文章编号:1000-0887(2015)09-1003-08

非线性扰动广义 NNV 系统的孤立子渐近行波解*

史娟荣[1,4], 吴钦宽[2], 莫嘉琪[3]

(1. 安徽机电职业技术学院, 安徽 芜湖 241002;
2. 南京工程学院 数理系, 南京 211167;
3. 安徽师范大学 数学系, 安徽 芜湖 241003;
4. 上海交通大学 数学系, 上海 200240)

摘要: 采用了一个简单而有效的技巧,研究一类非线性扰动广义 NNV(Nizhnik-Novikov-Veselov)系统.首先用待定系数法得到一个相应典型系统的孤立子解.其次构造一个广义泛函式,并对它进行变分计算,利用变分原理求出对应的 Lagrange 乘子,并由此构造一个特殊的变分迭代关系式.然后依次求出原非线性扰动广义 NNV 系统的孤立子渐近行波解.最后通过举例,说明了使用该方法得到的近似解具有简单而有效的优点.

关 键 词: 孤立子; 扰动 NNV 系统; 迭代方法
中图分类号: O175.29 **文献标志码:** A
doi: 10.3879/j.issn.1000-0887.2015.09.011

引 言

孤立子在流体力学、场论、光波散射、激波、神经网络和量子力学中都有许多应用[1-8].研究孤立子解已有许多新的方法,例如齐次平衡法、辅助方程法、双曲函数法、符号计算代数法、Jacobi 椭圆函数展开法 、Riccati 函数法和 (G'/G) 展开法[9-14]等.目前,求解一类非线性问题的方法不断改进[15].近来许多学者,例如 Hovhannisyan 和 Vulanovic[16],Barbu 和 Cosma[17]以及 Ramos[18]讨论了有关非线性问题.莫嘉琪等也研究了一类非线性问题的孤立子、激波、激光脉冲和大气物理等问题[19-29].本文讨论的是近代物理的一个非线性扰动 NNV 系统,利用简单而有效的迭代方法得到了相应系统的孤立子渐近行波解.

1 扰动 NNV 系统

考虑如下一个非线性扰动广义 NNV 系统[15]:

$$u_t + u_{xxx} - 3v_x u - 3vu_x = f(u,v), \tag{1}$$

* **收稿日期:** 2015-03-05; **修订日期:** 2015-06-15
基金项目: 国家自然科学基金(11202106); 安徽高校自然科学研究重点项目(KJ2015A418; KJ2015A347)
作者简介: 史娟荣(1981—), 女,安徽宣城人, 副教授, 硕士(E-mail: ahjdshjr@126.com);
莫嘉琪(1937—),男,浙江德清人,教授(通讯作者. E-mail: mojiaqi@mail.ahnu.edu.cn).

$$u_x - v_y = g(u,v), \tag{2}$$

其中f,g为扰动项,是关于其变量在相应的区域内为充分光滑的函数.在理论物理等学科中许多相关问题均涉及到系统(1)、(2).

作行波变换$s = ax + by - ct$,其中a,b,c为常数.得到关于s的微分系统:

$$a^3 u_{sss} - cu_s - 3av_s u - 3avu_s = f(u,v), \tag{3}$$

$$au_s - bv_s = g(u,v). \tag{4}$$

对应于系统(3)、(4)中扰动项$f(u,v)$,$g(u,v) = 0$的情形,可得

$$a^3 u_{ss} - cu - \frac{3a^2}{b} u^2 = C_1,$$

$$v = \frac{a}{b} u + C_2,$$

其中C_1,C_2为常数,不妨设它们为0.故有

$$a^3 u_{ss} - cu - \frac{3a^2}{b} u^2 = 0, \tag{5}$$

$$v = \frac{a}{b} u. \tag{6}$$

现寻找方程(5)具有如下形式的解:

$$u = A_1 w + A_2 w^2 + B_0 (1 - w^2)^{1/2}, \tag{7}$$

其中A_1,A_2,B_0为待定常数,而w是满足微分方程

$$w_s = - w(1 - w^2)^{1/2} \tag{8}$$

的解.不难知道,方程(8)有解

$$w = \operatorname{sech} s. \tag{9}$$

由式(7)得

$$u_s = - (A_1 w + 2A_2 w^2)(1 - w^2)^{1/2} + B_0 w^2, \tag{10}$$

$$u_{ss} = - (A_1 w + 2A_2 w^2) w^2 + (A_1 w + 4A_2 w^2)(1 - w^2) - 2B_0 w^2 (1 - w^2)^{1/2}. \tag{11}$$

