

多孔弹性层的刚性边界对扭转 表面波传播的影响*

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摘要: 根据介质的力学性能,正如 Cowin 及 Nunziato 一样,导出多孔弹性层覆盖在多孔弹性半空间上时,研究其刚性边界对扭转表面波传播的影响.导出了速度方程并对其结果进行了讨论.发现介质中可能存在两类扭转表面波阵面,而 Dey 等(*Tamkang Journal of Science and Engineering*, 2003, 6(4):241-249.)给出的没有刚性边界面时,存在3类扭转表面波阵面.研究还揭示,多孔弹性层中 Love 波也可能随同扭转表面波一起存在.值得注意的是,刚性边界面多孔弹性层中 Love 波的相速度,不同于自由边界面多孔弹性层中的相速度.实际观察到扭转波的色散性,以及速度随着振荡频率的增大而减小.

关键词: 扭转表面波; Love 波; 刚性边界; 多孔; 波阵面

中图分类号: O33;O347.4⁺1 **文献标志码:** A

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符号说明

H	弹性层的厚度	ρ	介质密度
τ_{ij}	应力分量	e_{ij}	应变分量
δ_{ij}	Kronecker δ 符号	λ, μ	Lamé 常数
$\alpha, \beta, \xi, \omega, \bar{k}$	空洞参数	k	波数
ϕ	根据参考体积率兑换的体积率	u, v, w	分别为径向、周向和轴向位移分量
m	波的圆频率	c	扭转波的速度
c_1	弹性层中剪切波的速度	c_2	弹性半空间中剪切波的速度
c_3	弹性层携带的兑换空洞率的波速	\bar{c}_3	弹性半空间携带的兑换空洞率的波速

引言

地震造成的影响是多方面的,有些变化是暂时性的,例如地面的震动、物体的晃动、窗户的卡塔卡塔的响声,以及树的摇摆.永久性的影响包括,建筑物、运输系统、供水系统的破坏,甚至

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本文原文为英文,吴承平译,张禄坤校.

于崩塌。地震经常在震中产生不同类型的地震波。Ewing 等^[1]指出,地震波的传播提供了大量完备的信息。在不同的刊物中,发表了大量的论文,其后出版了地震的书籍。在非均质的地球中,扭转表面波作为一种类型的表面波,正为大家所了解。扭转表面波传播,是偏离水平的振动,并造成介质的扭曲。Dey 等^[2]就非均匀各向异性介质,刚度呈多项式和指数变化,密度为常数时,讨论了扭转表面波的传播。Dey 和 Dutta^[3]首先着重圆柱体中扭转表面波的研究。

有一些专门描述多孔材料力学性能的理论,如大家知道的 Biot 固结理论——流体充满了多孔固体^[4-5]。典型地,当多孔介质中没有流体时,这些理论可简化为经典的弹性理论。这就是为什么 Cowin 和 Nunziato^[6]要提出一种的新理论,可以更加充分地解释空洞没有流体时,均匀弹性材料的动力学本质。当前,很多作者对这种材料^[7-9]提出了一般理论,但仅限于少数混凝土材料问题进行了求解,并没有对这个模型下的实际材料作实质性的评估。带空洞的线弹性材料,是一种由不含任何物体的小孔所组成的弹性材料,又是一种高能有效的弹性材料。这些小孔被称为空洞,若忽略这些小孔的体积,就变成经典的弹性材料。因为经典的弹性理论,并不适用具有内部结构的材料,需要将定向理论和带孔洞的弹性材料相结合。

一般来说,该理论是基于能量的平衡,当假设有空洞存在时,就增加了一个自由度,即单元体积率。因此,块质量密度将由两个场量的乘积得到,一个是空洞体积率,另一个是基体(弹性)材料的质量密度。带空洞弹性材料的线性理论,是一种特殊类型的非线性理论^[10]。在带空洞弹性材料的线性理论中,兑换的空洞体积率和应变是作为独立的运动学变量。材料中分布有不包含任何物质的小空洞,称为多孔材料。

在 Cowin-Nunziato 理论框架下,Puri 和 Cowin^[11]首先在具有空洞的线弹性材料中,研究所有可能传播的平面波类型。接着,Chandrasekharaiah^[12]研究了空洞体积率对 Rayleigh 表面波的影响。众多研究者进行了,不同的波动问题以及基于上述弹性理论振动的研究,并发表在公开的文献中。其中著名的论文有:Parfitt 和 Eringen 的文献[13]、Tomar 和 Gogna 的文献[14]、Singh 和 Kumar 的文献[15]、Midya 的文献[16]、Iesan 和 Nappa 的文献[17]、Golamhossen 的文献[18]、Wright 的文献[19]、Dhaliwal 和 Wang 的文献[20]、Iesan 和 Quintanilla 的文献[21]和 Quintanilla 的文献[22],少数几个文献例外。Dey 等在文献[23]中,研究了扭转表面波在有空洞弹性半空间中的传播,并得出结论:在该介质中可能出现两个波阵面。Dey 等在文献[24]中,对有空洞孔隙的弹性层,覆盖在有空洞孔隙的弹性半空间上时,研究扭转表面波的传播,并得出结论:在该弹性层中可能出现 3 个扭转表面波阵面。他们还指出,除 3 个扭转表面波阵面以外,弹性层中还可能有 Love 波的传播。

