{1\,(2)}\phi_2'(z) + e_{15}^{2\,(2)}\psi_2'(z) - \epsilon_{11}^{(2)}\eta_2'(z)), \end{cases} \tag{40}$$

其中 $K_{\text{Ⅲ}}^{\sigma}$ 表示声子场应力强度因子，$K_{\text{Ⅲ}}^{H}$ 表示相位子场应力强度因子，$K_{\text{Ⅲ}}^{D}$ 表示电位移强度因子.

5.1　一条界面裂纹

1）任意位置受集中载荷

如图 2，将式(19)和式(31)代入式(40)得

$$\begin{cases} K_{\mathrm{III}}^{\sigma} = -\operatorname{Im}\left[\dfrac{\sqrt{z_0^2 - l^2}/(l - z_0) + \sqrt{\bar{z}_0^2 - l^2}/(l - \bar{z}_0)}{2\sqrt{\pi l}}(I_1 \times J + L_1)\right], \\ K_{\mathrm{III}}^{H} = -\operatorname{Im}\left[\dfrac{\sqrt{z_0^2 - l^2}/(l - z_0) + \sqrt{\bar{z}_0^2 - l^2}/(l - \bar{z}_0)}{2\sqrt{\pi l}}(I_2 \times J + L_2)\right], \\ K_{\mathrm{III}}^{D} = -\operatorname{Im}\left[\dfrac{\sqrt{z_0^2 - l^2}/(l - z_0) + \sqrt{\bar{z}_0^2 - l^2}/(l - \bar{z}_0)}{2\sqrt{\pi l}}(I_3 \times J + L_3)\right], \end{cases} \tag{41}$$

其中 $I_1, I_2, I_3, J, L_1, L_2, L_3$ 的表达式见附录.

2）裂纹面上受集中载荷作用

如图 4 所示，当 $z_0 \to t$, t 为上半平面上任意点时，由式(41)可得

$$\begin{cases} K_{\mathrm{III}}^{\sigma} = \operatorname{Im}\left[\sqrt{\dfrac{t + l}{t - l}}\dfrac{I_1 \times J + L_1}{\sqrt{\pi l}}\right], \\ K_{\mathrm{III}}^{H} = \operatorname{Im}\left[\sqrt{\dfrac{t + l}{t - l}}\dfrac{I_2 \times J + L_2}{\sqrt{\pi l}}\right], \\ K_{\mathrm{III}}^{D} = \operatorname{Im}\left[\sqrt{\dfrac{t + l}{t - l}}\dfrac{I_3 \times J + L_3}{\sqrt{\pi l}}\right]. \end{cases} \tag{42}$$

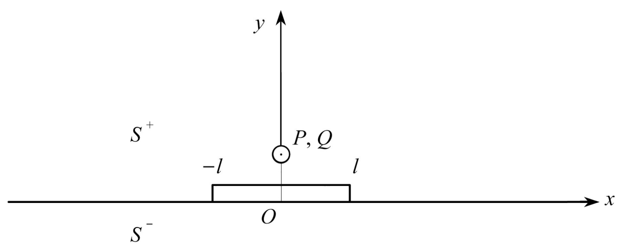

图 4　单个界面裂纹面上受集中载荷

Fig. 4　A single interfacial crack surface subjected to a concentrated load

令 $t \to 0$，由式(42)可得

$$\begin{cases} K_{\mathrm{III}}^{\sigma} = \operatorname{Im}\dfrac{I_1 \times J + L_1}{\sqrt{\pi l}}, \\ K_{\mathrm{III}}^{H} = \operatorname{Im}\dfrac{I_2 \times J + L_2}{\sqrt{\pi l}}, \\ K_{\mathrm{III}}^{D} = \operatorname{Im}\dfrac{I_3 \times J + L_3}{\sqrt{\pi l}}. \end{cases} \tag{43}$$

3）裂纹面上受均布集中载荷

如图 5 所示，在上半平面裂纹面上，任意区段 (a, b) 受均布集中载荷作用，由式(42)积分得

$$\begin{cases} K_{\mathrm{III}}^{\sigma} = -\operatorname{Im}\left[(I_1 \times J + L_1)\sqrt{\dfrac{l}{\pi}}\left(\sqrt{1 - \dfrac{a^2}{l^2}} - \sqrt{1 - \dfrac{b^2}{l^2}} + 2\left(\sin^{-1}\sqrt{\dfrac{l - a}{2l}} - \sin^{-1}\sqrt{\dfrac{l - b}{2l}}\right)\right)\right], \\ K_{\mathrm{III}}^{H} = -\operatorname{Im}\left[(I_2 \times J + L_2)\sqrt{\dfrac{l}{\pi}}\left(\sqrt{1 - \dfrac{a^2}{l^2}} - \sqrt{1 - \dfrac{b^2}{l^2}} + 2\left(\sin^{-1}\sqrt{\dfrac{l - a}{2l}} - \sin^{-1}\sqrt{\dfrac{l - b}{2l}}\right)\right)\right], \\ K_{\mathrm{III}}^{D} = -\operatorname{Im}\left[(I_3 \times J + L_3)\sqrt{\dfrac{l}{\pi}}\left(\sqrt{1 - \dfrac{a^2}{l^2}} - \sqrt{1 - \dfrac{b^2}{l^2}} + 2\left(\sin^{-1}\sqrt{\dfrac{l - a}{2l}} - \sin^{-1}\sqrt{\dfrac{l - b}{2l}}\right)\right)\right]. \end{cases} \tag{44}$$

当 $a = -l$, $b = l$，载荷布满裂纹面时，

$$\begin{cases} K_{\mathrm{III}}^{\sigma} = -\operatorname{Im}[(I_1 \times J + L_1)\sqrt{\pi l}], \\ K_{\mathrm{III}}^{H} = -\operatorname{Im}[(I_2 \times J + L_2)\sqrt{\pi l}], \\ K_{\mathrm{III}}^{D} = -\operatorname{Im}[(I_3 \times J + L_3)\sqrt{\pi l}]. \end{cases} \tag{45}$$

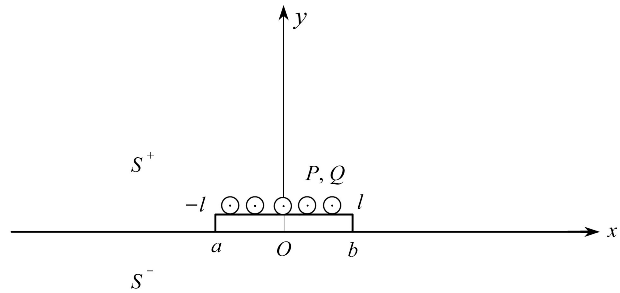

图 5　单个界面裂纹面上受均布集中载荷

Fig. 5　A single interfacial crack surface subjected to uniformly distributed concentrated loads