将式(7)、(10)、(11)代入式(5)得

$$\begin{aligned} & - a^3 [(A_1 w + 2A_2 w^2) w^2 + (A_1 w + 4A_2 w^2)(1 - w^2) - 2B_0 w^2 (1 - w^2)^{1/2}] - \\ & \quad c[A_1 w + A_2 w^2 + B_0 (1 - w^2)^{1/2}] - \\ & \quad \frac{3a^2}{b} [A_1 w + A_2 w^2 + B_0 (1 - w^2)^{1/2}]^2 = 0. \end{aligned} \tag{12}$$

合并式(12)等号左端w^i,$w^i(1 - w^2)^{1/2}$同次幂项,并令其系数为0,可得

$$c = 4a^3,\ A_1 = B_0 = 0,\ A_2 = - \frac{2ab}{3}.$$

因此可选定

$$A_1 = 0,\ A_2 = - \frac{2ab}{3},\ B_0 = 0,\ c = 4a^3. \tag{13}$$

由式(7)、(13),得到系统(5)、(6)的一组精确孤立子解:

$$u(s) = - \frac{2ab \operatorname{sech}^2 s}{3},\ s = ax + by - 4a^3 t. \tag{14}$$

$$v(s) = - \frac{2a^2 \operatorname{sech}^2 s}{3},\ s = ax + by - 4a^3 t. \tag{15}$$

在式(14)、(15)中,取 $a=1, b=2$,孤立子解$(u(s), v(s))$的曲线如图1,2所示.

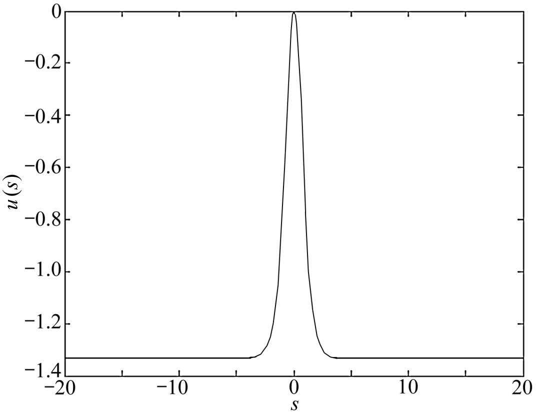

图1 孤立子 $u(s)$ 的曲线 $(a=1, b=2)$

Fig. 1 The curve of solitary wave $u(s)$ $(a=1, b=2)$

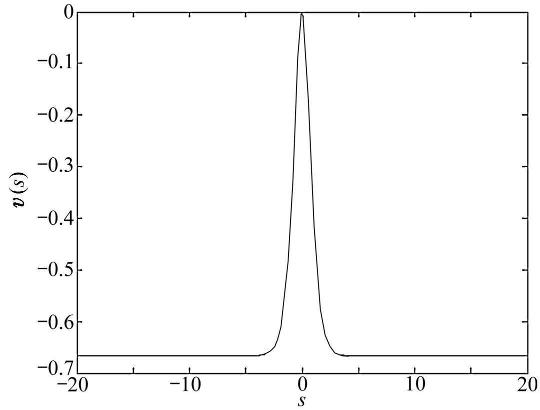

图2 孤立子 $v(s)$ 的曲线 $(a=1, b=2)$

Fig. 2 The curve of solitary wave $v(s)$ $(a=1, b=2)$

2 扰动 NNV 系统渐近解

由于式(1)、(2)为非线性系统,一般不能求得有限项初等函数的精确解.为此我们需要构造其近似解.

