

不对称柔性壁管道内幂律流体 蠕动传输的精确解*

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摘要: 在不对称管道内,研究了壁面柔性对非 Newton 流体蠕动流的影响.流变学性质由幂律流体本构方程表征.在数学表达中,采用了长波和低 Reynolds 数近似.得到了流函数和速度的精确解.给出了流线图及其俘获现象.对所讨论的流动,陈列了关键参数的显著特征,并最后给出了主要结论.

关键词: 蠕动流; 幂律流体; 柔性壁

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符号说明

V	速度	u	x 向速度分量
v	y 向速度分量	T	Cauchy 应力张量
S	外加应力张量	D	速度梯度的对称部分
A_1	Rivlin-Erickson 第一张量	L	速度梯度
L^T	速度梯度的转置	C	粘性阻尼系数
B	板的抗挠刚度	a_1	沿管道上壁传播波的幅值
a_2	沿管道下壁传播波的幅值	E_1	每单位面积板的无量纲质量
E_3	板的无量纲抗挠刚度	E_5	无量纲弹性刚度系数
\tilde{m}	单位面积板质量	t	时间
x	沿管道壁的空间坐标	y	垂直管道壁的空间坐标
m	幂律指数	$-\rho I$	球面应力
c	波速	k	弹性刚度系数
p	压力	d_1	上半部管道宽度
d_2	下半部管道宽度	a	沿上壁面无量纲波幅值比

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E_2	无量纲粘性阻尼系数	b	沿下壁面无量纲波幅值比
h	管道长宽比	E_4	薄膜的无量纲弹性张力
η_1	上壁面竖直位移	Re	Reynolds 数
μ	动力学粘度	η_2	下壁面竖直位移
ν	运动学粘度	μ_{app}	表观粘度
ρ	液体密度	Ψ	流函数
θ	相位差	δ	无量纲波数
λ	波长	τ	薄膜弹性张力

引言

大家知道,蠕动的机理有如用泵抽吸生物和工业流体.即使在日常生活里,蠕动已有长达多个世纪的观测,Shapiro 等^[1]的波理论,Fung 和 Yih^[2]的实验理论,被认为开创了蠕动传输的数学模型.很多研究者跟随着 Shapiro 或者 Fung,涉足蠕动传输领域的研究.还应注意的是,多数关系蠕动流的研究,都按 Newton 流体来处理.受到合成生物流体流变学的推动,开始出现了变化,按包括非 Newton 流体^[3-14]来处理.Raju 和 Devanathan^[15]将血液作为幂律流体,研究了血管中非 Newton 流体的蠕动流.他们所得到的流函数解,是变形幅值项的幂级数.Radhakrishnamacharya^[16]在长波近似下,研究了管道中幂律流体的蠕动流.Lew 等^[17]及 Misra 和 Pandey^[18]以幂律流体为模型,研究小肠和食道内食糜的蠕动传输.Srivastava 等^[19]就男性生殖问题,展开了幂律流体蠕动传输的讨论.润滑法出现在管道宽度远小于长度时.与润滑法相类似,Mishra 和 Rao^[20]应用长波法,研究了不对称管道中的蠕动流,不对称性是由于蠕动波的不同幅值,不同相位造成的.

上面提到的所有研究,都按壁面没有柔曲性的管道来处理问题.有一些研究^[21-25],按柔性壁面的 Newton 流体和非 Newton 流体来处理蠕动传输问题.迄今未发现在非对称柔性壁面管道内,研究幂律流体的论文.因此,本文的目的就是对此作一探讨.流动分析采用了长波近似和低 Reynolds 数假定.导出了重要流动量的结果,并进行了讨论.

1 数学模型

假定幂律流体被约束在宽度为 $d_1 + d_2$ 不对称的管道中.管道内充满了不可压缩的幂律流体,具不同幅值和相位的无数波群,以速度 c 沿管壁传播,导致不对称性.而且管道壁是有柔性的,像反冲弹力板一样,且运动限于竖向.设 η_1 和 η_2 分别表示上管壁和下管壁的竖向位移,则不对称上下管道壁的竖向位移为

$$\eta_1 = d_1 + a_1 \cos\left(\frac{2\pi}{\lambda}(x - ct)\right), \quad \eta_2 = d_2 + a_2 \cos\left[\frac{2\pi}{\lambda}(x - ct) + \theta\right],$$

其中, λ 为波长, c 为波速, a_1 和 a_2 为波幅, θ 为相位差 ($0 \leq \theta \leq \pi$), x 和 y 为直角坐标, x 为波的传播方向, y 方向垂直于管道壁的平均位置.值得注意的是, $\theta = 0$ 表示对称管道波异相, $\theta = \pi$ 表示对称管道波同相,并且 a_1, a_2, d_1, d_2 和 θ 满足条件 $a_1^2 + a_2^2 + 2a_1 a_2 \cos\theta \leq (d_1 + d_2)^2$.