5.2　两条界面裂纹

1) 任意位置 z_0 受集中载荷作用

结合两条有限长界面裂纹的情形, 将式(19)代入式(40), 并考虑到式(38), 在裂纹尖端 $x = a$ 处

$$\begin{cases} K_{\mathrm{III}}^{\sigma} = -\operatorname{Im}\dfrac{(I_1 \times J + L_1)X_1}{2\sqrt{a\pi(a^2 - b^2)}}, \\ K_{\mathrm{III}}^{H} = -\operatorname{Im}\dfrac{(I_2 \times J + L_2)X_1}{2\sqrt{a\pi(a^2 - b^2)}}, \\ K_{\mathrm{III}}^{D} = -\operatorname{Im}\dfrac{(I_3 \times J + L_3)X_1}{2\sqrt{a\pi(a^2 - b^2)}}, \end{cases} \tag{46}$$

其中

$$X_1 = \frac{b}{\mathrm{E}(k)}\left[\frac{z_0X(z_0)\Pi(\varphi,h,k)}{b(z_0^2 - b^2)} + \frac{\bar{z}_0X(\bar{z}_0)\Pi(\varphi,h,k)}{b(\bar{z}_0^2 - b^2)} + \pi\right] + \left[\frac{\sqrt{(z_0^2 - a^2)(z_0^2 - b^2)}}{a - z_0} + \frac{\sqrt{(\bar{z}_0^2 - a^2)(\bar{z}_0^2 - b^2)}}{a - \bar{z}_0}\right].$$

在裂纹尖端 $x = b$ 处

$$\begin{cases} K_{\mathrm{III}}^{\sigma} = -\operatorname{Im}\dfrac{(I_1 \times J + L_1)X_2}{2\sqrt{b\pi(b^2 - a^2)}}, \\ K_{\mathrm{III}}^{H} = -\operatorname{Im}\dfrac{(I_2 \times J + L_2)X_2}{2\sqrt{b\pi(b^2 - a^2)}}, \\ K_{\mathrm{III}}^{D} = -\operatorname{Im}\dfrac{(I_3 \times J + L_3)X_2}{2\sqrt{b\pi(b^2 - a^2)}}, \end{cases} \tag{47}$$

其中

$$X_2 = \frac{b}{\mathrm{E}(k)}\left[\frac{z_0X(z_0)\Pi(\varphi,h,k)}{b(z_0^2 - b^2)} + \frac{\bar{z}_0X(\bar{z}_0)\Pi(\varphi,h,k)}{b(\bar{z}_0^2 - b^2)} + \pi\right] + \left[\frac{\sqrt{(z_0^2 - a^2)(z_0^2 - b^2)}}{b - z_0} + \frac{\sqrt{(\bar{z}_0^2 - a^2)(\bar{z}_0^2 - b^2)}}{b - \bar{z}_0}\right].$$

2) 裂纹面上受集中载荷作用

如图 6，设界面上有两条等长裂纹，集中载荷作用在上半平面裂纹面 $x=t$ 处，注意到在裂纹面上，$z_0=\bar{z}_0=t, X(z_0)=X(\bar{z}_0)=X(t)$. 在裂纹尖端 $x=a$ 处，由式(46)，令 $z_0\to t$，得

$$\begin{cases} K_{\mathrm{III}}^{\sigma}=-\mathrm{Im}\left\{\dfrac{I_1\times J+L_1}{\sqrt{a\pi(a^2-b^2)}}\left[\dfrac{bX(t)}{\mathrm{E}(k)}\left(\dfrac{t\Pi(\varphi,h,k)}{b(t^2-b^2)}+\dfrac{\pi}{2X(t)}\right)+\dfrac{X(t)}{a-t}\right]\right\}, \\ K_{\mathrm{III}}^{H}=-\mathrm{Im}\left\{\dfrac{I_2\times J+L_2}{\sqrt{a\pi(a^2-b^2)}}\left[\dfrac{bX(t)}{\mathrm{E}(k)}\left(\dfrac{t\Pi(\varphi,h,k)}{b(t^2-b^2)}+\dfrac{\pi}{2X(t)}\right)+\dfrac{X(t)}{a-t}\right]\right\}, \\ K_{\mathrm{III}}^{D}=-\mathrm{Im}\left\{\dfrac{I_3\times J+L_3}{\sqrt{a\pi(a^2-b^2)}}\left[\dfrac{bX(t)}{\mathrm{E}(k)}\left(\dfrac{t\Pi(\varphi,h,k)}{b(t^2-b^2)}+\dfrac{\pi}{2X(t)}\right)+\dfrac{X(t)}{a-t}\right]\right\}, \end{cases} \tag{48}$$

式中

$$\varphi=\frac{\pi}{2},\ h=\frac{b^2-a^2}{b^2-t^2},\ k=\sqrt{1-\frac{a^2}{b^2}}.$$

在裂纹尖端 $x=b$ 处，令 $z_0\to t$，得

$$\begin{cases} K_{\mathrm{III}}^{\sigma}=-\mathrm{Im}\left\{\dfrac{I_1\times J+L_1}{\sqrt{b\pi(b^2-a^2)}}\left[\dfrac{bX(t)}{\mathrm{E}(k)}\left(\dfrac{t\Pi(\varphi,h,k)}{b(t^2-b^2)}+\dfrac{\pi}{2X(t)}\right)+\dfrac{X(t)}{(b-t)}\right]\right\}, \\ K_{\mathrm{III}}^{H}=-\mathrm{Im}\left\{\dfrac{I_2\times J+L_2}{\sqrt{b\pi(b^2-a^2)}}\left[\dfrac{bX(t)}{\mathrm{E}(k)}\left(\dfrac{t\Pi(\varphi,h,k)}{b(t^2-b^2)}+\dfrac{\pi}{2X(t)}\right)+\dfrac{X(t)}{(b-t)}\right]\right\}, \\ K_{\mathrm{III}}^{D}=-\mathrm{Im}\left\{\dfrac{I_3\times J+L_3}{\sqrt{b\pi(b^2-a^2)}}\left[\dfrac{bX(t)}{\mathrm{E}(k)}\left(\dfrac{t\Pi(\varphi,h,k)}{b(t^2-b^2)}+\dfrac{\pi}{2X(t)}\right)+\dfrac{X(t)}{(b-t)}\right]\right\}. \end{cases} \tag{49}$$

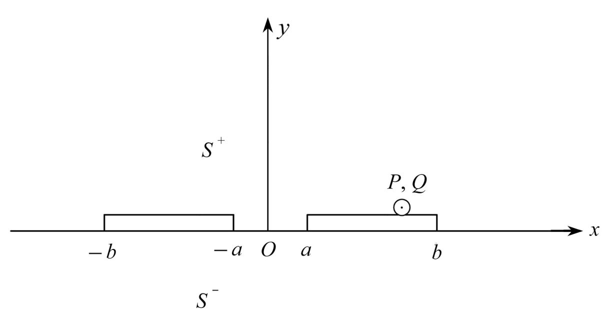

图 6 集中载荷作用在界面裂纹面

Fig. 6 A concentrated load applied to the interface crack surface

6 数 值 算 例

从式(49)中可以看出，裂纹尖端场强度因子与弹性常数、外载荷以及裂纹长度的取值有关. 数值算例分别讨论集中力和电荷、双材料耦合系数之比不同时，场强度因子随裂纹长度的变化规律. 本节仅讨论一条界面裂纹受反平面集中载荷作用下的情形. 一维六方压电准晶双材料弹性常数如表 1 所示[19].