为了得到系统(1)、(2)的孤立子近似解析解,首先引入一组泛函 $F_i(u,v)(i=1,2)$:

$$F_1[u,v]=u-\int_{-\infty}^{s}\lambda_1(r)[a^3u_{rrr}-cu_r-3a\bar{v}_r\bar{u}-3a\bar{v}\bar{u}_r-f(\bar{u},\bar{v})]\mathrm{d}r, \quad (16)$$

$$F_2[u,v]=v-\int_{-\infty}^{s}\lambda_2(r)[a\bar{u}_r-bv_r-g(\bar{u},\bar{v})]\mathrm{d}r, \quad (17)$$

其中 $\bar{u},\bar{v}$ 为 u,v 的限制变量[15],而 λ_i 为 Lagrange 乘子.

计算泛函式(16)、(17)的广义变分 $\delta F_i(i=1,2)$:

$$\delta F_1=\delta u-[\lambda_1(a^3\delta u_{rr}-c\delta u_r)]|_{r=s}+[\lambda_{1r}a^3\delta u_r]|_{r=s}-[\lambda_{1rr}a^3\delta u]_{r=s}+\int_{-\infty}^{s}[a^3\lambda_{1rrr}-c\lambda_1]\delta u\mathrm{d}r,$$

$$\delta F_2=\delta v-[\lambda_2\delta v]|_{r=s}-\int_{-\infty}^{s}\lambda_{2r}\delta v\mathrm{d}r.$$

令 $\delta F_i=0(i=1,2)$, 得

$$a^3\lambda_{1rrr}-c\lambda_1=0,\ \lambda_1|_{\eta=s}=\lambda_r|_{r=s}=0,\ \lambda_{rr}|_{\eta=r}=\frac{1}{a^3}, \quad (18)$$

$$\lambda_{2r}=0,\ \lambda_2|_{r=s}=1. \quad (19)$$

由式(18)、(19)得

$$\lambda_1=\frac{2}{(1-\sqrt{3})a\sqrt[3]{c^2}}\left[\exp\left(\frac{\sqrt[3]{c}}{a}(r-s)\right)+\exp\left[\left(-\frac{\sqrt{3}\sqrt[3]{c}}{2a}(r-s)\right)\left(-\cos\frac{\sqrt{3}\sqrt[3]{c}}{2a}+\sin\frac{\sqrt{3}\sqrt[3]{c}}{2a}\right)\right]\right], \quad (20)$$

$$\lambda_2=1. \quad (21)$$

由式(16)、(17)、(20)、(21),构造如下迭代关系式:

$$u_{n+1}(s) = u_n(s) - \frac{2}{(1-\sqrt{3})a\sqrt[3]{c^2}}\int_{-\infty}^{s}\left[\exp\left(\frac{\sqrt[3]{c}}{a}(r-s)\right) + \right.$$
$$\left.\exp\left[\left(-\frac{\sqrt{3}\sqrt[3]{c}}{2a}(r-s)\right)\left(-\cos\frac{\sqrt{3}\sqrt[3]{c}}{2a} + \sin\frac{\sqrt{3}\sqrt[3]{c}}{2a}\right)\right]\right] \times$$
$$[a^3 u_{nrrr} - cu_{nr} - 3av_{nr}u_n - 3av_n u_{nr} - f(u_n, v_n)]\mathrm{d}r, \tag{22}$$

$$v_{n+1} = v_n - \int_{-\infty}^{s}[au_{nr} - bv_{nr} - g(u_n, v_n)]\mathrm{d}r. \tag{23}$$

选取初始迭代 $u_0(z)$ 为系统(5)、(6)的一组孤立子解(14)、(15),即

$$u_0(s) = -\frac{2ab\mathrm{sech}^2 s}{3}, \tag{24}$$

$$v_0(s) = -\frac{2a^2\mathrm{sech}^2 s}{3}. \tag{25}$$

再由迭代关系式(22)、(23),可得到一组序列 $\{u_n(s), v_n(s)\}$.由泛函分析不动点原理可以证明 $\lim_{n\to\infty} u_n$ 及 $\lim_{n\to\infty} v_n$ 在 $s \in [-M, +M]$ 上一致地成立,其中 M 为任意大的正常数.再由迭代式(22)、(23),不难知道 $u(s) = \lim_{n\to\infty} u_n(s)$, $v(s) = \lim_{n\to\infty} v_n(s)$ 就是微分系统(3)、(4)的一组孤立子精确解.而 $u(ax + by - 4a^3t)$,$v(ax + by - 4a^3t)$ 为非线性广义扰动 NNV 系统(1)、(2)的一组孤立子行波精确解.且 $u_n(ax + by - 4a^3t)$,$v_n(ax + by - 4a^3t)$ 为非线性广义扰动 NNV 系统(1)、(2)的一组 n 次孤立子渐近行波解,由变分原理知,得到的渐近解具有较高的近似度.