二维流动方程为

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \quad (1)$$

$$\rho \left(\frac{\partial}{\partial t} + u \frac{\partial}{\partial x} + v \frac{\partial}{\partial y} \right) u = - \frac{\partial p}{\partial x} + \frac{\partial S_{xx}}{\partial x} + \frac{\partial S_{xy}}{\partial y}, \quad (2)$$

$$\rho \left(\frac{\partial}{\partial t} + u \frac{\partial}{\partial x} + v \frac{\partial}{\partial y} \right) v = - \frac{\partial p}{\partial y} + \frac{\partial S_{xy}}{\partial x} + \frac{\partial S_{yy}}{\partial y}, \quad (3)$$

$$S_{xx} = 2\mu \left[4 \left(\frac{\partial u}{\partial x} \right)^2 + 2 \left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right) + 4 \left(\frac{\partial v}{\partial y} \right)^2 \right]^m \frac{\partial u}{\partial x}, \quad (4)$$

$$S_{xy} = \mu \left[4 \left(\frac{\partial u}{\partial x} \right)^2 + 2 \left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right) + 4 \left(\frac{\partial v}{\partial y} \right)^2 \right]^m \left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right), \quad (5)$$

$$S_{yy} = 2\mu \left[4 \left(\frac{\partial u}{\partial x} \right)^2 + 2 \left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right) + 4 \left(\frac{\partial v}{\partial y} \right)^2 \right]^m \frac{\partial v}{\partial y}; \quad (6)$$

边界条件为

$$u = 0, \quad \text{当 } y = \left\{ \begin{array}{l} d_1 + a_1 \cos \left(\frac{2\pi}{\lambda} (x - ct) \right) \\ -d_2 - a_2 \cos \left[\frac{2\pi}{\lambda} (x - ct) + \theta \right] \end{array} \right\}, \quad (7)$$

$$\begin{aligned} \frac{\partial p}{\partial x} = \frac{\partial}{\partial x} \left[\tilde{m} \frac{\partial^2}{\partial t^2} + C \frac{\partial}{\partial t} + B \frac{\partial^4}{\partial x^4} - \tau \frac{\partial^2}{\partial x^2} + k \right] \left\{ \begin{array}{l} \eta_1 \\ \eta_2 \end{array} \right\} = \\ \frac{\partial S_{xx}}{\partial x} + \frac{\partial S_{xy}}{\partial y} - \rho \left(\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} \right), \end{aligned} \quad (8)$$

其中, u 和 v 分别为 x 和 y 方向的速度分量, ρ, t, p 和 μ 分别为流体密度、时间、压力和动粘度. 进而 $L = \tilde{m}(\partial^2/\partial t^2) + C(\partial/\partial t) + B(\partial^4/\partial x^4) - \tau(\partial^2/\partial x^2) + k$, τ 为薄膜内的弹性张力, \tilde{m} 为单位面积质量, C 为粘性阻尼系数, B 为板的弯曲刚度, k 为弹性刚度系数. 进一步, 方程(1) ~ (6) 所描述的流体: 当 $m < 0$ 时, 为稀释的剪切流体, 当 $m = 0$ 时, 为 Newton 流体, 当 $m > 0$ 时, 为浓缩的剪切流体.