表 1 一维六方压电准晶双材料弹性常数

Table 1 Elastic constants of 1D hexagonal piezoelectric quasicrystals

material	C_{44} /GPa	K_2 /GPa	R_3 /GPa	e_{15}^1 /(C·m^{-2})	e_{15}^2 /(C·m^{-2})	ϵ_{11} /(10^{-9} C^2·N^{-1}·m^{-2})
1	70.19	24	0.884 6	11.6	1.16	5
2	50	0.3	1.2	−0.318	−0.16	0.082 6

图 7—9 给出了两种材料耦合系数之比不同时，$K_{\mathrm{III}}^{\sigma}, K_{\mathrm{III}}^{H}, K_{\mathrm{III}}^{D}$ 随着裂纹长度改变的变化趋势，这里取 $P_1=1$ mN, $P_2=0.6$ mN, $Q=1$ mC. 从图 7—9 中可以看出，当耦合系数之比给定时，随着裂纹长度的增大，$K_{\mathrm{III}}^{\sigma}$, $K_{\mathrm{III}}^{H}, K_{\mathrm{III}}^{D}$ 都随之增加. 而当裂纹长度一定时，双材料耦合系数之比增大(固定 R_3^2 不变，耦合系数之比增大等

价于 R_3^1 增大)，$K_{\mathrm{III}}^{\sigma}$，$K_{\mathrm{III}}^{H}$ 随之增加，而 K_{III}^{D} 几乎不变.这表明耦合系数 R_3 只对 $K_{\mathrm{III}}^{\sigma}$，$K_{\mathrm{III}}^{H}$ 有影响，而对 K_{III}^{D} 几乎无影响，这与文献[20]所得结论完全一致.

图 10—12 给出了声子场载荷 P_1 取不同值时，场强度因子随着裂纹长度的变化趋势，这里取 $P_2 = 0.6$ mN，$Q = 1$ mC.从图 10—12 中可以看出，随着 P_1 增加，$K_{\mathrm{III}}^{\sigma}$ 增大而 K_{III}^{H} 减小，K_{III}^{D} 几乎不变.

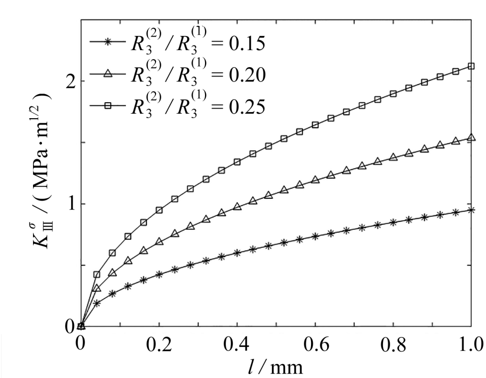

图 7　$R_3^{(2)}/R_3^{(1)}$ 不同时，$K_{\mathrm{III}}^{\sigma}$ 随 l 变化

Fig. 7　The variation of $K_{\mathrm{III}}^{\sigma}$ with l for different $R_3^{(2)}/R_3^{(1)}$ values

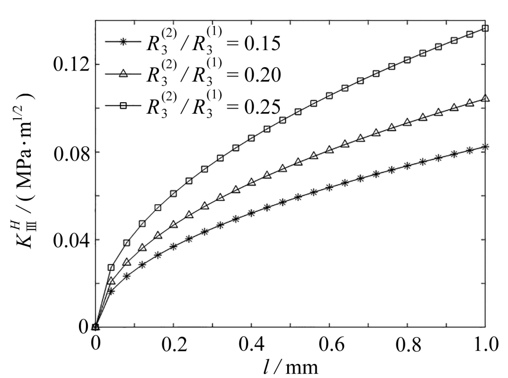

图 8　$R_3^{(2)}/R_3^{(1)}$ 不同时，K_{III}^{H} 随 l 变化

Fig. 8　The variation of K_{III}^{H} with l for different $R_3^{(2)}/R_3^{(1)}$ values

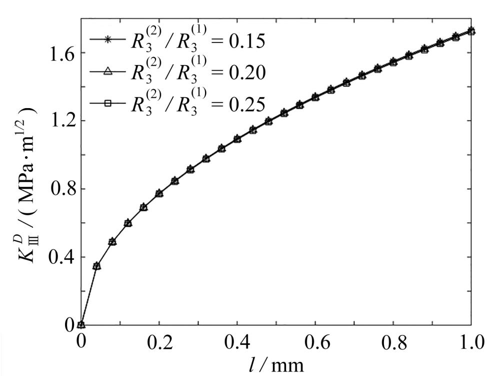

图 9　$R_3^{(2)}/R_3^{(1)}$ 不同时，K_{III}^{D} 随 l 变化

Fig. 9　The variation of K_{III}^{D} with l for different $R_3^{(2)}/R_3^{(1)}$ values

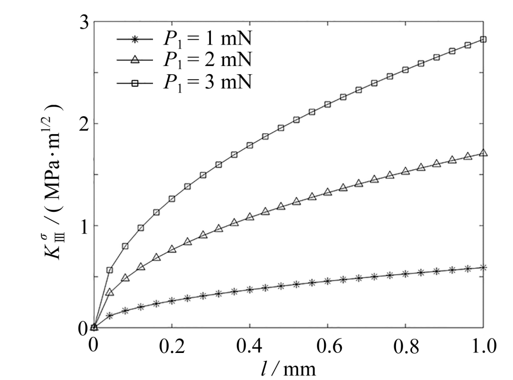

图 10　P_1 不同时，$K_{\mathrm{III}}^{\sigma}$ 随 l 变化

Fig. 10　The variation of $K_{\mathrm{III}}^{\sigma}$ with l for different P_1 values