3 举　　例

讨论一个特殊的广义 NNV 系统(1)、(2).它的扰动项为 $f = \sin v$, $g = \cos u$,而 $a = b = 1$.这时系统(1)、(2)为

$$u_t + u_{xxx} - 3v_x u - 3vu_x = \sin v, \tag{26}$$
$$u_x - v_y = \cos u. \tag{27}$$

在行波变换 $s = x + y - 4t$ 下,系统(3)、(4)对应为

$$u_{sss} - 4u_s - 3v_s u - 3vu_s = \sin v, \tag{28}$$
$$u_s - v_s = \cos u. \tag{29}$$

现求系统(28)、(29)的一组孤立子渐近解 $u_n(s)$,$v_n(s)$.利用迭代方法,由式(24)、(25),取零次孤立子渐近解 u_0,v_0 为

$$u_0(s) = -\frac{2}{3}\mathrm{sech}^2 s, \tag{30}$$

$$v_0(s) = -\frac{2}{3}\mathrm{sech}^2 s. \tag{31}$$

再由关系式(22)、(23),可得一组 1 次孤立子渐近解 u_1,v_1:

$$u_1(s) = -\frac{2}{3}\mathrm{sech}^2 s - \frac{2}{2(1-\sqrt{3})\sqrt[3]{4}}\int_{-\infty}^{s}\left[\exp(\sqrt[3]{4}(r-s)) + \right.$$
$$\left.\exp\left[\left(-\frac{\sqrt[3]{36}}{2}(r-s)\right)\left(-\cos\frac{\sqrt[3]{36}}{2} + \sin\frac{\sqrt[3]{36}}{2}\right)\right]\right] \times$$
$$[-3v_{0r}u_0 - 3v_0 u_{0r} - \sin u_0]\mathrm{d}r, \tag{32}$$

$$v_1(s) = -\frac{2}{3}\operatorname{sech}^2 s + \int_{-\infty}^{s} \cos v_0 \mathrm{d}r, \tag{33}$$

其中 u_0, v_0 分别由式(30)、(31)表示.再由迭代式(22)、(23), 可得一组 2 次孤立子渐近解 u_2, v_2:

$$u_2(s) = u_1(s) - \frac{2}{(1-\sqrt{3})\sqrt[3]{4}}\int_{-\infty}^{s}\left[\exp(\sqrt[3]{4}(r-s)) + \exp\left[\left(-\frac{\sqrt[3]{36}}{2}(r-s)\right)\left(-\cos\frac{\sqrt[3]{36}}{2} + \sin\frac{\sqrt{3}\sqrt[3]{4}}{2}\right)\right]\right] \times [u_{1rrr} - 4u_{1r} - 3v_{1r}u_1 - 3v_1u_{1r} - \sin v_1]\mathrm{d}r, \tag{34}$$

$$v_2(s) = v_1 - \int_{-\infty}^{s}[u_{1r} - v_{1r} - \cos u_1]\mathrm{d}r, \tag{35}$$

其中 u_1, v_1 分别由式(32)、(33)表示.

将行波变换 $s = x + y - 4t$ 代入式(30)~(35),便分别得到广义扰动 NNV 系统(26)、(27)的一组 0 次、1 次孤立子渐近行波解:

$$u_0(x,y,t) = -\frac{2}{3}\operatorname{sech}^2(x+y-4t), \tag{36}$$

$$v_0(x,y,t) = -\frac{2}{3}\operatorname{sech}^2(x+y-4t), \tag{37}$$

$$u_1(x,y,t) = -\frac{2}{3}\operatorname{sech}^2(x+y-4t) - \frac{2}{2(1-\sqrt{3})\sqrt[3]{4}}\int_{-\infty}^{x+y+4t}\left[\exp[\sqrt[3]{4}(r-(x+y-4t))] + \exp\left[\left(-\frac{\sqrt[3]{36}}{2}(r-(x+y-4t))\right)\left(-\cos\frac{\sqrt[3]{36}}{2} + \sin\frac{\sqrt[3]{36}}{2}\right)\right]\right] \times [-3v_{0r}u_0 - 3v_0u_{0r} - \sin u_0]\mathrm{d}r, \tag{38}$$

$$v_1(x,y,t) = -\frac{2}{3}\operatorname{sech}^2(x+y-4t) + \int_{-\infty}^{x+y+4t}\cos v_0 \mathrm{d}r, \tag{39}$$

