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一类非线性双曲型发展方程的孤子解*

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摘要: 研究了一类非线性发展方程.首先在无扰动情形下,利用待定函数和泛函同伦映射方法得到了非扰动发展方程的孤子精确解和扰动方程的任意次近似行波孤子解.接着引入一个同伦映射,并选取初始近似函数,再用同伦映射理论,依次求出非线性双曲型发展扰动方程孤子解的各次近似解析解.再利用摄动理论举例说明了用该方法得到的近似解析解的有效性和各次近似解的近似度.最后,简述了用同伦映射方法得到的近似解的意义,指出了用上述方法得到的各次近似解具有便于求解、精度高等优点.

关 键 词: 孤子解; 扰动; 非线性双曲型方程
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引 言

非线性微分方程孤子解广泛地存在于物理学、力学和其它自然科学的许多领域的应用中,该理论是当前国际上十分活跃的研究课题.许多学者在光波散射、爆炸、燃烧、激波理论、神经网络、量子力学、大气物理等方面都做了大量研究成果,提出了非线性孤子的许多求解方法,如椭圆函数展开法、(G'/G) 展开法、Riccati 函数法、齐次平衡法、双曲正切函数法、F-展开法和辅助方程法等[1-4].在研究过程中,Zhang 等,马松华等及范恩贵等提出了各种类型的孤子,譬如分形孤子、方孤子、泡孤子、峰孤子、环孤子等[5-8].本文讨论了一类非线性双曲型发展方程,得到非线性扰动方程的孤子精确解和行波渐近解.当前对非线性问题已有深入研究,渐近方法也大量涌现,其中包括多重尺度法,边界层、内部层方法,匹配展开法等.许多学者,例如 Barbu 等[9-10],D'Aprile 等[11],Suzuki[12],Ei 等[13],Kellogg 等[14]做了很多工作.利用渐近理论,笔者等也讨论了一类非线性问题[15-24].本文利用一个特殊而有效的待定函数和泛函分析同伦映射理论和方法对一类非线性扰动双曲型发展方程构造其孤子解.

1 非线性双曲型发展方程

今讨论如下一类扰动双曲型发展方程:

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$$u_{tt}-u_{xx}+mu+nuu_x+ku^3=f(u),\tag{1}$$

其中，m,n,k 为参数；t,x 分别为时间、空间自变量；f 为扰动项，它是关于其变量的有界解析函数.我们用待定函数法和扰动理论[8,25-26]来寻求非线性双曲型方程(1)的孤子解.

首先考虑在方程(1)中当 $f=0$ 时的无扰动双曲型发展方程：

$$u_{tt}-u_{xx}+mu+nuu_t+ku^3=0.\tag{2}$$

现利用一个特殊的待定函数法，来求出方程(2)的孤子解.作函数变换：

$$u=y'(w)\frac{\partial w}{\partial x}+a,\ \frac{\partial w}{\partial t}=b\frac{\partial w}{\partial x},\tag{3}$$

其中 y,w 为待定函数，而 a,b 为待定常数.由式(3)得

$$u_{tt}=b^2y'''\left(\frac{\partial w}{\partial x}\right)^3+3b^2y''\frac{\partial w}{\partial x}\frac{\partial^2 w}{\partial x^2}+b^2y'\frac{\partial^3 w}{\partial x^3},$$

$$u_{xx}=y'''\left(\frac{\partial w}{\partial x}\right)^3+3y''\frac{\partial w}{\partial x}\frac{\partial^2 w}{\partial x^2}+y'\frac{\partial^3 w}{\partial x^3},$$

$$uu_t=by'y''\left(\frac{\partial w}{\partial x}\right)^3+b(y')^2\frac{\partial w}{\partial x}\frac{\partial^2 w}{\partial x^2}+aby''\left(\frac{\partial w}{\partial x}\right)^2+aby'\frac{\partial^2 w}{\partial x^2},$$

$$u^3=(y')^3\left(\frac{\partial w}{\partial x}\right)^3+3a(y')^2\left(\frac{\partial w}{\partial x}\right)^2+3a^2y'\frac{\partial w}{\partial x}+a^3,$$