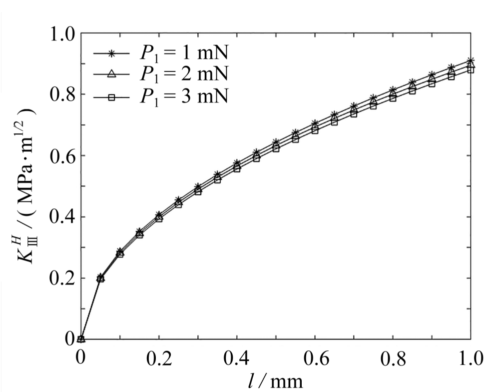

图 11　P_1 不同时，K_{III}^{H} 随 l 变化

Fig. 11　The variation of K_{III}^{H} with l for different P_1 values

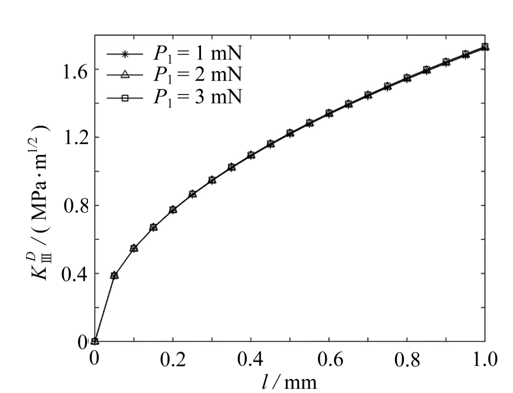

图 12　P_1 不同时，K_{III}^{D} 随 l 变化

Fig. 12　The variation of K_{III}^{D} with l for different P_1 values

图 13—15 给出了相位子场载荷 P_2 取不同值时，场强度因子随着裂纹长度的变化趋势，这里取 $P_1 = 1$ mN，$Q = 1$ mC.观察图 13—15 可以发现，随着 P_2 增加，K_{III}^{H} 增加而 $K_{\mathrm{III}}^{\sigma}$ 减小，K_{III}^{D} 几乎不变.图 10、11 和图 13、

14 所得结论与准晶的物理结构有关[21].声子场和相位子场相互垂直，因此在对其中一个场施加载荷时，对同场应力强度因子起促进作用，而对其耦合场的应力强度因子产生了抑制作用.图 16—18 给出了电载荷 Q 取不同值时，场强度因子随着裂纹长度的变化趋势，这里取 $P_1 = 1$ mN, $P_2 = 0.6$ mN .从图 16—18 中可以看出，随着 Q 增加，$K_{\mathrm{III}}^{\sigma}$, K_{III}^{D} 增大而 K_{III}^{H} 减小.这是因为电载荷作用方向与声子场载荷作用方向一致，因此电载荷对应力强度因子的影响趋势与声子场载荷的影响趋势一致.图 10—18 表明，尽管声子场载荷和相位子场载荷对电位移强度因子几乎无影响，但是电载荷却对应力强度因子影响显著.

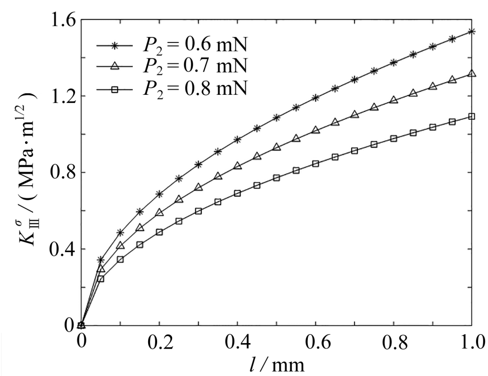

图 13 P_2 不同时，$K_{\mathrm{III}}^{\sigma}$ 随 l 变化

Fig. 13 The variation of $K_{\mathrm{III}}^{\sigma}$ with l for different P_2 values

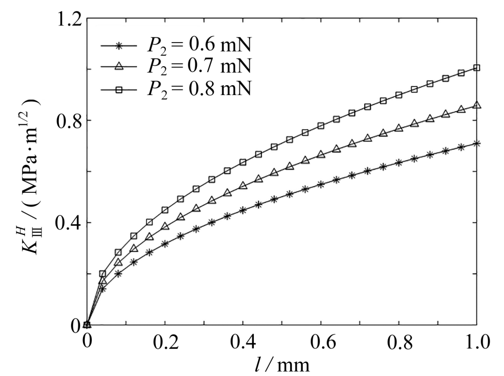

图 14 P_2 不同时，K_{III}^{H} 随 l 变化

Fig. 14 The variation of K_{III}^{H} with l for different P_2 values

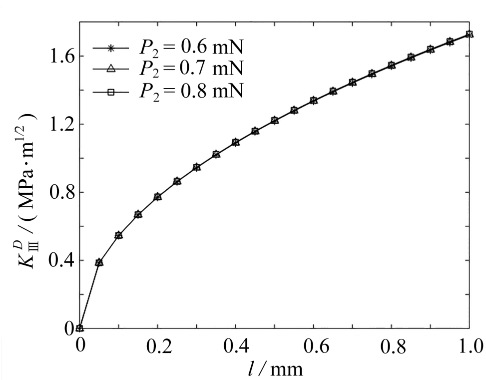

图 15 P_2 不同时，K_{III}^{D} 随 l 变化

Fig. 15 The variation of K_{III}^{D} with l for different P_2 values

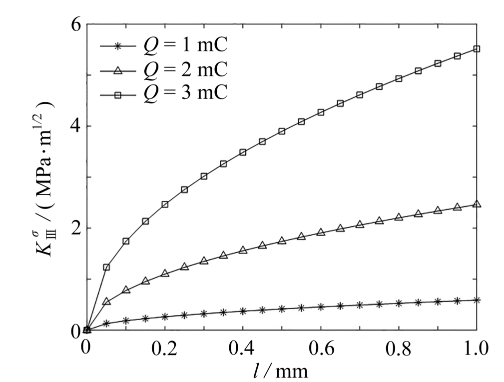

图 16 Q 不同时，$K_{\mathrm{III}}^{\sigma}$ 随 l 变化

Fig. 16 The variation of $K_{\mathrm{III}}^{\sigma}$ with l for different Q values

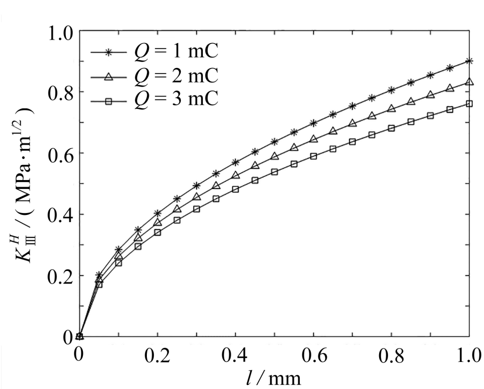

图 17 Q 不同时，K_{III}^{H} 随 l 变化

Fig. 17 The variation of K_{III}^{H} with l for different Q values

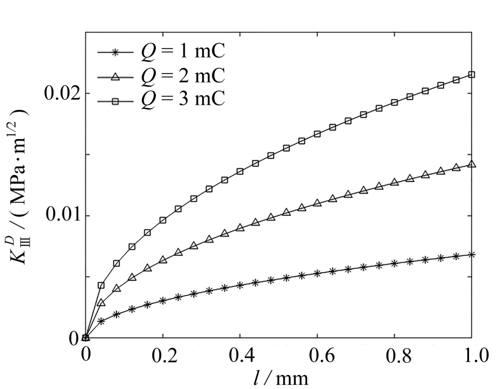

图 18 Q 不同时，K_{III}^{D} 随 l 变化

Fig. 18 The variation of K_{III}^{D} with l for different Q values

7 结　　论

本文研究了一维六方压电准晶双材料在集中载荷(力和电荷)作用下共线裂纹的反平面弹性问题.基于复变函数理论，把弹性问题转化为解析函数边值问题进行求解，得到了裂纹尖端场强度因子的解析解.作为特例，给出了不同载荷作用下含有一条和两条界面裂纹的封闭解.数值算例分析了场强度因子随裂纹长度、外载荷以及双材料耦合系数之比改变时的变化趋势.