设

$$u = \frac{\partial \Psi}{\partial y}, \quad v = - \frac{\partial \Psi}{\partial x},$$

并定义如下无量纲变量和参数:

$$\begin{aligned} x^* = \frac{x}{\lambda}, \quad y^* = \frac{y}{d_1}, \quad \Psi^* = \frac{\Psi}{cd_1}, \quad t^* = \frac{ct}{\lambda}, \quad \eta_1^* = \frac{\eta_1}{d_1}, \quad \eta_2^* = \frac{\eta_2}{d_1}, \quad p^* = \frac{d_1}{\mu c \lambda} \left(\frac{d_1^2}{2c^2} \right)^m, \\ S^* = \frac{d_1}{\mu c} \left(\frac{d_1^2}{2c^2} \right)^m, \quad \delta = \frac{d_1}{\lambda}, \quad E_1 = \frac{\tilde{m} c d_1^3}{\lambda^3 \mu} \left(\frac{d_1^2}{2c^2} \right)^m, \quad E_2 = \frac{C d_1^3}{\lambda^2 \mu} \left(\frac{d_1^2}{2c^2} \right)^m, \\ E_3 = \frac{B d_1^3}{\lambda^5 \mu c} \left(\frac{d_1^2}{2c^2} \right)^m, \quad E_4 = \frac{\tau d_1^3}{\lambda^3 \mu c} \left(\frac{d_1^2}{2c^2} \right)^m, \quad E_5 = \frac{k d_1^3}{\lambda \mu c} \left(\frac{d_1^2}{2c^2} \right)^m, \quad Re = \frac{c d_1}{\nu} \left(\frac{d_1^2}{2c^2} \right)^m. \end{aligned}$$

方程(1)自动满足, 在长波和低 Reynolds 假设下, 方程(2) ~ (8)化简为

$$\frac{\partial^2 S_{xy}}{\partial y^2} = 0, \quad (9)$$

$$S_{xx} = 0, \quad (10)$$

$$S_{xy} = \left(\frac{\partial^2 \Psi}{\partial y^2} \right)^{2m+1}, \quad (11)$$

$$S_{yy} = 0, \quad (12)$$

$$\frac{\partial S_{xy}}{\partial y} = \frac{\partial}{\partial x} \left[E_1 \frac{\partial^2}{\partial t^2} + E_2 \frac{\partial}{\partial t} + E_3 \frac{\partial^4}{\partial x^4} - E_4 \frac{\partial^2}{\partial x^2} + E_5 \right] \left\{ \begin{array}{l} \eta_1 \\ \eta_2 \end{array} \right\}, \quad \text{当 } y = \left\{ \begin{array}{l} \eta_1 \\ \eta_2 \end{array} \right\}, \quad (13)$$

$$\frac{\partial \Psi}{\partial y} = 0, \quad \text{当 } y = \pm \left\{ \begin{matrix} \eta_1 \\ \eta_2 \end{matrix} \right\} = \pm \left[\left\{ \begin{matrix} 1 + a \cos 2\pi(x-t) \\ h + b \cos[2\pi(x-t) + \theta] \end{matrix} \right\} \right], \quad (14)$$

其中,省略了字母上的星号, Ψ 为流函数, $a = a_1/d_1$, $b = a_2/d_1$ 且 $h = d_2/d_1$.

2 问题的解

方程(9)~(14)的闭式解为

$$\Psi = \frac{2m+1}{2m+2} Q_1^{1/(2m+1)} \left[\frac{2m+1}{4m+3} \left[y + \frac{\eta_2 - \eta_1}{2} \right]^{(2m+2)/(2m+1)} - y \left(\frac{\eta_1 + \eta_2}{2} \right)^{(2m+2)/(2m+1)} \right], \quad (15)$$

其中

$$Q_1 = 8a\pi^3 \left\{ \left[E_1 - E_4 - 4\pi^2 E_3 - \frac{E_5}{4\pi^2} \right] \sin 2\pi(x-t) + \frac{E_2}{2\pi} \cos 2\pi(x-t) \right\}.$$

纵向速度为

$$u = \frac{\partial \Psi}{\partial y} = \frac{2m+1}{2m+2} Q_1^{1/(2m+1)} \left\{ \left[y + \frac{\eta_2 - \eta_1}{2} \right]^{(2m+2)/(2m+1)} - \left(\frac{\eta_1 + \eta_2}{2} \right)^{(2m+2)/(2m+1)} \right\}. \quad (16)$$

3 结果和讨论

本节利用图形分析纵向速度和流函数.图1~18显示了不同参数的影响.图1~3示出了相位差 θ 对纵向速度 u 的影响.我们发现,与管道上半部相比较,管道下半部的速度减小了.图1~3还分别给出了相位差 θ 增大时,稀释剪切流、Newton 流和浓缩剪切流的纵向速度变化.可是,对所有的流体来说,随着 θ 的增大,速度是增大的.还观察到,与浓缩剪切流(图3)相比较,稀释剪切流的速度(图1)最大值.