地球内有一层非常硬的地层(也称刚性层)存在。由于地球是由非常硬的地层和多孔性介质合成,并且刚性界面在地震波的传播中起着非常重要的作用。考虑到上述因素,本文就具有刚性边界、有限厚(H)多孔弹性层,覆盖在多孔弹性半空间上时,研究扭转表面波的传播。在该问题的处理时,具有空洞空隙弹性基的力学性能沿用 Cowin 和 Nunziato 模型。获得了波的速度方程,并能够观察到两个波阵面的扭转表面波,而在自由界面时,出现 3 个扭转表面波的波阵面。本研究还显示,在多孔弹性层中,Love 波也能够连同扭转表面波一起出现。可以看到,在具有刚性边界面的多孔弹性层中 Love 波的相速度,不同于自由界面多孔弹性层^[24]中的相速度。同时得到了所假定弹性层中 Love 波的速度方程。

1 场方程和本构关系

Cowin 和 Nunziato 在文献[6]中给出了, 不计体力时均匀各向同性多孔弹性介质的运动方程

$$\mu \nabla^2 \mathbf{v} + (\lambda + \mu) \nabla(\nabla \cdot \mathbf{v}) + \beta \nabla \phi = \rho \frac{\partial^2 \mathbf{v}}{\partial t^2}, \quad (1)$$

$$\alpha \nabla^2 \phi - \xi \phi - \omega \frac{\partial \phi}{\partial t} - \beta \nabla \cdot \mathbf{v} = \rho \bar{k} \frac{\partial^2 \phi}{\partial t^2}, \quad (2)$$

其中, λ 和 μ 为 Lamé 参数; $\alpha, \beta, \xi, \omega$ 和 \bar{k} 为空洞参数; $\mathbf{v}(x, t)$ 为位移向量; ϕ 为根据参考体积率兑换的体积率; ρ 为介质密度, t 为时间.

Cowin 和 Nunziato 还在文献[6]中给出了, 均匀各向同性多孔弹性固体介质的本构关系:

$$\tau_{ij} = \lambda \delta_{ij} e_{kk} + 2\mu e_{ij} + \beta \phi \delta_{ij} \quad (i, j = 1, 2, 3), \quad (3)$$

其中, τ_{ij} 为应力张量分量; e_{ij} 为应变张量分量, 如下给出:

$$e_{ij} = \frac{1}{2}(u_{i,j} + u_{j,i}), \quad (4)$$

其中, $u_i = (u, v, w)$ 为位移向量分量, 下角逗号表示对特定坐标的偏导数, 例如

$$u_{i,j} = \frac{\partial u_i}{\partial x_j}. \quad (5)$$

2 上述问题的公式化表示及其求解

考虑在一个多孔弹性半空间上, 覆盖着带刚性边界、厚度为 H 的弹性多孔层(见图1). 柱面坐标系的原点位于弹性多孔层与半空间的界面上, z 轴向下为正; 设 r 和 θ 分别为径向和周向坐标. 扭转波 $u = w = 0$ 和 $v = v(r, z, t)$, 其运动方程为

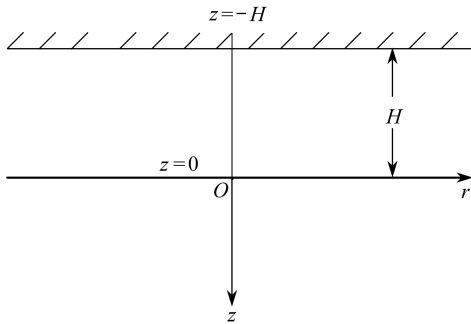


图1 本问题的几何关系

Fig.1 Geometry of the problem

$$\mu \left[\frac{\partial^2 v}{\partial r^2} + \frac{1}{r} \frac{\partial v}{\partial r} + \frac{\partial^2 v}{\partial z^2} \right] + \beta \left[\frac{\partial \phi}{\partial r} + \frac{\partial \phi}{\partial z} \right] = \rho \frac{\partial^2 v}{\partial t^2}, \quad (6)$$

$$\alpha \left[\frac{\partial^2 \phi}{\partial r^2} + \frac{1}{r} \frac{\partial \phi}{\partial r} + \frac{\partial^2 \phi}{\partial z^2} \right] - \omega \frac{\partial \phi}{\partial t} - \xi \phi = \bar{k} \rho \frac{\partial^2 \phi}{\partial t^2}. \quad (7)$$

应用无量纲的径向和深度参数

$$s = \frac{r}{L}, \quad P = \xi_0 + \frac{z}{L}, \quad \xi_0 = \text{const}, \quad (8)$$