致谢　本文作者衷心感谢宁夏大学新华学院科学研究基金项目重点项目(23XHKY01)对本文的资助.

附　　录

① 式(7)中的 M_1, M_2, M_3 表达式如下:

$$
\left\{
\begin{aligned}
M_1 &= \frac{-P'[K_2^{(1)}\epsilon_{11}^{(1)} + (e_{15}^{2\,(1)})^2]}{[C_{44}^{(1)}K_2^{(1)}\epsilon_{11}^{(1)} + C_{44}^{(1)}(e_{15}^{2\,(1)})^2 - (R_3^{(1)})^2\epsilon_{11}^{(1)} - 2R_3^{(1)}e_{15}^{1\,(1)}e_{15}^{2\,(1)} + (e_{15}^{1})^2K_2^{(1)}]2\pi},\\
M_2 &= \frac{P'(R_3^{(1)}\epsilon_{11}^{(1)} + e_{15}^{1\,(1)}e_{15}^{2\,(1)})}{[C_{44}^{(1)}K_2^{(1)}\epsilon_{11}^{(1)} + C_{44}^{(1)}(e_{15}^{2\,(1)})^2 - (R_3^{(1)})^2\epsilon_{11}^{(1)} - 2R_3^{(1)}e_{15}^{1\,(1)}e_{15}^{2\,(1)} + (e_{15}^{1})^2K_2^{(1)}]2\pi} -\\
&\quad \frac{P_2\epsilon_{11}^{(1)} + Qe_{15}^{2\,(1)}}{2\pi[K_2^{(1)}\epsilon_{11}^{(1)} + (e_{15}^{2\,(1)})^2]},\\
M_3 &= \frac{-P'(e_{15}^{1\,(1)}K_2^{(1)} - R_3^{(1)}e_{15}^{2\,(1)})}{[C_{44}^{(1)}K_2^{(1)}\epsilon_{11}^{(1)} + C_{44}^{(1)}(e_{15}^{2\,(1)})^2 - (R_3^{(1)})^2\epsilon_{11}^{(1)} - 2R_3^{(1)}e_{15}^{1\,(1)}e_{15}^{2\,(1)} + (e_{15}^{1})^2K_2^{(1)}]2\pi} +\\
&\quad \frac{QK_2^{(1)} - P_2e_{15}^{2\,(1)}}{2\pi[K_2^{(1)}\epsilon_{11}^{(1)} + (e_{15}^{2\,(1)})^2]},
\end{aligned}
\right.
\tag{A1}
$$

其中

$$
P' = [P_1K_2\epsilon_{11} + P_1\,(e_{15}^2)^2 - P_2R_3\epsilon_{11} - QR_3e_{15}^2 + QK_2e_{15}^1 - P_2e_{15}^1e_{15}^2]/[K_2\epsilon_{11} + (e_{15}^2)^2].
$$

② 式(14)中的 $N_1, N_2, N_3, D_3, D_4, D_5$ 表达式如下:

$$
\left\{
\begin{aligned}
N_1 &= -A_1[(K_2^{(1)} + K_2^{(2)})(\epsilon_{11}^{(1)} + \epsilon_{11}^{(2)}) + (e_{15}^{2\,(1)} + e_{15}^{2\,(2)})^2] +\\
&\quad A_2[(K_2^{(1)} + K_2^{(2)})(\epsilon_{11}^{(1)} + \epsilon_{11}^{(2)}) + (e_{15}^{2\,(1)} + e_{15}^{2\,(2)})^2],\\
N_2 &= A_1[(R_3^{(1)} + R_3^{(2)})(\epsilon_{11}^{(1)} + \epsilon_{11}^{(2)}) + (e_{15}^{1\,(1)} + e_{15}^{1\,(2)})(e_{15}^{2\,(1)} + e_{15}^{2\,(2)})] -\\
&\quad A_2[(R_3^{(1)} + R_3^{(2)})(\epsilon_{11}^{(1)} + \epsilon_{11}^{(2)}) + (e_{15}^{1\,(1)} + e_{15}^{1\,(2)})(e_{15}^{2\,(1)} + e_{15}^{2\,(2)})] -\\
&\quad \frac{Q(e_{15}^{2\,(1)} + e_{15}^{2\,(2)}) + P_2(\epsilon_{11}^{(1)} + \epsilon_{11}^{(2)})}{\pi[(K_2^{(1)} + K_2^{(2)})(\epsilon_{11}^{(1)} + \epsilon_{11}^{(2)}) + (e_{15}^{2\,(1)} + e_{15}^{2\,(2)})^2]},\\
N_3 &= A_1[(R_3^{(1)} + R_3^{(2)})(e_{15}^{2\,(1)} + e_{15}^{2\,(2)}) - (K_2^{(1)} + K_2^{(2)})(e_{15}^{1\,(1)} + e_{15}^{1\,(2)})] -\\
&\quad A_2[(R_3^{(1)} + R_3^{(2)})(e_{15}^{2\,(1)} + e_{15}^{2\,(2)}) - (K_2^{(1)} + K_2^{(2)})(e_{15}^{1\,(1)} + e_{15}^{1\,(2)})] +\\
&\quad \frac{Q(K_2^{(1)} + K_2^{(2)}) - P_2(e_{15}^{2\,(1)} + e_{15}^{2\,(2)})}{\pi[(e_{15}^{2\,(1)} + e_{15}^{2\,(2)})^2 + (K_2^{(1)} + K_2^{(2)})(\epsilon_{11}^{(1)} + \epsilon_{11}^{(2)})]},
\end{aligned}
\right.
\tag{A2}
$$