$$u=y'\frac{\partial w}{\partial x}+a.$$

将上述各式代入方程(2)，得

$$\begin{aligned}&[(b^2-1)y'''+(y')^3+bny'y'']\left(\frac{\partial w}{\partial x}\right)^3+\\&\quad(3b^2y''-3y''+bn(y')^2)\frac{\partial w}{\partial x}\frac{\partial^2 w}{\partial x^2}+\\&\quad abny'\frac{\partial^2 w}{\partial x^2}+3ak(y')^2\left(\frac{\partial w}{\partial x}\right)^2+abny''\left(\frac{\partial w}{\partial x}\right)^2+\\&\quad\left[(b^2-1)\frac{\partial^3 w}{\partial x^3}+3a^2k\frac{\partial w}{\partial x}+m\frac{\partial w}{\partial x}\right]y'+\\&\quad a(m+a^2k)=0.\end{aligned}\tag{4}$$

令

$$(b^2-1)y'''+k(y')^3+bny'y''=0.\tag{5}$$

由式(5)解得

$$y=\pm\left(\frac{2(1-b^2)}{k}\right)^{1/2}\ln w,\ (y')^2=\mp\left(\frac{2(1-b^2)}{k}\right)^{1/2}y''.\tag{6}$$

将式(6)代入式(4)，便有

$$\begin{aligned}&\left[3(b^2-1)\frac{\partial^2 w}{\partial x^2}\mp\left(3ak\frac{\partial w}{\partial x}+bn\frac{\partial^2 w}{\partial x^2}\right)\left(\frac{2(1-b^2)}{k}\right)^{1/2}+abn\frac{\partial w}{\partial x}\right]\frac{\partial w}{\partial x}y''+\\&\quad\left[(b^2-1)\frac{\partial^3 w}{\partial x^3}+abn\frac{\partial^2 w}{\partial x^2}+(3a^2k+m)\frac{\partial w}{\partial x}\right]y'+a(m+a^2k)=0.\end{aligned}\tag{7}$$

在式(7)中,令 y^0, y, y' 的系数为 0:

$$\left[3(b^2-1)\frac{\partial^2 w}{\partial x^2} \mp \left(3ak\frac{\partial w}{\partial x} + bn\frac{\partial^2 w}{\partial x^2}\right)\left(\frac{2(1-b^2)}{k}\right)^{1/2} + abn\frac{\partial w}{\partial x}\right]\frac{\partial w}{\partial x} = 0, \tag{8}$$

$$(b^2-1)\frac{\partial^3 w}{\partial x^3} + abn\frac{\partial^2 w}{\partial x^2} + (3a^2k+m)\frac{\partial w}{\partial x} = 0, \tag{9}$$

$$a(m+a^2k) = 0. \tag{10}$$

设待定函数 w 具有如下形式:

$$w = c(x+bt) + \exp[l(x+bt)], \tag{11}$$

其中, c, l 为待定常数.将式(11)代入式(8)~(10),有

$$3(b^2-1)l^2 \mp (3ak(c+l) + bnl^2)\left(\frac{2(1-b^2)}{k}\right)^{1/2} + abn(c+l) = 0, \tag{12}$$

$$(b^2-1)l^3 + abnl^2 + (3a^2k+m)(c+l) = 0, \tag{13}$$

$$a(m+a^2k) = 0. \tag{14}$$

由式(12)~(14)可得

$$a = \pm\sqrt{\frac{-m}{k}}, \tag{15}$$

$$l = \left[-3(b^2-1) \pm \left(3ak\left(\frac{abn}{3a^2k+m}\right) + bn\right)\left(\frac{2(1-b^2)}{k}\right)^{1/2} + \frac{(abn)^2}{3a^2k+m}\right] \times \left[\left(3k\left(\frac{2(1-b^2)}{k}\right)^{1/2} + bn\right)\frac{a(1-b^2)}{3a^2k+m}\right]^{-1/2}, \tag{16}$$

$$c = \frac{(1-b^2)l^3 + abnl^2}{3a^2k+m} - l. \tag{17}$$

再由式(3)、(6)、(11),便得到发展方程(2)如下形式的孤子解:

$$u(t,x) = \pm\left(\frac{2(1-b^2)}{k}\right)^{1/2}\frac{c + l\exp[l(x+bt)]}{c(x+bt) + \exp[l(x+bt)]} + a, \tag{18}$$

其中常数 a, c, l 分别由式(15)~(17)表示.