其中 $1/L$ 为波数, 则方程(6)和(7)可写为

$$\mu \left[\frac{\partial^2 v}{\partial s^2} + \frac{1}{s} \frac{\partial v}{\partial s} + \frac{\partial^2 v}{\partial P^2} \right] + \beta L \left[\frac{\partial \phi}{\partial s} + \frac{\partial \phi}{\partial P} \right] = \rho L^2 \frac{\partial^2 v}{\partial t^2}, \quad (9)$$

$$\alpha \left[\frac{\partial^2 \phi}{\partial s^2} + \frac{1}{s} \frac{\partial \phi}{\partial s} + \frac{\partial^2 \phi}{\partial P^2} \right] - L^2 \omega \frac{\partial \phi}{\partial t} - L^2 \xi \phi = L^2 \bar{k} \rho \frac{\partial^2 \phi}{\partial t^2}. \quad (10)$$

关于 z 轴对称的正弦波, 方程(9)和(10)的解, 可以表达为

$$V = V_0(P)J_0(s)e^{imt}, \quad (11)$$

$$\phi = \phi_0(P)J_0(s)e^{imt}, \quad (12)$$

其中, J_0 为第一类零阶 Bessel 函数, m 为波的圆频率.

将方程(11)和(12)代入方程(9)和(10), 得到

$$V_0''(P) - (1 - \nu)V_0(P) + \frac{\beta L}{\mu} \left[\phi_0 \frac{J_0'(s)}{J_0(s)} + \phi_0'(P) \right] = 0, \quad (13)$$

$$\phi_0''(P) - A\phi_0(P) = 0, \quad (14)$$

其中撇号表示方程中相应自变量的导数, 且

$$A = 1 + \frac{L^2 \xi}{\alpha} - \frac{\rho L^2 \bar{k} m^2}{\alpha} + \frac{L^2 m \omega i}{\alpha}.$$

对正弦波而言, 阻尼项 ω 非常小, 可以忽略, A 的值可取为

$$A = 1 + \frac{L^2 \xi}{\alpha} - \frac{\rho L^2 \bar{k} m^2}{\alpha}, \quad (15)$$

又

$$\nu = \frac{\rho L^2 m^2}{\mu} = \frac{c^2}{\bar{c}_1^2}, \quad c = Lm, \quad \bar{c}_1 = \sqrt{\frac{\mu}{\rho}} \quad (\text{弹性介质中剪切波的速度}).$$

将方程(15)的 A 代入方程(14), 得到方程(14)的解

$$\phi_0(P) = B e^{\sqrt{A}P} + G e^{-\sqrt{A}P}, \quad (16)$$

利用方程(16), 得到方程(13)的解

$$V_0(P) = R e^{\sqrt{1-\nu}P} + T e^{-\sqrt{1-\nu}P} - B \frac{\beta L \left[\frac{J_0'(s)}{J_0(s)} + \sqrt{A} \right] e^{\sqrt{A}P}}{\mu [D^2 - (1 - \nu)]} - G \frac{\beta L \left[\frac{J_0'(s)}{J_0(s)} - \sqrt{A} \right] e^{-\sqrt{A}P}}{\mu [D^2 - (1 - \nu)]},$$

其中 $D = \frac{d}{dP}.$

情况 1

若 $A - (1 - \nu) = 0,$

$$V_0(P) = R e^{\sqrt{1-\nu}P} + T e^{-\sqrt{1-\nu}P} - B \frac{\beta L \left[\frac{J_0'(s)}{J_0(s)} + \sqrt{A} \right] P e^{\sqrt{A}P}}{2\mu \sqrt{A}} + G \frac{\beta L \left[\frac{J_0'(s)}{J_0(s)} - \sqrt{A} \right] P e^{-\sqrt{A}P}}{2\mu \sqrt{A}}, \quad (17)$$

于是方程(9)和(10)的解为

$$V = \left[R e^{\sqrt{1-\nu}P} + T e^{-\sqrt{1-\nu}P} - B \frac{\beta L \left[\frac{J_0'(s)}{J_0(s)} + \sqrt{A} \right] P e^{\sqrt{A}P}}{2\mu \sqrt{A}} + G \frac{\beta L \left[\frac{J_0'(s)}{J_0(s)} - \sqrt{A} \right] P e^{-\sqrt{A}P}}{2\mu \sqrt{A}} \right] J_0(s) e^{imt}, \quad (18)$$

$$\phi = [Be^{\sqrt{A}P} + Ge^{-\sqrt{A}P}]J_0(s)e^{imt}. \quad (19)$$

情况2

若 $A - (1 - \nu) \neq 0$,

方程(13)的解简化为

$$V_0(P) = Re^{\sqrt{1-\nu}P} + Te^{-\sqrt{1-\nu}P} - B \frac{\beta L \left[\frac{J'_0(s)}{J_0(s)} + \sqrt{A} \right] e^{\sqrt{A}P}}{\mu[A - (1 - \nu)]} - G \frac{\beta L \left[\frac{J'_0(s)}{J_0(s)} - \sqrt{A} \right] e^{-\sqrt{A}P}}{\mu[A - (1 - \nu)]}, \quad (20)$$