$$
\left\{
\begin{aligned}
D_3 &= A\{2[(K_2^{(1)} + K_2^{(2)})(\epsilon_{11}^{(1)} + \epsilon_{11}^{(2)}) + (e_{15}^{2\,(1)} + e_{15}^{2\,(2)})^2]D_0 -\\
&\quad 2[(R_3^{(1)} + R_3^{(2)})(\epsilon_{11}^{(1)} + \epsilon_{11}^{(2)}) + (e_{15}^{2\,(1)} + e_{15}^{2\,(2)})(e_{15}^{1\,(1)} + e_{15}^{1\,(2)})]D_1 -\\
&\quad 2[(R_3^{(1)} + R_3^{(2)})(e_{15}^{2\,(1)} + e_{15}^{2\,(2)}) - (K_2^{(1)} + K_2^{(2)})(e_{15}^{1\,(1)} + e_{15}^{1\,(2)})]D_2\},\\
D_4 &= A\{-2[(R_3^{(1)} + R_3^{(2)})(\epsilon_{11}^{(1)} + \epsilon_{11}^{(2)}) + (e_{15}^{1\,(1)} + e_{15}^{1\,(2)})(e_{15}^{2\,(1)} + e_{15}^{2\,(2)})]D_0 +\\
&\quad 2[(C_{44}^{(1)} + C_{44}^{(2)})(\epsilon_{11}^{(1)} + \epsilon_{11}^{(2)}) + (e_{15}^{1\,(1)} + e_{15}^{1\,(2)})^2]D_1 +\\
&\quad 2[(C_{44}^{(1)} + C_{44}^{(2)})(e_{15}^{2\,(1)} + e_{15}^{2\,(2)}) - (R_3^{(1)} + R_3^{(2)})(e_{15}^{1\,(1)} + e_{15}^{1\,(2)})]D_2\},\\
D_5 &= A\{-2[(R_3^{(1)} + R_3^{(2)})(e_{15}^{2\,(1)} + e_{15}^{2\,(2)}) - (K_2^{(1)} + K_2^{(2)})(e_{15}^{1\,(1)} + e_{15}^{1\,(2)})]D_0 +\\
&\quad 2[(R_3^{(1)} + R_3^{(2)})^2 - (C_{44}^{(1)} + C_{44}^{(2)})(K_2^{(1)} + K_2^{(2)})]D_2 +\\
&\quad 2[(C_{44}^{(1)} + C_{44}^{(2)})(e_{15}^{2\,(1)} + e_{15}^{2\,(2)}) - (R_3^{(1)} + R_3^{(2)})(e_{15}^{1\,(1)} + e_{15}^{1\,(2)})]D_1\},
\end{aligned}
\right.
\tag{A3}
$$

其中

$$
\begin{aligned}
A =\ & [(K_2^{(1)}+K_2^{(2)})(C_{44}^{(1)}+C_{44}^{(2)})(\epsilon_{11}^{(1)}+\epsilon_{11}^{(2)})+(C_{44}^{(1)}+C_{44}^{(2)})(e_{15}^{2\,(1)}+e_{15}^{2\,(2)})^2-\\
& (R_3^{(1)}+R_3^{(2)})^2(\epsilon_{11}^{(1)}+\epsilon_{11}^{(2)})-2(R_3^{(1)}+R_3^{(2)})(e_{15}^{1\,(1)}+e_{15}^{1\,(2)})(e_{15}^{2\,(1)}+e_{15}^{2\,(2)})+\\
& (K_2^{(1)}+K_2^{(2)})(e_{15}^{1\,(1)}+e_{15}^{1\,(2)})^2]^{-1},\\
A_1 =\ & A\{P'[(K_2^{(1)}+K_2^{(2)})(\epsilon_{11}^{(1)}+\epsilon_{11}^{(2)})+(e_{15}^{2\,(1)}+e_{15}^{2\,(2)})^2]+Q[(e_{15}^{1\,(1)}+e_{15}^{1\,(2)})(K_2^{(1)}+K_2^{(2)})-\\
& (R_3^{(1)}+R_3^{(2)})(e_{15}^{2\,(1)}+e_{15}^{2\,(2)})]-P_2[(e_{15}^{1\,(1)}+e_{15}^{1\,(2)})(e_{15}^{2\,(1)}+e_{15}^{2\,(2)})+\\
& (R_3^{(1)}+R_3^{(2)})(\epsilon_{11}^{(1)}+\epsilon_{11}^{(2)})]\}/\{\pi[(K_2^{(1)}+K_2^{(2)})(\epsilon_{11}^{(1)}+\epsilon_{11}^{(2)})+(e_{15}^{2\,(1)}+e_{15}^{2\,(2)})^2]\},\\
A_2 =\ & \frac{A(QK_2^{(1)}e_{15}^{1\,(1)}-P_2e_{15}^{1\,(1)}e_{15}^{2\,(1)}-P_2R_3^{(1)}\epsilon_{11}^{(1)}-QR_3^{(1)}e_{15}^{2\,(1)})}{\pi[K_2^{(1)}\epsilon_{11}^{(1)}+(e_{15}^{2\,(1)})^2]}.
\end{aligned}
$$

③ 式(32)中的 $B_1,B_2,B_3,E_1,E_2,E_3,G_1,G_2,G_3$ 表达式如下：

$$
\begin{cases}
B_1=\dfrac{C_{44}^{(2)}K_2^{(1)}\epsilon_{11}^{(1)}+C_{44}^{(2)}(e_{15}^{2\,(1)})^2-R_3^{(1)}R_3^{(2)}\epsilon_{11}^{(1)}-R_3^{(1)}e_{15}^{1\,(2)}e_{15}^{2\,(1)}-e_{15}^{1\,(1)}R_3^{(2)}e_{15}^{2\,(1)}+e_{15}^{1\,(1)}e_{15}^{1\,(2)}K_2^{(1)}}{C_{44}^{(1)}K_2^{(1)}\epsilon_{11}^{(1)}+C_{44}^{(1)}(e_{15}^{2\,(1)})^2-(R_3^{(1)})^2\epsilon_{11}^{(1)}-2R_3^{(1)}e_{15}^{1\,(1)}e_{15}^{2\,(1)}+(e_{15}^{1\,(1)})^2K_2^{(1)}},\\
E_1=\dfrac{R_3^{(2)}K_2^{(1)}\epsilon_{11}^{(1)}+R_3^{(2)}(e_{15}^{2\,(1)})^2-R_3^{(1)}K_2^{(2)}\epsilon_{11}^{(1)}-R_3^{(1)}e_{15}^{2\,(2)}e_{15}^{2\,(1)}+e_{15}^{1\,(1)}e_{15}^{2\,(2)}K_2^{(1)}-K_2^{(2)}e_{15}^{1\,(1)}e_{15}^{2\,(1)}}{C_{44}^{(1)}K_2^{(1)}\epsilon_{11}^{(1)}+C_{44}^{(1)}(e_{15}^{2\,(1)})^2-(R_3^{(1)})^2\epsilon_{11}^{(1)}-2R_3^{(1)}e_{15}^{1\,(1)}e_{15}^{2\,(1)}+(e_{15}^{1\,(1)})^2K_2^{(1)}},\\
G_1=\dfrac{K_2^{(1)}\epsilon_{11}^{(1)}e_{15}^{1\,(2)}+e_{15}^{1\,(2)}(e_{15}^{2\,(1)})^2+R_3^{(1)}\epsilon_{11}^{(2)}e_{15}^{2\,(1)}-R_3^{(1)}e_{15}^{2\,(2)}\epsilon_{11}^{(1)}-e_{15}^{2\,(2)}e_{15}^{2\,(1)}e_{15}^{1\,(1)}-\epsilon_{11}^{(2)}K_2^{(1)}e_{15}^{1\,(1)}}{C_{44}^{(1)}K_2^{(1)}\epsilon_{11}^{(1)}+C_{44}^{(1)}(e_{15}^{2\,(1)})^2-(R_3^{(1)})^2\epsilon_{11}^{(1)}-2R_3^{(1)}e_{15}^{1\,(1)}e_{15}^{2\,(1)}+(e_{15}^{1\,(1)})^2K_2^{(1)}},
\end{cases}
\tag{A4}
$$