其中 $u_0 = u_0(s)$, $v_0 = v_0(s)$ 分别由式(30)、(31)表示.再由迭代式(22)、(23),可得一组 2 次孤立子渐近行波解 u_2, v_2:

$$u_2(x,y,t) = u_1(x,y,t) - \frac{2}{(1-\sqrt{3})\sqrt[3]{4}}\int_{-\infty}^{x+y+4t}\left[\exp[\sqrt[3]{4}(r-(x+y-4t))] + \exp\left[\left(-\frac{\sqrt[3]{36}}{2}(r-(x+y-4t))\right)\left(-\cos\frac{\sqrt[3]{36}}{2} + \sin\frac{\sqrt{3}\sqrt[3]{4}}{2}\right)\right]\right] \times [u_{1rrr} - 4u_{1r} - 3v_{1r}u_1 - 3v_1u_{1r} - \sin v_1]\mathrm{d}r,$$

$$v_2(x,y,t) = v_1(x,y,t) - \int_{-\infty}^{x+y+4t}[u_{1r} - v_{1r} - \cos u_1]\mathrm{d}r,$$

其中 $u_1 = u_1(s)$, $v_1 = v_1(s)$ 分别由式(32)、(33)表示.

继续利用迭代式(22)、(23),可以得到非线性扰动广义 NNV 系统(26)、(27)的一组更高次孤立子渐近行波解 $u_n(x+y-4t)$, $v_n(x+y-4t)$ $(n = 3,4,\cdots)$.

4 结束语

用迭代方法求解广义扰动 NNV 系统的孤立子渐近行波解是一个简单而有效的方法**.**由本方法得到的渐近解不是离散型的数值解**.**它还能继续进行解析运算,作相应的定性和定量方面的分析**.**同时,本文选取的一组初始近似解 (u_0, v_0) 是采用非扰动情形下典型系统的孤立子解,由于变分原理,它保证了非线性扰动广义 NNV 系统(1)、(2)具有较快地求得对应孤立子在要求的精度范围内的渐近解**.**

参考文献(References):

[1] McPhaden M J, Zhang D. Slowdown of the meridional overturning circulation in the upper Pacific Pcean[J]. *Nature*, 2002, **415**(6872): 603-608.

[2] GU Dai-fang, Philander S G H. Interdecadal climate fluctuations that depend on exchanges between the tropics and extratropics[J]. *Science*, 1997, **275**(7): 805-807.

[3] 马松华, 强继业, 方建平. (2+1)维 Boiti-Leon-Pempinelli 系统的混沌行为及孤立子间的相互作用[J]. 物理学报, 2007, **56**(2): 620-626.(MA Song-hua, QIANG Ji-ye, FANG Jian-ping. The interaction between solitons and chaotic behaviours of (2+1)-dimensional Boiti-Leon-Pempinelli system[J]. *Acta Physica Sinica*, 2007, **56**(2): 620-626.(in Chinese))

[4] Loutsenko I. The variable coefficient Hele-Shaw problem, integrability and quadrature identities[J]. *Communications in Mathematical Physics*, 2006, **268**(2): 465-479.

[5] Gedalin M. Low-frequency nonlinear stationary waves and fast shocks: hydrodynamical description[J]. *Physics of Plasmas*, 1998, **5**(1): 127-132.

[6] Parkes E J. Some periodic and solitary travelling-wave solutions of the short-pulse equation [J]. *Chaos Solitons Fractals*, 2008, **38**(1): 154-159.