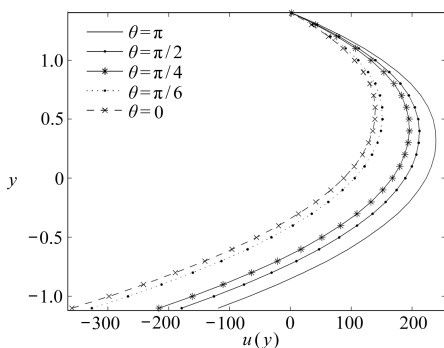


图1 不同相位差 θ 时,纵向速度 u 的变化
(稀释剪切流 ($m = -0.05$), $a = 0.5$,
 $b = 0.3, h = 0.5, E_1 = 0.7, E_2 = 0.5$,
 $E_3 = 0.1, E_4 = 0.3, E_5 = 0.1$,
 $x = 0.3, t = 0.2$)

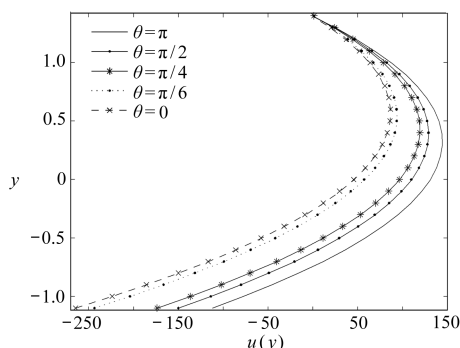


图2 不同相位差 θ 时,纵向速度 u 的变化
(Newton 流 ($m = 0$), $a = 0.5$,
 $b = 0.3, h = 0.5, E_1 = 0.7$,
 $E_2 = 0.5, E_3 = 0.1, E_4 = 0.3$,
 $E_5 = 0.1, x = 0.3, t = 0.2$)

图4~6给出了壁面柔性参数 E_1, E_2, E_3, E_4 和 E_5 的影响.这些图表明,纵向速度随着这些

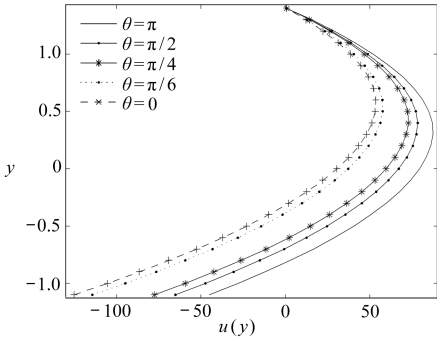


图3 不同相位差 θ 时,纵向速度 u 的变化 (浓缩剪切流 ($m = 0.05$), $a = 0.5$, $b = 0.3, h = 0.5, E_1 = 0.7$, $E_2 = 0.5, E_3 = 0.1, E_4 = 0.3$, $E_5 = 0.1, x = 0.3, t = 0.2$)

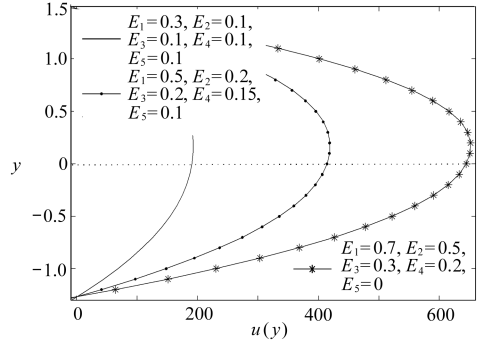


图4 不同壁面参数时,纵向速度 u 的变化 (稀释剪切流 ($m = -0.05$), $\theta = \pi/4, a = 0.5, b = 0.3$, $h = 1, x = 0.3, t = 0.25$)

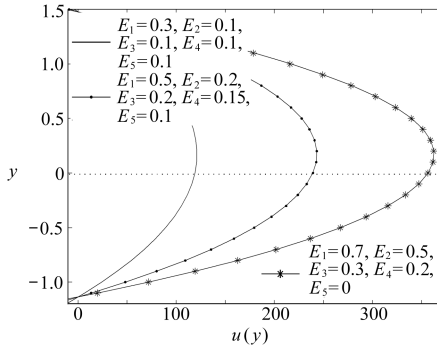


图5 不同壁面参数时,纵向速度 u 的变化 (Newton 流 ($m = 0$), $\theta = \pi/4, a = 0.5, b = 0.3, h = 1$, $x = 0.3, t = 0.25$)