$$
\begin{cases}
B_2=\dfrac{R_3^{(1)}\epsilon_{11}^{(1)}C_{44}^{(2)}+R_3^{(1)}e_{15}^{1\,(1)}e_{15}^{1\,(2)}+C_{44}^{(2)}e_{15}^{1\,(1)}e_{15}^{2\,(1)}-R_3^{(2)}\epsilon_{11}^{(1)}C_{44}^{(1)}-R_3^{(2)}(e_{15}^{1\,(1)})^2-C_{44}^{(1)}e_{15}^{1\,(2)}e_{15}^{2\,(1)}}{C_{44}^{(1)}K_2^{(1)}\epsilon_{11}^{(1)}+C_{44}^{(1)}(e_{15}^{2\,(1)})^2-(R_3^{(1)})^2\epsilon_{11}^{(1)}-2R_3^{(1)}e_{15}^{1\,(1)}e_{15}^{2\,(1)}+(e_{15}^{1\,(1)})^2K_2^{(1)}},\\
E_2=\dfrac{R_3^{(1)}R_3^{(2)}\epsilon_{11}^{(1)}+R_3^{(1)}e_{15}^{1\,(1)}e_{15}^{1\,(2)}+R_3^{(2)}e_{15}^{1\,(1)}e_{15}^{2\,(1)}-K_2^{(2)}\epsilon_{11}^{(1)}C_{44}^{(1)}-K_2^{(2)}(e_{15}^{1\,(1)})^2-C_{44}^{(1)}e_{15}^{2\,(2)}e_{15}^{2\,(1)}}{C_{44}^{(1)}K_2^{(1)}\epsilon_{11}^{(1)}+C_{44}^{(1)}(e_{15}^{2\,(1)})^2-(R_3^{(1)})^2\epsilon_{11}^{(1)}-2R_3^{(1)}e_{15}^{1\,(1)}e_{15}^{2\,(1)}+(e_{15}^{1\,(1)})^2K_2^{(1)}},\\
G_2=\dfrac{R_3^{(1)}e_{15}^{1\,(2)}\epsilon_{11}^{(1)}-R_3^{(1)}e_{15}^{1\,(1)}\epsilon_{11}^{(2)}+e_{15}^{1\,(1)}e_{15}^{1\,(2)}e_{15}^{2\,(1)}+e_{15}^{2\,(1)}\epsilon_{11}^{(2)}C_{44}^{(1)}-C_{44}^{(1)}\epsilon_{11}^{(1)}e_{15}^{2\,(2)}-(e_{15}^{1\,(1)})^2e_{15}^{2\,(2)}}{C_{44}^{(1)}K_2^{(1)}\epsilon_{11}^{(1)}+C_{44}^{(1)}(e_{15}^{2\,(1)})^2-(R_3^{(1)})^2\epsilon_{11}^{(1)}-2R_3^{(1)}e_{15}^{1\,(1)}e_{15}^{2\,(1)}+(e_{15}^{1\,(1)})^2K_2^{(1)}},
\end{cases}
\tag{A5}
$$

$$
\begin{cases}
B_3=\dfrac{C_{44}^{(2)}R_3^{(1)}e_{15}^{2\,(1)}-C_{44}^{(2)}e_{15}^{1\,(1)}K_2^{(1)}+R_3^{(1)}R_3^{(2)}e_{15}^{1\,(1)}+C_{44}^{(1)}K_2^{(1)}e_{15}^{1\,(2)}-(R_3^{(1)})^2e_{15}^{1\,(2)}-C_{44}^{(1)}R_3^{(2)}e_{15}^{2\,(1)}}{C_{44}^{(1)}K_2^{(1)}\epsilon_{11}^{(1)}+C_{44}^{(1)}(e_{15}^{2\,(1)})^2-(R_3^{(1)})^2\epsilon_{11}^{(1)}-2R_3^{(1)}e_{15}^{1\,(1)}e_{15}^{2\,(1)}+(e_{15}^{1\,(1)})^2K_2^{(1)}},\\
E_3=\dfrac{R_3^{(1)}R_3^{(2)}e_{15}^{2\,(1)}-R_3^{(2)}e_{15}^{1\,(1)}K_2^{(1)}+R_3^{(1)}K_2^{(2)}e_{15}^{1\,(1)}+C_{44}^{(1)}K_2^{(1)}e_{15}^{2\,(2)}-(R_3^{(1)})^2e_{15}^{2\,(2)}-C_{44}^{(1)}K_2^{(2)}e_{15}^{2\,(1)}}{C_{44}^{(1)}K_2^{(1)}\epsilon_{11}^{(1)}+C_{44}^{(1)}(e_{15}^{2\,(1)})^2-(R_3^{(1)})^2\epsilon_{11}^{(1)}-2R_3^{(1)}e_{15}^{1\,(1)}e_{15}^{2\,(1)}+(e_{15}^{1\,(1)})^2K_2^{(1)}},\\
G_3=\dfrac{R_3^{(1)}e_{15}^{1\,(2)}e_{15}^{2\,(1)}-e_{15}^{1\,(2)}e_{15}^{1\,(1)}K_2^{(1)}+R_3^{(1)}e_{15}^{1\,(1)}e_{15}^{2\,(2)}-C_{44}^{(1)}e_{15}^{2\,(1)}e_{15}^{2\,(2)}-C_{44}^{(1)}K_2^{(1)}\epsilon_{11}^{(2)}+(R_3^{(1)})^2\epsilon_{11}^{(2)}}{C_{44}^{(1)}K_2^{(1)}\epsilon_{11}^{(1)}+C_{44}^{(1)}(e_{15}^{2\,(1)})^2-(R_3^{(1)})^2\epsilon_{11}^{(1)}-2R_3^{(1)}e_{15}^{1\,(1)}e_{15}^{2\,(1)}+(e_{15}^{1\,(1)})^2K_2^{(1)}}.
\end{cases}
\tag{A6}
$$