[7] 潘留仙, 左伟明, 颜家壬. Landau-Ginzburg-Higgs 方程的微扰理论[J]. 物理学报, 2005, **54**(1): 1-5.(PAN Liu-xian, ZUO Wei-ming, YAN Jia-ren. The theory of the perturbation for Landau-Ginzburg-Higgs equation[J]. *Acta Physica Sinica*, 2005, **54**(1): 1-5.(in Chinese))

[8] 封国林, 戴新刚, 王爱慧, 丑纪范. 混沌系统中可预报性的研究[J]. 物理学报, 2001, **50**(4): 606-611.(FENG Guo-lin, DAI Xin-gang, WANG Ai-hui , CHOU Ji-fan. On the numerical predictability in the chaos system[J]. *Acta Physica Sinica*, 2001, **50**(4): 606-611.(in Chinese))

[9] WENG Ming-liang. Solitary wave solutions for variant Boussinesq equations[J]. *Physics Letters A*, 1995, **199**(3/4): 169-172.

[10] 吴国将, 韩家骅, 史良马, 张苗. 一般变换下双 Jacobi 椭圆函数展开法及应用[J]. 物理学报, 2006, **55**(8): 3858-3863.(WU Guo-jiang, HAN Jia-hua, SHI Liang-ma, ZHANG Miao. Double Jacobian elliptic function expansion method under a general function transform and its applications[J]. *Acta Physica Sinica*, 2006, **55**(8): 3858-3863.(in Chinese))

[11] 李向正, 李修勇, 赵丽英, 张金良. Gerdjikov-Ivanov 方程的精确解[J]. 物理学报, 2008, **57**(4): 2031-2034.(LI Xiang-zheng, LI Xiu-yong, ZHAO Li-ying, ZHANG Jin-liang. Exact solutions of Gerdjikov-Ivanov equation[J]. *Acta Physica Sinica*, 2008, **57**(4): 2031-2034.(in Chinese))

[12] 高亮, 徐伟, 唐亚宁, 申建伟. 一类广义 Boussinesq 方程和 Boussinesq-Burgers 方程新的显式精确解[J]. 物理学报, 2007, **56**(4): 1860-1869.(GAO Liang, XU Wei, TANG Ya-ning, SHEN Jian-wei. New explicit exact solutions of one type of generalized Boussinesq equations and the Boussinesq-Burgers equation[J]. *Acta Physica Sinica*, 2007, **56**(4): 1860-1869.(in Chi-

nese))

[13] 马松华, 吴小红, 方建平, 郑春龙. (3+1)维 Burgers 系统的新精确解及其特殊孤立子结构[J]. 物理学报, 2008, **57**(1): 11-17.(MA Song-hua, WU Xiao-hong, FANG Jian-ping, ZHENG Chun-long. New exact solutions and special soliton structures for the (3+1)-dimensional Burgers system[J]. *Acta Physica Sinica*, 2008, **57**(1): 11-17.(in Chinese))

[14] 李帮庆, 马玉兰. (G'/G) 展开法和(2+1)维非对称 Nizhnik-Novikov-Veselov 系统的新精确解[J]. 物理学报, 2009, **58**(7): 4373-4378.(LI Bang-qing, MA Yu-lan. (G'/G)-expansion method and new exact solutions for (2+1)-dimensional asymmetrical Nizhnik-Novikov-Veselov system[J]. *Acta Physica Sinica*, 2009, **58**(7): 4373-4378.(in Chinese))

[15] 何吉欢. 工程和科学计算中的近似非线性分析方法[M]. 郑州: 河南科学技术出版社, 2002.(HE Ji-huan. *Approximate Analytical Methods in Engineering and Sciences*[M]. Zhengzhou: Henan Science and Technology Press, 2002.(in Chinese))

[16] Hovhannisyan G, Vulanovic R. Stability inequalities for one-dimensional singular perturbation problems[J]. *Nonlinear Studies*, 2008, **15**(4): 297-322.

[17] Barbu L, Cosma E. Elliptic regularization for the nonlinear heat equation[J]. *Journal of Mathematical Analysis and Applications*, 2009, **351**(1): 392-399.

[18] Ramos M. On singular perturbation of superlinear elliptic systems[J]. *Journal of Mathematical Analysis and Applications*, 2009, **352**(1): 246-258.

[19] MO Jia-qi. Singular perturbation for a class of nonlinear reaction diffusion systems[J]. *Science in China Ser A*, 1989, **32**(11): 1306-1315.