因此,方程(9)和(10)的解为

$$V = \left[Re^{\sqrt{1-\nu}P} + Te^{-\sqrt{1-\nu}P} - B \frac{\beta L \left[\frac{J'_0(s)}{J_0(s)} + \sqrt{A} \right] e^{\sqrt{A}P}}{\mu[A - (1 - \nu)]} - G \frac{\beta L \left[\frac{J'_0(s)}{J_0(s)} - \sqrt{A} \right] e^{-\sqrt{A}P}}{\mu[A - (1 - \nu)]} \right] J_0(s)e^{imt}, \quad (21)$$

$$\phi = [Be^{\sqrt{A}P} + Ge^{-\sqrt{A}P}]J_0(s)e^{imt}. \quad (22)$$

上部多孔弹性层的解

上部多孔弹性层的量用下角标1表示,其解为

(i) 当 $A - (1 - \nu) = 0$,

$$V_1 = \left[Re^{\sqrt{1-\nu_1}P} + Te^{-\sqrt{1-\nu_1}P} - B \frac{\beta_1 L \left[\frac{J'_0(s)}{J_0(s)} + \sqrt{A_1} \right] Pe^{\sqrt{A_1}P}}{2\mu_1 \sqrt{A_1}} + G \frac{\beta_1 L \left[\frac{J'_0(s)}{J_0(s)} - \sqrt{A_1} \right] Pe^{-\sqrt{A_1}P}}{2\mu_1 \sqrt{A_1}} \right] J_0(s)e^{imt}. \quad (23)$$

(ii) 当 $A - (1 - \nu) \neq 0$,

$$\bar{V}_1 = \left[Re^{\sqrt{1-\nu_1}P} + Te^{-\sqrt{1-\nu_1}P} - B \frac{\beta_1 L \left[\frac{J'_0(s)}{J_0(s)} + \sqrt{A_1} \right] e^{\sqrt{A_1}P}}{\mu_1 [A_1 - (1 - \nu_1)]} - G \frac{\beta_1 L \left[\frac{J'_0(s)}{J_0(s)} - \sqrt{A_1} \right] e^{-\sqrt{A_1}P}}{\mu_1 [A_1 - (1 - \nu_1)]} \right] J_0(s)e^{imt} \quad (24)$$

以及

$$\phi_1 = [Be^{\sqrt{A_1}P} + Ge^{-\sqrt{A_1}P}]J_0(s)e^{imt}. \quad (25)$$

在(i)和(ii)两种情况下,都有

$$A_1 = 1 + \frac{L^2 \xi_1}{\alpha_1} - \frac{\rho_1 L^2 \bar{k}_1 m^2}{\alpha_1}, \quad \nu_1 = \frac{\rho_1 L^2 m^2}{\mu_1} = \frac{c^2}{c_1^2}, \quad c = Lm, \quad c_1 = \left(\frac{\mu_1}{\rho_1} \right)^{1/2}.$$

弹性半空间的解

弹性半空间的量用下角标2表示,同时隐含着,当 $z \rightarrow \infty$ 时, $\phi \rightarrow 0$ 和当 $z \rightarrow \infty$ 时, $V \rightarrow 0$, 其解为

(i) 当 $A - (1 - \nu) = 0$,

$$V_2 = \left[Ee^{-\sqrt{1-v_2}P} + F \frac{\beta_2 L \left[\frac{J_0'(s)}{J_0(s)} - \sqrt{A_2} \right] P e^{\sqrt{A_2}P}}{2\mu_2 \sqrt{A_2}} \right] J_0(s) e^{imt}. \quad (26)$$

(ii) 当 $A - (1 - v) \neq 0$,

$$\bar{V}_2 = \left[Ee^{-\sqrt{1-v_2}P} - F \frac{\beta_2 L \left[\frac{J_0'(s)}{J_0(s)} - \sqrt{A_2} \right] e^{-\sqrt{A_2}P}}{\mu_2 [A_2 - (1 - v_2)]} \right] J_0(s) e^{imt} \quad (27)$$

以及 $\phi_2 = [F e^{-\sqrt{A_2}P}] J_0(s) e^{imt}. \quad (28)$

在(i)和(ii)两种情况下,都有

$$A_2 = 1 + \frac{L^2 \xi_2}{\alpha_2} - \frac{\rho_2 L^2 \bar{k}_2 m^2}{\alpha_2}, \quad v_2 = \frac{\rho_2 L^2 m^2}{\mu_2} = \frac{c^2}{c_2^2}, \quad c = Lm, \quad c_2 = \left(\frac{\mu_2}{\rho_2} \right)^{1/2}.$$

边界条件

考虑到界面上应力和位移的连续,其相应的边界条件为

在 $z = 0$ 处:

(i) $(\tau_{z\theta})_1 = (\tau_{z\theta})_2,$

(ii) $(\bar{n} \cdot \nabla \phi)_1 = (\bar{n} \cdot \nabla \phi)_2,$

(iii) $(\phi)_1 = (\phi)_2,$

(iv) $(V)_1 = (V)_2,$

又,在刚性边界 $z = -H$ 处,位移为 0,根据参考体积率兑换的体积率不出现,即

在 $z = -H$ 处:

(v) $(V)_1 = 0,$

(vi) $(\phi)_1 = 0.$

情况 1

当 $A_1 - (1 - v_1) = 0$ 和 $A_2 - (1 - v_2) = 0$ 时,在方程(23)、(25)、(26)和(28)中,利用边界条件,得到

$$R \frac{\mu_1 k_1 e^{k_1 \xi_0}}{L} - T \frac{\mu_1 k_1 e^{-k_1 \xi_0}}{L} - B \frac{\beta_1 M z_1}{2 \sqrt{A_1}} + G \frac{\beta_1 N z_2}{2 \sqrt{A_1}} + E \frac{\mu_2 k_2 e^{-k_2 \xi_0}}{L} - F \frac{\beta_2 X z_3}{2 \sqrt{A_2}} = 0, \quad (29)$$

$$B \sqrt{A_1} e^a - G \sqrt{A_1} e^{-a} + F \sqrt{A_2} e^{-b} = 0, \quad (30)$$

$$B e^a + G e^{-a} - F e^{-b} = 0, \quad (31)$$

$$R e^{k_1 \xi_0} + T e^{-k_1 \xi_0} - B \frac{\beta_1 L M \xi_0 e^a}{2\mu_1 \sqrt{A_1}} + G \frac{\beta_1 L N \xi_0 e^{-a}}{2\mu_1 \sqrt{A_1}} - E e^{-k_2 \xi_0} - F \frac{\beta_2 L X \xi_0 e^{-b}}{2\mu_2 \sqrt{A_2}} = 0, \quad (32)$$

$$B \frac{\sqrt{A_1} e^Q}{L} - G \frac{\sqrt{A_1} e^{-Q}}{L} = 0, \quad (33)$$

$$R \frac{\mu_1 k_1 e^{k_1 U}}{L} - T \frac{\mu_1 k_1 e^{-k_1 U}}{L} - B \frac{\beta_1 M z_4}{2 \sqrt{A_1}} + G \frac{\beta_1 N z_5}{2 \sqrt{A_1}} = 0. \quad (34)$$

情况 2

当 $A_1 - (1 - \nu_1) \neq 0$ 和 $A_2 - (1 - \nu_2) \neq 0$ 时,在方程(24)、(25)、(27)和(28)中,利用边界条件,得到

$$R \frac{\mu_1 k_1 e^{k_1 \xi_0}}{L} - T \frac{\mu_1 k_1 e^{-k_1 \xi_0}}{L} - B \frac{\beta_1 M \sqrt{A_1} e^a}{A_1 - (1 - \nu_1)} + G \frac{\beta_1 N \sqrt{A_1} e^{-a}}{A_1 - (1 - \nu_1)} + E \frac{\mu_2 k_2 e^{-k_2 \xi_0}}{L} - F \frac{\beta_2 X \sqrt{A_2} e^{-b}}{A_2 - (1 - \nu_2)} = 0, \quad (35)$$

$$B \sqrt{A_1} e^a - G \sqrt{A_1} e^{-a} + F \sqrt{A_2} e^{-b} = 0, \quad (36)$$

$$B e^a + G e^{-a} - F e^{-b} = 0, \quad (37)$$

$$R e^{k_1 \xi_0} + T e^{-k_1 \xi_0} - B \frac{\beta_1 L M e^a}{\mu_1 [A_1 - (1 - \nu_1)]} - G \frac{\beta_1 L N e^{-a}}{\mu_1 [A_1 - (1 - \nu_1)]} - E e^{-k_2 \xi_0} + F \frac{\beta_2 L X e^{-b}}{\mu_2 [A_2 - (1 - \nu_2)]} = 0, \quad (38)$$

$$B \frac{\sqrt{A_1} e^Q}{L} - G \frac{\sqrt{A_1} e^{-Q}}{L} = 0, \quad (39)$$

$$R \frac{\mu_1 k_1 e^{k_1 U}}{L} - T \frac{\mu_1 k_1 e^{-k_1 U}}{L} - B \frac{\beta_1 M \sqrt{A_1}}{A_1 - (1 - \nu_1)} e^Q + G \frac{\beta_1 N \sqrt{A_1}}{A_1 - (1 - \nu_1)} e^{-Q} = 0, \quad (40)$$