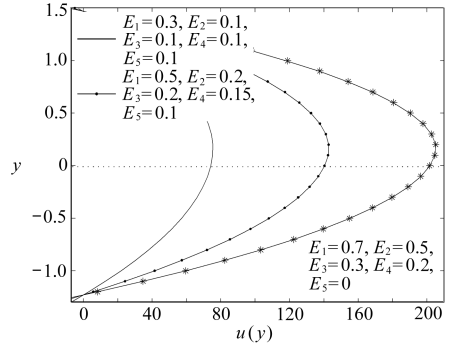


图6 不同壁面参数时,纵向速度 u 的变化 (浓缩剪切流 ($m = 0.05$), $\theta = \pi/4, a = 0.5, b = 0.3, h = 1$, $x = 0.3, t = 0.25$)

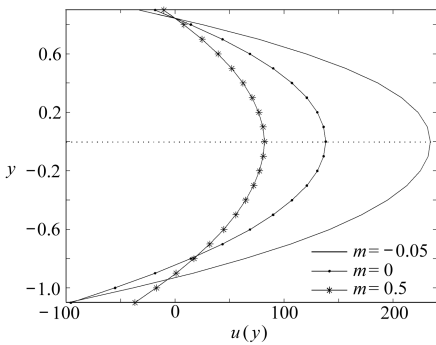


图7 不同幂律指数 m 时,纵向速度 u 的变化 ($\theta = 0, a = 0.5, b = 0.5$, $h = 1, E_1 = 1, E_2 = 0.5, E_3 = 0.1$, $E_4 = 0.3, E_5 = 0.1, x = 0.5, t = 0.2$)

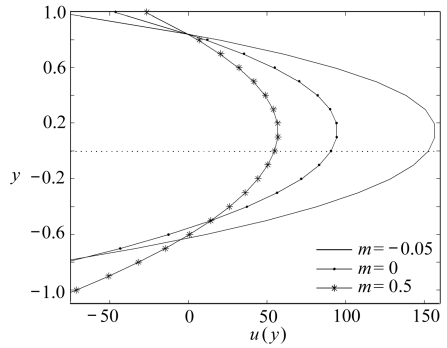


图8 不同幂律指数 m 时,纵向速度 u 的变化 ($\theta = \pi/4, a = 0.5, b = 0.5$, $h = 1, E_1 = 1, E_2 = 0.5, E_3 = 0.1$, $E_4 = 0.3, E_5 = 0.1, x = 0.5, t = 0.2$)

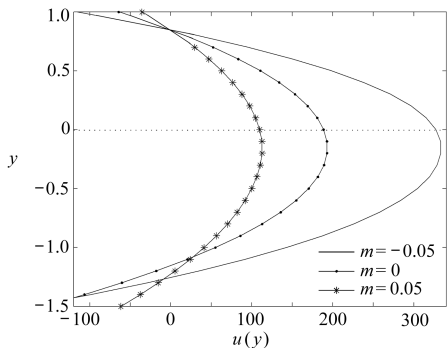


图 9 不同幂律指数 m 时, 纵向速度 u 的变化 ($\theta = \pi, a = 0.5, b = 0.5, h = 1, E_1 = 1, E_2 = 0.5, E_3 = 0.1, E_4 = 0.3, E_5 = 0.1, x = 0.5, t = 0.2$)

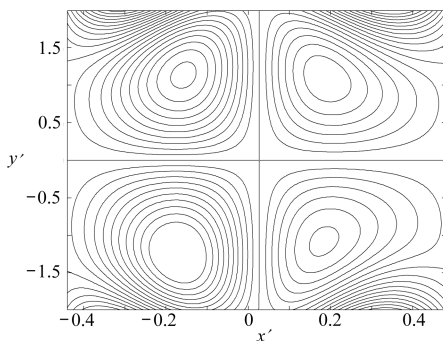


图 10 $m = -0.05$ 时的流线图 (其中 $\theta = \pi/6, a = 0.25, b = 0.25, h = 1, t = 0, E_1 = 0.5, E_2 = 0.05, E_3 = 0.01, E_4 = 0.14, E_5 = 0.5$)