2 非线性 LGH 发展方程

在方程(2)中,选取参数

$$n = 0,\ m = -p^2 < 0,\ k = q^2 > 0,$$

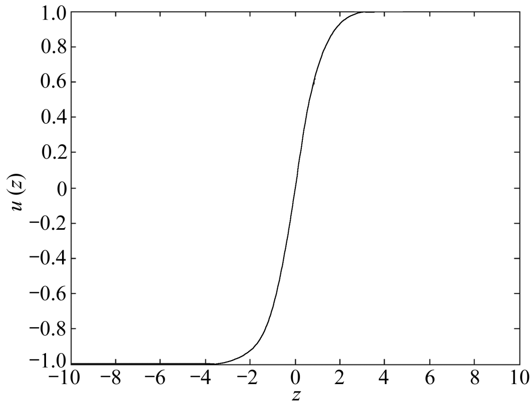
这时方程为典型的非线性 Landau-Ginzburg-Higgs(LGH)发展方程[8]:

$$u_{tt} - u_{xx} - p^2u + q^2u^3 = 0. \tag{19}$$

由上节叙述的待定函数方法得到的孤子解式(18),经过简单的计算简化后可得到非线性 LGH 发展方程具有如下的孤子解:

$$u(t,x) = \pm\frac{p}{q}\tanh\left[\frac{p}{\sqrt{2(1-b^2)}}(x+bt)\right], \qquad b^2 < 1. \tag{20}$$

取参数 $p = q = 1, b = 1/2$,且设 $z = x + t/2$.这时由式(20)知,非线性 LGH 发展方程(19)具有的行波孤子解 $u(z)$ 的曲线图形如图 1 所示.



$(p = q = 1,\ b = 1/2,\ z = x + t/2)$

图 1　LGH 发展方程(19)的行波孤子解 $u(z)$ 曲线

Fig. 1　Travelling wave soliton solution $u(z)$ to LGH evolution equation(19)

3　泛函同伦映射

为了进一步求得非线性扰动发展方程(1)的解.首先引入一个 $\mathbf{R}\times[0,1]\to\mathbf{R}$ 的泛函映射[25]：

$$H[u,r]=L[u]-L[\bar{u}]+r[L[\bar{u}]+nuu_t+ku^3-f(u)],\tag{21}$$

其中 r 为人工参数[25]，$\bar{u}$ 为原方程(1) 的初始近似,算子 $L[u]$ 定义为

$$L[u]\equiv\frac{\partial^2u}{\partial t^2}-\frac{\partial^2u}{\partial x^2}+mu.$$

由映射关系式(21)不难看出，$H[u,s]=0$ 的解 $u(x,t,r)$，当 $r\to1$ 的极限情形就是方程(1)的解.

令 $H[u,r]=0$ 的解 $u(x,t,r)$ 为

$$u(x,t,r)=\sum_{i=0}^{\infty}u_i(x,t)r^i.\tag{22}$$

将式(22)代入映射关系式(21)，按其中的非线性项按 r 的幂展开，合并 $r^i(i=0,1,2,\cdots)$ 的同次幂的系数，并将对应合并的系数分别令其为 0.可依次得到

$$L[u_0]-L[\bar{u}]=0,\tag{23}$$

$$L[u_1]=f(u_0),\tag{24}$$

$$L[u_2]=[f_u(u_0)]u_1-f(u_0),\tag{25}$$

$$L[u_i]=G_i(t,x),\qquad i=3,4,\cdots,\tag{26}$$

这里

$$G_i(t,x)=\frac{1}{i!}\frac{\partial^{i-1}}{\partial r^{i-1}}\left[f\left(\sum_{j=0}^{\infty}u_jr^j\right)-n\left(\sum_{j=0}^{\infty}u_jr^j\right)\left(\sum_{j=0}^{\infty}(u_j)_tr^j\right)-k\left(\sum_{j=0}^{\infty}u_jr^j\right)^3\right]_{r=0}.\tag{27}$$

由式(23)知，可取 u_0 为由式(18) 决定的一个孤子解 $\bar{u}$：

$$u_0(t,x)=\left(\frac{2(1-b^2)}{k}\right)^{1/2}\frac{c+l\exp[l(x+bt)]}{c(x+bt)+\exp[l(x+bt)]}+a,\tag{28}$$

其中常数 a,c,l 分别由式(15)~(17)表示.