其中

$$M = \frac{J'_0(s)}{J_0(s)} + \sqrt{A_1}, \quad N = \frac{J'_0(s)}{J_0(s)} - \sqrt{A_1}, \quad X = \frac{J'_0(s)}{J_0(s)} - \sqrt{A_2},$$

$$k_1 = \sqrt{1 - \nu_1}, \quad k_2 = \sqrt{1 - \nu_2},$$

$$Z_1 = e^{\sqrt{A_1} \xi_0} + \sqrt{A_1} \xi_0 e^{\sqrt{A_1} \xi_0}, \quad Z_2 = e^{-\sqrt{A_1} \xi_0} - \sqrt{A_1} \xi_0 e^{-\sqrt{A_1} \xi_0},$$

$$Z_3 = e^{-\sqrt{A_2} \xi_0} - \sqrt{A_2} \xi_0 e^{-\sqrt{A_2} \xi_0}, \quad Z_4 = e^{\sqrt{A_1} U} + \sqrt{A_1} U e^{\sqrt{A_1} U},$$

$$Z_5 = e^{-\sqrt{A_1} U} - \sqrt{A_1} U e^{-\sqrt{A_1} U},$$

$$U = \xi_0 - H/L, \quad a = \sqrt{A_1} \xi_0, \quad b = \sqrt{A_2} \xi_0, \quad Q = \sqrt{A_1} U.$$

因此,情况 1 时,消去 R, T, B, G, E, F , 得到速度方程:

$$\begin{vmatrix} \mu_1 k_1 e^{k_1 \xi_0} & -\mu_1 k_1 e^{-k_1 \xi_0} & -\beta_1 M Z_1 & \beta_1 N Z_2 & \mu_2 k_2 e^{-k_2 \xi_0} & -\beta_2 X Z_3 \\ L & L & 2\sqrt{A_1} & 2\sqrt{A_1} & L & 2\sqrt{A_2} \\ 0 & 0 & \sqrt{A_1} e^a & -\sqrt{A_1} e^{-a} & 0 & \sqrt{A_2} e^{-b} \\ 0 & 0 & e^a & e^{-a} & 0 & -e^{-b} \\ e^{k_1 \xi_0} & e^{-k_1 \xi_0} & \frac{-\beta_1 L M \xi_0 e^a}{2\mu_1 \sqrt{A_1}} & \frac{\beta_1 L N \xi_0 e^{-a}}{2\mu_1 \sqrt{A_1}} & -e^{-k_2 \xi_0} & \frac{-\beta_2 L X \xi_0 e^{-b}}{2\mu_2 \sqrt{A_2}} \\ \frac{\mu_1 e^{k_1 U}}{L} & \frac{\mu_1 e^{-k_1 U}}{L} & \frac{-\beta_1 M U e^Q}{2\sqrt{A_1}} & \frac{\beta_1 N U e^{-Q}}{2\sqrt{A_1}} & 0 & 0 \\ 0 & 0 & e^Q & e^{-Q} & 0 & 0 \end{vmatrix} = 0, \quad (41)$$

情况 2 时的速度方程为

$$\left| \begin{array}{ccc}
 \frac{\mu_1 k_1 e^{k_1 \xi_0}}{L} & \frac{-\mu_1 k_1 e^{-k_1 \xi_0}}{L} & \frac{-\beta_1 M \sqrt{A_1} e^a}{A_1 - (1 - v_1)} \\
 0 & 0 & \sqrt{A_1} e^a \\
 0 & 0 & e^a \\
 e^{k_1 \xi_0} & e^{-k_1 \xi_0} & \frac{-\beta_1 L M e^a}{\mu_1 [A_1 - (1 - v_1)]} \\
 \frac{\mu_1 e^{k_1 U}}{L} & \frac{\mu_1 e^{-k_1 U}}{L} & \frac{-\beta_1 M e^Q}{A_1 - (1 - v_1)} \\
 0 & 0 & e^Q \\
 \\
 \frac{\beta_1 N \sqrt{A_1} e^{-a}}{A_1 - (1 - v_1)} & \frac{\mu_2 k_2 e^{-k_2 \xi_0}}{L} & \frac{-\beta_2 X \sqrt{A_2} e^{-b}}{A_2 - (1 - v_2)} \\
 -\sqrt{A_1} e^{-a} & 0 & \sqrt{A_2} e^{-b} \\
 e^{-a} & 0 & -e^{-b} \\
 \frac{-\beta_1 L N e^{-a}}{\mu_1 [A_1 - (1 - v_1)]} & -e^{-k_2 \xi_0} & \frac{\beta_2 L X e^{-b}}{\mu_2 [A_2 - (1 - v_2)]} \\
 \frac{-\beta_1 N e^{-Q}}{A_1 - (1 - v_1)} & 0 & 0 \\
 e^{-Q} & 0 & 0
 \end{array} \right| = 0. \quad (42)$$