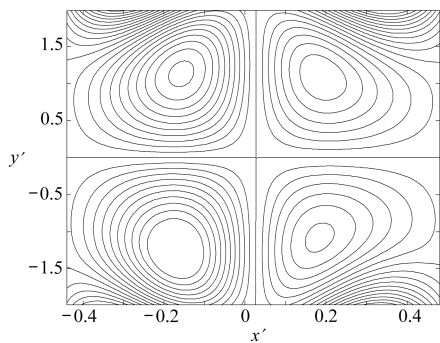


图 11 $m = 0$ 时的流线图 (其中 $\theta = \pi/6, a = 0.25, b = 0.25, h = 1, t = 0, E_1 = 0.5, E_2 = 0.05, E_3 = 0.01, E_4 = 0.14, E_5 = 0.5$)

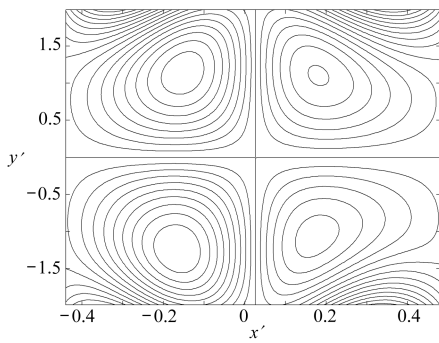


图 12 $m = 0.05$ 时的流线图 (其中 $\theta = \pi/6, a = 0.25, b = 0.25, h = 1, t = 0, E_1 = 0.5, E_2 = 0.05, E_3 = 0.01, E_4 = 0.14, E_5 = 0.5$)

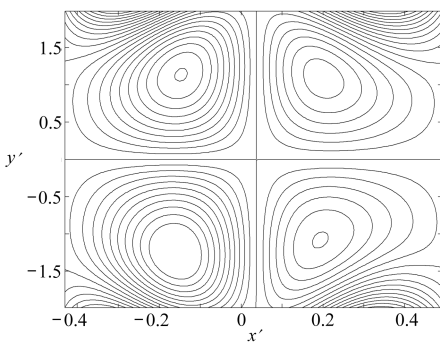


图 13 稀释剪切流 $m = -0.05$ 时的流线图 (其中 $\theta = \pi/6, a = 0.25, b = 0.25, h = 1, t = 0.01, E_1 = 0.44, E_2 = 0.05, E_3 = 0.01, E_4 = 0.08, E_5 = 0.45$)

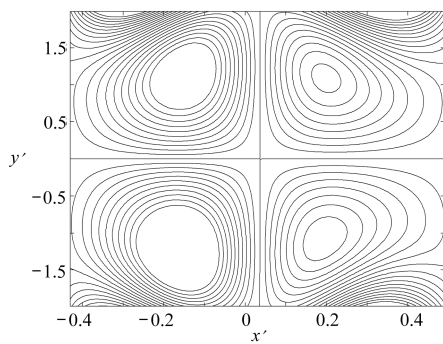


图 14 稀释剪切流 $m = -0.05$ 时的流线图 (其中 $\theta = \pi/6, a = 0.25, b = 0.25, h = 1, t = 0.01, E_1 = 0.47, E_2 = 0.06, E_3 = 0.01, E_4 = 0.12, E_5 = 0.5$)

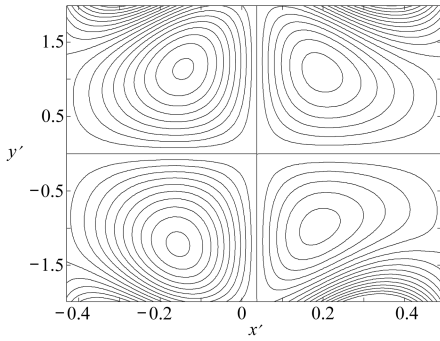


图 15 Newton 流 $m = 0$ 时的流线图
(其中 $\theta = \pi/6, a = 0.25, b = 0.25,$
 $h = 1, t = 0.01, E_1 = 0.44, E_2 = 0.05,$
 $E_3 = 0.01, E_4 = 0.08, E_5 = 0.45$)

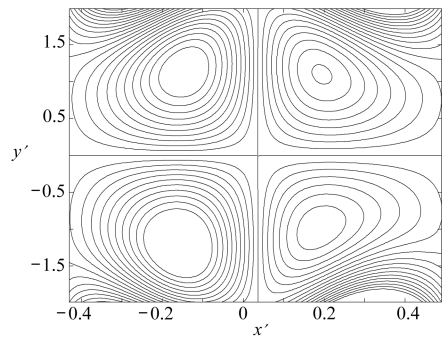


图 16 Newton 流 $m = 0$ 时的流线图
(其中 $\theta = \pi/6, a = 0.25, b = 0.25,$
 $h = 1, t = 0.01, E_1 = 0.47, E_2 = 0.06,$
 $E_3 = 0.01, E_4 = 0.12, E_5 = 0.5$)