考虑式(23),不难由 Fourier 变换和偏微分方程理论得线性双曲型方程(24)的解为

$$u_1(x,t)=\frac{1}{2\pi}\int_0^t\int_{-\infty}^{\infty}\frac{f(u_0(\tau,\xi))}{\sqrt{\xi^2+m}}[\cos(\sqrt{\xi^2+m}\ \xi)-\sin(\sqrt{\xi^2+m}\ \xi)]\exp(\mathrm{i}\xi x)\mathrm{d}\xi\mathrm{d}\tau,\tag{29}$$

其中 u_0 由式(28)表示.

同样,可依次得到双曲型方程(25)、(26)的解为

$$u_2(x,t)=\frac{1}{2\pi}\int_0^t\int_{-\infty}^{\infty}\frac{f_u(u_0(\tau,\xi))u_1(\tau,\xi)-f(u_0(\tau,\xi))}{\sqrt{\xi^2+m}}[\cos(\sqrt{\xi^2+m}\ \xi)-\sin(\sqrt{\xi^2+m}\ \xi)]\exp(\mathrm{i}\xi x)\mathrm{d}\xi\mathrm{d}\tau,\tag{30}$$

$$u_i(x,t)=\frac{1}{2\pi}\int_0^t\int_{-\infty}^{\infty}\frac{G_i(\tau,\xi)}{\sqrt{\xi^2+m}}[\cos(\sqrt{\xi^2+m}\ \xi)-\sin(\sqrt{\xi^2+m}\ \xi)]\exp(\mathrm{i}\xi x)\mathrm{d}\xi\mathrm{d}\tau,\qquad i=2,3,\cdots.\tag{31}$$

于是,将式(28)~(31)代入级数(22),用级数收敛性理论能够证明级数(22)在有限时段和区域内关于 $p\in[0,1]$ 是一致收敛的[9,25-26].所以令 $p=1$ 便得到非线性扰动发展方程(1)的解为

$$u(t,x)=\left(\frac{2(1-b^2)}{k}\right)^{1/2}\frac{c+l\exp[l(x+bt)]}{c(x+bt)+\exp[l(x+bt)]}+a+\frac{1}{2\pi}\int_0^t\int_{-\infty}^{\infty}\frac{f_u[u_0(\tau,\xi)]u_1(\tau,\xi)+\sum_{i=2}^{\infty}G_i(\tau,\xi)}{\sqrt{\xi^2+m}}\times[\cos(\sqrt{\xi^2+m}\ \xi)-\sin(\sqrt{\xi^2+m}\ \xi)]\exp(\mathrm{i}\xi x)\mathrm{d}\xi\mathrm{d}\tau,\tag{32}$$

而

$$U_M(t,x)=\left(\frac{2(1-b^2)}{k}\right)^{1/2}\frac{c+l\exp[l(x+bt)]}{c(x+bt)+\exp[l(x+bt)]}+a+\frac{1}{2\pi}\int_0^t\int_{-\infty}^{\infty}\frac{f_u[u_0(\tau,\xi)]u_1(\tau,\xi)+\sum_{i=2}^{M}G_i(\tau,\xi)}{\sqrt{\xi^2+m}}\times[\cos(\sqrt{\xi^2+m}\ \xi)-\sin(\sqrt{\xi^2+m}\ \xi)]\exp(\mathrm{i}\xi x)\mathrm{d}\xi\mathrm{d}\tau\tag{33}$$

为非线性扰动发展方程(1)的第 M 次近似解.