经简化,由方程(41)和(42),得到一个共同方程:

$$\frac{\mu_1}{L^2} e^{-[\sqrt{A_2} + k_2] \xi_0} \left[\left(\frac{\mu_2 k_2}{\mu_1 k_1} \right) + \coth \left(\frac{k_1 H}{L} \right) \right] \left[\coth \left(\frac{\sqrt{A_1} H}{L} \right) + \sqrt{\frac{A_2}{A_1}} \right] = 0. \quad (43)$$

对于情况 1 和情况 2,由下列方程:

$$(i) \coth \left(\frac{\sqrt{A_1} H}{L} \right) + \sqrt{\frac{A_2}{A_1}} = 0, \quad (44)$$

$$(ii) \frac{\mu_2 k_2}{\mu_1 k_1} + \coth \left(\frac{k_1 H}{L} \right) = 0, \quad (45)$$

在下列条件下:

$$A_1 - (1 - v_1) = 0, \quad (46)$$

$$A_2 - (1 - v_2) = 0, \quad (47)$$

$$A_1 - (1 - v_1) \neq 0, \quad (48)$$

$$A_2 - (1 - v_2) \neq 0, \quad (49)$$

得到扭转表面波的速度.由方程(44)得

$$\cot \left[\left(\frac{c^2}{c_3^2} - 1 - \frac{1}{(km_1)^2} \right)^{1/2} kH \right] = \left[1 - \left(\frac{c^2}{c_3^2} \right) q + \frac{1}{(km_2)^2} \right]^{1/2} / \left[\frac{c^2}{c_3^2} - 1 - \frac{1}{(km_1)^2} \right]^{1/2}, \quad (50)$$

其中, $m_1 = (\alpha_1 / \xi_1)^{1/2}$ 和 $m_2 = (\alpha_2 / \xi_2)^{1/2}$ 是位移参数; $c_3 = (\alpha_1 / (\rho_1 \bar{k}_1))^{1/2}$ 为上部弹性层携带

的兑换空洞率的波速; $\bar{c}_3 = (\alpha_2/(\rho_2\bar{k}_2))^{1/2}$ 为弹性半空间携带的兑换空洞率的波速; $q = (c_3/\bar{c}_3)^2$; 以及 $k = 1/L$ 为波数. 假定多孔弹性层有着刚性的上边界面, 并覆盖在多孔弹性半空间上, 则方程(50)给出了 Love 波的相速度.

Dey 等在文献[24]中给出了 Love 型波的色散方程(51), 是针对没有刚性边界的多孔弹性层覆盖的多孔弹性半空间:

$$\tan \left[\left(\frac{c^2}{c_3^2} - 1 - \frac{1}{(km_1)^2} \right)^{1/2} kH \right] = \left[1 - \left(\frac{c^2}{c_3^2} \right) q + \frac{1}{(km_2)^2} \right]^{1/2} / \left[\left(\frac{c^2}{c_3^2} - 1 - \frac{1}{(km_1)^2} \right) \right]^{1/2}. \quad (51)$$

现由方程(45), 得到

$$\cot \left[\left(\frac{c^2}{c_1^2} - 1 \right)^{1/2} kH \right] = \mu_2 \left(1 - \frac{c^2}{c_2^2} \right)^{1/2} / \left(\mu_1 \left(\frac{c^2}{c_1^2} - 1 \right)^{1/2} \right). \quad (52)$$

这就是大家熟知的 Love 波方程, 是针对均匀各向同性弹性层覆盖在均匀各向同性弹性半空间上.

由方程(46), 得到

$$c = \left[1 / \left((km_1)^2 \left(\frac{1}{c_3^2} - \frac{1}{c_1^2} \right) \right) \right]^{1/2}. \quad (53)$$

可以理解为多孔弹性层覆盖在多孔弹性半空间时, 第一类扭转波的速度, 假定多孔弹性层的上边界面是刚性的.

由方程(47), 得到

$$c = \left[1 / \left((km_2)^2 \left(\frac{1}{c_3^2} - \frac{1}{c_2^2} \right) \right) \right]^{1/2}. \quad (54)$$

可以理解为多孔弹性层覆盖在多孔弹性半空间时, 第二类扭转表面波的速度, 假定多孔弹性层的上边界面是刚性的.

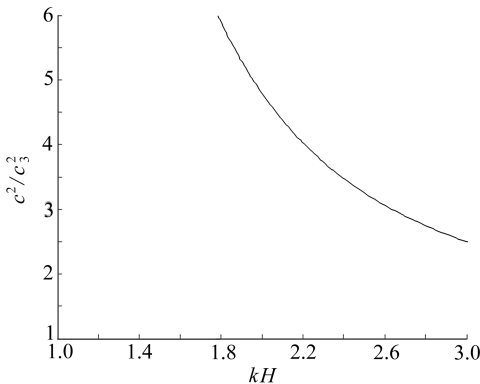


图2 当参数 $km_1 = 4$, $km_2 = 0.4$, $q = 0.5$, 刚性上边界时, 下部多孔弹性介质中 Love 波的色散曲线

Fig. 2 Love wave dispersion curve in a porous elastic medium under rigid boundary for a set of parameters $km_1 = 4$, $km_2 = 0.4$, $q = 0.5$