[20] MO Jia-qi, LIN Wan-tao. Asymptotic solution for a class of sea-air oscillator model for El-Nino-southern oscillation[J]. *Chinese Physics B*, 2008, **17**(2): 370-372.

[21] MO Jia-qi, LIN Wan-tao. Asymptotic solution of activator inhibitor systems for nonlinear reaction diffusion equations[J]. *Journal of Systems Science and Complexity*, 2008, **20**(1): 119-128.

[22] MO Jia-qi, LIN Yi-hua, LIN Wan-tao, CHEN Li-hua. Perturbed solving method for interdecadal sea-air oscillator model[J]. *Chinese Geographical Science*, 2012, **22**(1): 42-47.

[23] MO Jia-qi. A variational iteration solving method for a class of generalized Boussinesq equation[J]. *Chinese Physics Letters*, 2009, **26**(6): 060202.

[24] 莫嘉琪, 张伟江, 陈贤峰. 一类强非线性发展方程孤波变分迭代解法[J]. 物理学报, 2009, **58**(11): 7397-7401.(MO Jia-qi, ZHANG Wei-jiang, CHEN Xian-feng. Variational iteration method for solving a class of strongly nonlinear evolution equations[J]. *Acta Physica Sinica*, 2009, **58**(11): 7397-7401.(in Chinese))

[25] MO Jia-qi. Homotopiv mapping solving method for gain fluency of a laser pulse amplifier[J]. *Science in China Ser G*, 2009, **52**(7): 1007-1010.

[26] MO Jia-qi, LIN Wan-tao, WANG Hui. Variational iteration solution of a sea-air oscillator model for the ENSO[J]. *Progress in Natural Science*, 2007, **17**(2): 230-232.

[27] 史娟荣, 石兰芳, 莫嘉琪. 一类非线性强阻尼扰动发展方程的解[J]. 应用数学和力学, 2014, **35**(9): 1046-1054.(SHI Juan-rong, SHI Lan-fang, MO Jia-qi. The solutions for a class of nonlinear disturbed evolution equations[J]. *Applied Mathematics and Mechanics*, 2014, **35**(9): 1046-1054.(in Chinese))

[28] 吴钦宽. 一类非线性方程激波解的 Sinc-Galerkin 方法[J]. 物理学报, 2006, **55**(4): 1562-1564.(WU Qin-kuan. The shock solution for a class of the nonlinear equations by the Sinc-Galerkin method[J]. *Acta Physica Sinica*, 2006, **55**(4): 1562-1564.(in Chinese))

[29] ZHOU Xin-chun, SHI Lan-fang, HAN Xiang-lin, MO Jia-qi. Homotopic mapping solitary traveling wave solutions for the disturbed BKK mechanism physical model[J]. *Chinese Physics B*, 2014, **23**(9): 090204.

Asymptotic Travelling Wave Soliton Solutions for Nonlinear Disturbed Generalized NNV Systems

SHI Juan-rong[1,4], WU Qin-kuan[2], MO Jia-qi[3]

(1. *Anhui Technical College of Mechanical and Electrical Engineering*, *Wuhu*, *Anhui* 241002, *P.R.China*;
2. *Department of Mathematics & Physics*, *Nanjing Institute of Technology*, *Nanjing* 211167, *P.R.China*;
3. *Department of Mathematics*, *Anhui Normal University*, *Wuhu*, *Anhui* 241003, *P.R.China*;
4. *Department of Mathematics*, *Shanghai Jiao Tong University*, *Shanghai* 200240, *P.R.China*)

Abstract: A class of nonlinear disturbed generalized NNV (Nizhnik-Novikov-Veselov) system was addressed with a simple and valid technique. Firstly, the soliton solution to the corresponding typical differential system was obtained by means of the undetermined coefficient method. Secondly, a generalized functional equation was built and variationally calculated, and the corresponding Lagrange multiplier was derived according to the variation principle. Thereby, a special variational iteration relation expression was constructed. Then, the asymptotic travelling wave soliton solution for the original nonlinear disturbed generalized NNV system was attained successively. Finally, through an example, the proposed approximate analysis method is proved to be convenient and effective.

Key words: soliton; disturbed Nizhnik-Novikov-Veselov system; iterative method

Foundation item: The National Natural Science Foundation of China(11202106)