4 扰动方程近似解精度的讨论

为了简单起见,我们来考虑以 $f=\varepsilon\exp(-u^2)$ 为扰动项的非线性 LGH 发展方程:

$$u_{tt}-u_{xx}-p^2u+q^2u^3=\varepsilon\exp(-u^2),\tag{34}$$

其中 ε 为小参数.由于对应的典型 LGH 发展方程

$$u_{tt}-u_{xx}-p^2u+q^2u^3=0$$

具有形如

$$\bar{u}(t,x)=\frac{p}{q}\tanh\left[\frac{p}{\sqrt{2(1-b^2)}}(x+bt)\right],\qquad b^2<1$$

的孤子解.现设

$$u(x,t,r)=\sum_{i=0}^{\infty}u_i(x,t)r^i.$$

选取初始函数

$$u_0(t,x)=\bar{u}(t,x).$$

即

$$u_0(t,x)=\frac{p}{q}\tanh\left[\frac{p}{\sqrt{2(1-b^2)}}(x+bt)\right],\qquad b^2<1. \tag{35}$$

利用本文的泛函映射式(21)的方法,可得

$$u_1(x,t)=\frac{\varepsilon}{2\pi}\int_0^t\int_{-\infty}^{\infty}\frac{\exp(-u_0^2(\tau,\xi))}{\sqrt{\xi^2+m}}[\cos(\sqrt{\xi^2+m}\,\xi)-\sin(\sqrt{\xi^2+m}\,\xi)]\exp(\mathrm{i}\xi x)\mathrm{d}\xi\mathrm{d}\tau. \tag{36}$$

并不难得到非线性 LGH 扰动发展方程(34)的二次近似 $U_2(t,x)$ 为

$$U_2(t,x)=\frac{p}{q}\tanh\left[\frac{p}{\sqrt{2(1-b^2)}}(x+bt)\right]-\frac{\varepsilon}{2\pi}\int_0^t\int_{-\infty}^{\infty}\frac{u_0u_1\exp(-u_0^2)}{\sqrt{\xi^2+m}}[\cos(\sqrt{\xi^2+m}\,\xi)-\sin(\sqrt{\xi^2+m}\,\xi)]\exp(\mathrm{i}\xi x)\mathrm{d}\xi\mathrm{d}\tau, \tag{37}$$

其中 u_0,u_1 分别由式(35)、(36)表示.

因为非线性扰动 LGH 发展方程(34)是摄动方程,故它可以用摄动方法来得到其渐近解.不难用摄动理论证明[9,26],由式(37)得到的近似解 $U_2(t,x)$ 与方程(34)相应的精确解 $u(t,x)$ 具有如下的关系:

$$u(t,x)=U_2(t,x)+O(\varepsilon^2),\qquad 0<\varepsilon\ll 1.$$

由上式可以间接看出,用本文求出的近似解具有良好的精度.

5 结　　论

众所周知,非线性问题一般不能用有限的初等函数项来得到其精确解,本文用泛函分析同伦映射方法,选择合理的初始近似函数,以较快速度得到较高精度的近似解析解.

本文用泛函分析同伦映射方法得到具有扰动的一类非线性发展方程孤子近似解析解,它不同于单纯用数值模拟方法得到的模拟解,因为它还可进行微分、积分等运算.得到的近似解还可对非线性发展方程进一步研究,使得非线性发展方程的解具有更好的物理性态.同时,用泛函分析同伦映射方法还可解其它相应的非线性方程.

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The Soliton Solutions to a Class of Nonlinear Hyperbolic Evolution Equations

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Abstract: A class of nonlinear evolution equations were investigated. With the undetermined functions and functional homotopic mapping methods, the exact solitary solution to the non-disturbed evolution equation and the arbitrary order approximate travelling wave solitary solution to the disturbed evolution equation were obtained. A homotopic mapping was introduced, and an initial approximate function was chosen to find out successively the arbitrary order solitary approximate analytic solutions to the nonlinear hyperbolic evolution equation based on the homotopic mapping theory. With the perturbation method, the examples illustrated the validity and approximation degree of the arbitrary order approximate solutions. A discussion shows the practicability and high accuracy of the approximate solutions obtained with the proposed homotopic mapping method.

Key words: soliton solution; disturbed; nonlinear hyperbolic equation

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