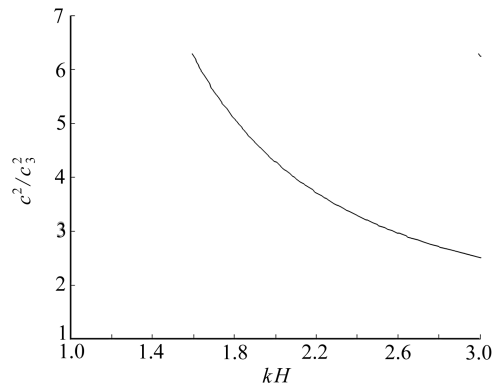


图3 当参数 $km_1 = 2$, $km_2 = 0.2$, $q = 0.3$, 刚性上边界时, 下部多孔弹性介质中 Love 波的色散曲线

Fig. 3 Love wave dispersion curve in a porous elastic medium under rigid boundary for a second set of parameters $km_1 = 2$, $km_2 = 0.2$, $q = 0.3$

3 数值计算

对两组 km_1, km_2, q 参数值,按 kH 的不同取值,由方程(50)数值地计算 c^2/c_3^2 值.计算结果示于图2和图3.由图可以看出,在低 kH 值时,Love 波阵面速度依赖于兑换的空洞空隙的体积,多数时间里,远高于 Love 波携带空隙的空洞率时的速度.根据方程(53)和(54)分别得到,两个扭转表面波阵面速度的数值结果,分别如图4和图5所示.实质观察到波的色散性,同时,波速随着振动频率的增大而减小.

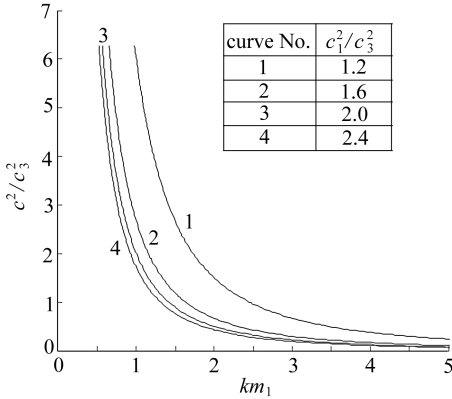


图4 第一类扭转表面波的色散曲线

Fig.4 Dispersion curve for the torsional surface wave of the first kind

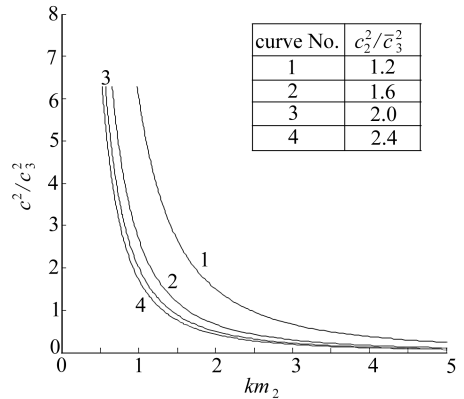


图5 第二类扭转表面波的色散曲线

Fig.5 Dispersion curve for the torsional surface wave of the second kind

4 结 论

(i) 刚性边界对扭转表面波传播的主要影响是,仅存在2个扭转表面波阵面,而没有刚性边界时^[24],则存在3个扭转表面波阵面.

(ii) 除2个扭转表面波阵面外,Love波也可在弹性层中传播.由方程(50)和(51)可以看到,刚性边界面弹性层中Love波的相速度,与自由边界面弹性层中的相速度是不相同的.因此,扭转表面波在此介质中的实际传播是相当复杂的,多重原因影响着位移的传播,既由于所有的波阵面,也由于Love波阵面.

(iii) 在低 kH 值时,Love波阵面速度依赖于兑换的空洞空隙的体积,多数时间里,远高于携带空隙的空洞体积率时的波速.

(iv) 实际观察到的扭转波是色散的,并且随着振荡频率的增大,扭转速度是减小的.

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Effect of Rigid Boundary on Propagation of Torsional Surface Waves in Porous Elastic Layer

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Abstract: The effect of rigid boundary on the propagation of torsional surface waves in a porous elastic layer over a porous elastic half space was presented using the mechanics of the medium as derived by Cowin and Nunziato. The velocity equation was derived and the results were discussed. It is observed that there may be two torsional surface wave fronts in the medium whereas there exists three wave fronts of torsional surface waves in the absence of rigid boundary plane given by Dey *et al*(*Tamkang Journal of Science and Engineering*, 2003, 6(4): 241-249.). The results also reveals that in the porous layer, the Love wave is also available along with the torsional surface waves. It is remarkable that phase speed of Love wave in a porous layer with rigid surface is different from that in a porous layer with a free surface. The torsional waves are observed to be dispersive in nature, and the velocity decreases as the frequency of oscillation increases.

Key words: torsional surface waves; Love wave; rigid boundary; porous; wave front