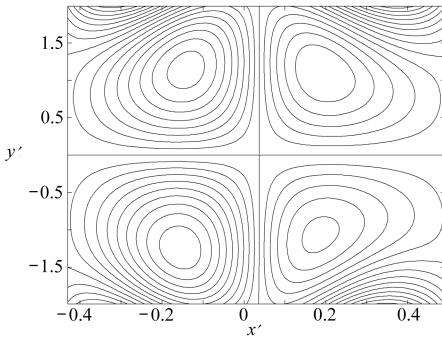


图 17 浓缩剪切流 $m = 0.05$ 时的流线图
(其中 $\theta = \pi/6, a = 0.25, b = 0.25,$
 $h = 1, t = 0.01, E_1 = 0.44, E_2 = 0.05,$
 $E_3 = 0.01, E_4 = 0.08, E_5 = 0.45$)

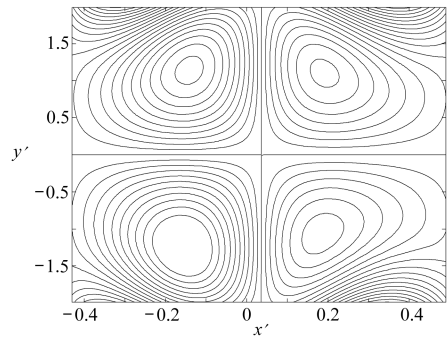


图 18 浓缩剪切流 $m = 0.05$ 时的流线图
(其中 $\theta = \pi/6, a = 0.25, b = 0.25,$
 $h = 1, t = 0.01, E_1 = 0.47, E_2 = 0.06,$
 $E_3 = 0.01, E_4 = 0.12, E_5 = 0.5$)

参数的增大而增大. 与 Newton 流(图 5)相比较, 稀释剪切流(图 4)的速度最大.

图 7~9 示出了其他参数固定时, 幂律指数 m 对纵向速度的影响. 可以看出, 与 Newton 流和浓缩剪切流相比, 稀释剪切流的速度更大. 因此, 与管道的上半部相比, 速度的最大值出现在管道的下半部中. 所有这些图中, 速度曲线呈抛物线性质.

图 10~18 示出了俘获现象. 图 10~12 给出了在其他参数值固定时, 幂律指数 m 对流函数的影响. 这些图形表明, 稀释剪切流(图 10)比 Newton 流和浓缩剪切流(图 11~12), 俘获流线的数量更大.

图 13~18 示出了不同幂律指数 m 时, 壁面柔性参数对流函数的影响. 图 13~14 清楚表明, 对稀释剪切流($m < 0$)来说, 当壁参数 E_1, E_2, E_3, E_4 和 E_5 增大时, 俘获的流线数量也增大.

图 15~16 给出了 Newton 流 ($m = 0$) 的流线图, 有着图 13~14 类似的特点.

图 17~18 是浓缩剪切流 ($m > 0$) 的流线图, 有着图 13~14 类似的特点.

4 结 论

本文在柔性壁的不对称管道内,研究了幂律流体的蠕动流.获得了速度和流函数封闭形式的解.

主要结果为:

- (i) 速度大小随相位差 θ 的增大而增大;
- (ii) 当弹性/柔性壁参数增大时,稀释剪切流体获得速度最大值;
- (iii) 对所有 3 种流体,俘获流线的数量随着弹性参数的增大而增大;
- (iv) 与对称管道相比,不对称管道俘获流线的数量是减少的.

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Exact Solution for Peristaltic Transport of Power-Law Fluid in an Asymmetric Channel With Compliant Walls

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Abstract: Effects of compliant wall properties on the peristaltic flow of a non-Newtonian fluid in an asymmetric channel were investigated. The rheological characteristics were characterized by the constitutive equations of a power-law fluid. Long wavelength and low Reynolds number approximations were adopted in the presentation of mathematical developments. Exact solutions were established for the stream function and velocity. The streamlines pattern and trapping are given due attention. The salient features of the key parameters entering into the present flow are displayed and important conclusions have been pointed out.

Key words: peristaltic flow; power-law fluid; compliant walls