④ 式(41)中的 $J,I_1,I_2,I_3,L_1,L_2,L_3$ 表达式如下：

$$
\begin{aligned}
J =\ & \{P'[(K_2^{(1)}+K_2^{(2)})(\epsilon_{11}^{(1)}+\epsilon_{11}^{(2)})+(e_{15}^{2\,(1)}+e_{15}^{2\,(2)})^2]+Q[(e_{15}^{1\,(1)}+e_{15}^{1\,(2)})(K_2^{(1)}+K_2^{(2)})-\\
& (R_3^{(1)}+R_3^{(2)})(e_{15}^{2\,(1)}+e_{15}^{2\,(2)})]-P_2[(e_{15}^{1\,(1)}+e_{15}^{1\,(2)})(e_{15}^{2\,(1)}+e_{15}^{2\,(2)})+\\
& (R_3^{(1)}+R_3^{(2)})(\epsilon_{11}^{(1)}+\epsilon_{11}^{(2)})]\}/[(K_2^{(1)}+K_2^{(2)})(\epsilon_{11}^{(1)}+\epsilon_{11}^{(2)})+(e_{15}^{2\,(1)}+e_{15}^{2\,(2)})^2]-\\
& \frac{QK_2^{(1)}e_{15}^{1\,(1)}-P_2e_{15}^{1\,(1)}e_{15}^{2\,(1)}-P_2R_3^{(1)}\epsilon_{11}^{(1)}-QR_3^{(1)}e_{15}^{2\,(1)}}{K_2^{(1)}\epsilon_{11}^{(1)}+(e_{15}^{2\,(1)})^2},
\end{aligned}
\tag{A7}
$$

$$
\begin{cases}
\begin{aligned}
I_1 =\ & A\{C_{44}^{(2)}[(K_2^{(1)}+K_2^{(2)})(\epsilon_{11}^{(1)}+\epsilon_{11}^{(2)})+(e_{15}^{2\,(1)}+e_{15}^{2\,(2)})^2]-R_3^{(2)}[(R_3^{(1)}+R_3^{(2)})(\epsilon_{11}^{(1)}+\epsilon_{11}^{(2)})+\\
& (e_{15}^{1\,(1)}+e_{15}^{1\,(2)})(e_{15}^{2\,(1)}+e_{15}^{2\,(2)})]-e_{15}^{1\,(2)}[(R_3^{(1)}+R_3^{(2)})(e_{15}^{2\,(1)}+e_{15}^{2\,(2)})-\\
& (K_2^{(1)}+K_2^{(2)})(e_{15}^{1\,(1)}+e_{15}^{1\,(2)})]\},
\end{aligned}\\
L_1=\dfrac{R_3^{(2)}Q(e_{15}^{2\,(1)}+e_{15}^{2\,(2)})+R_3^{(2)}P_2(\epsilon_{11}^{(1)}+\epsilon_{11}^{(2)})-e_{15}^{1\,(2)}Q(K_2^{(1)}+K_2^{(2)})+e_{15}^{1\,(2)}P_2(e_{15}^{2\,(1)}+e_{15}^{2\,(2)})}{(K_2^{(1)}+K_2^{(2)})(\epsilon_{11}^{(1)}+\epsilon_{11}^{(2)})+(e_{15}^{2\,(1)}+e_{15}^{2\,(2)})^2},
\end{cases}
\tag{A8a}
$$

$$
\begin{cases}
\begin{aligned}
I_2 =\ & A\{R_3^{(2)}[(K_2^{(1)}+K_2^{(2)})(\epsilon_{11}^{(1)}+\epsilon_{11}^{(2)})+(e_{15}^{2\,(1)}+e_{15}^{2\,(2)})^2]-K_2^{(2)}[(R_3^{(1)}+R_3^{(2)})(\epsilon_{11}^{(1)}+\epsilon_{11}^{(2)})+\\
& (e_{15}^{1\,(1)}+e_{15}^{1\,(2)})(e_{15}^{2\,(1)}+e_{15}^{2\,(2)})]-e_{15}^{2\,(2)}[(R_3^{(1)}+R_3^{(2)})(e_{15}^{2\,(1)}+e_{15}^{2\,(2)})-\\
& (K_2^{(1)}+K_2^{(2)})(e_{15}^{1\,(1)}+e_{15}^{1\,(2)})]\},
\end{aligned}\\
L_2=\dfrac{QK_2^{(2)}(e_{15}^{2\,(1)}+e_{15}^{2\,(2)})+P_2K_2^{(2)}(\epsilon_{11}^{(1)}+\epsilon_{11}^{(2)})-Qe_{15}^{2\,(2)}(K_2^{(1)}+K_2^{(2)})+P_2e_{15}^{2\,(2)}(e_{15}^{2\,(1)}+e_{15}^{2\,(2)})}{(K_2^{(1)}+K_2^{(2)})(\epsilon_{11}^{(1)}+\epsilon_{11}^{(2)})+(e_{15}^{2\,(1)}+e_{15}^{2\,(2)})^2},
\end{cases}
\tag{A8b}
$$

$$\begin{cases} I_3 = A\{ {e^1_{15}}^{(2)}[(K_2^{(1)}+K_2^{(2)})(\epsilon_{11}^{(1)}+\epsilon_{11}^{(2)})+({e^2_{15}}^{(1)}+{e^2_{15}}^{(2)})^2] - {e^2_{15}}^{(2)}[(R_3^{(1)}+R_3^{(2)})(\epsilon_{11}^{(1)}+\epsilon_{11}^{(2)})+ \\ \quad ({e^1_{15}}^{(1)}+{e^1_{15}}^{(2)})({e^2_{15}}^{(1)}+{e^2_{15}}^{(2)})] + \epsilon_{11}^{(2)}[(R_3^{(1)}+R_3^{(2)})({e^2_{15}}^{(1)}+{e^2_{15}}^{(2)}) - \\ \quad (K_2^{(1)}+K_2^{(2)})({e^1_{15}}^{(1)}+{e^1_{15}}^{(2)})]\}, \\ L_3 = \dfrac{Q{e^2_{15}}^{(2)}({e^2_{15}}^{(1)}+{e^2_{15}}^{(2)})+P_2{e^2_{15}}^{(2)}(\epsilon_{11}^{(1)}+\epsilon_{11}^{(2)})+Q\epsilon_{11}^{(2)}(K_2^{(1)}+K_2^{(2)})-P_2\epsilon_{11}^{(2)}({e^2_{15}}^{(1)}+{e^2_{15}}^{(2)})}{(K_2^{(1)}+K_2^{(2)})(\epsilon_{11}^{(1)}+\epsilon_{11}^{(2)})+({e^2_{15}}^{(1)}+{e^2_{15}}^{(2)})^2}. \end{cases} \tag{A8c}$$

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