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时间标度上时滞脉冲复值神经网络的全局稳定性*

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摘要:　研究了时间标度上具有时滞和脉冲影响的复值神经网络的全局稳定性问题.利用时间标度上的微积分理论,将连续时间型复值神经网络和离散时间型复值神经网络统一在同一个框架下进行研究.在不要求激励函数有界的条件下,运用同胚映射原理,建立了确保时滞复值神经网络平衡点存在性和唯一性的判定条件.通过构造合适的 Lyapunov-Krasovskii 泛函,并使用自由权矩阵方法和矩阵不等式技巧,获得了时间标度上具有时滞和脉冲影响的复值神经网络平衡点全局稳定性的充分条件.给出的判据是由复值线性矩阵表示的,易于 MATLAB 软件的 YALMIP Toolbox 实现.数值仿真实例验证了获得结果的有效性.

关　键　词:　复值神经网络;　时间标度;　时滞;　脉冲;　复值线性矩阵不等式;　全局稳定性

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引　　言

神经网络是人工智能领域的重要分支,现已广泛应用于信号处理、模式识别、联想记忆、优化控制、保密通讯、工程计算等领域[1].为了更好地使用神经网络解决实际问题,首要问题是要分析所设计的神经网络的稳定性[2].然而,在用硬件实现神经网络的过程中,由于放大器转换速度的限制,传递延迟是不可避免的[3].时滞的出现不仅会降低网络的传递速度,而且会导致本来稳定的网络变成不稳定的网络,并有可能引起震荡甚至出现混沌现象[4].因此,将时滞引入神经网络,建立时滞神经网络模型并研究其稳定性具有重要的理论和实用价值[5].另一方面,在现实世界中,许多自然现象在发展的某些阶段会出现快速的变化.例如,气候的突变对生物种群生长的影响,突发的社会事件对股票市场和金融市场产生的波动,经济环境的突变对商

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品供给和需求的冲击等.由于这个快速变化的持续时间同整个发展过程相比是非常短暂的,因此可以认为是瞬间发生的,这种瞬间突变现象通常称之为脉冲现象[6].脉冲会对神经网络产生巨大的影响,它既能使稳定的神经网络不稳定,也能使不稳定的神经网络稳定[7].因此,对具有脉冲效应的神经网络稳定性进行研究,也具有重要的理论和实用价值[8].

以上文献研究的神经网络,其神经元的状态、输出、权值和活动函数都取实数值,因此称之为实值神经网络.虽然实值神经网络能够应用于许多领域,但也有其应用的局限性[9].自然地,复值神经网络应运而生.例如,在信号处理中,由于复值信号能够携带振幅和相位信息,因此能够处理复值数据的复值神经网络具有解决该领域问题的先天优势[10].近年来,一些学者研究了复值神经网络的稳定性问题[10-18].文献[10-12]研究了几类离散时间型复值神经网络的有界性和稳定性问题,建立了模型有界性和完全稳定性的一些充分条件.文献[13-18]给出了连续时间型时滞复值神经网络稳定性的一些判定条件.基于时间标度上的微积分理论[19],Bohner等建立了时间标度上复值神经网络模型,并研究了模型平衡点的存在性和全局指数稳定性问题[20].在文献[21]中,笔者考虑了时滞情形,建立了时间标度上既具有泻漏时滞(leakage time delay)又具有离散时滞的复值神经网络模型,并给出了模型平衡点的存在性、唯一性和全局稳定性的线性矩阵不等式判据,改进和推广了文献[20]的结果.最近,Rakkiyappan等考虑了脉冲对复值神经网络的影响问题,建立了脉冲复值神经网络模型,并研究了模型平衡点的稳定性[22].不同于已有工作,本文同时考虑了脉冲和时滞对复值神经网络稳定性的影响,建立了时间标度上的脉冲时滞复值神经网络模型,并给出了模型平衡点的存在性、唯一性和全局稳定性的充分判据,改进和推广了文献[22]的结果.

1 预备知识

本文考虑如下时间标度上离散时滞脉冲复值神经网络模型:

$$\begin{cases} \boldsymbol{z}^{\Delta}(t) = -\boldsymbol{C}\boldsymbol{z}(t) + \boldsymbol{D}\boldsymbol{f}(\boldsymbol{z}(t)) + \boldsymbol{E}\boldsymbol{f}(\boldsymbol{z}(t-\tau)) + \boldsymbol{H}, & t \neq t_k,\ t \in T_0^+, \\ \Delta \boldsymbol{z}(t_k) = \boldsymbol{z}(t_k) - \boldsymbol{z}(t_k^-) = \boldsymbol{J}_k(\boldsymbol{z}(t_k^-),\ \boldsymbol{z}_{t_k^-}), & t = t_k,\ k \in \mathbf{Z}_+, \end{cases} \tag{1}$$

其中,T 表示一个时间标度,它是实数集 $\mathbf{R}$ 的任意非空闭子集,$T_0^+ = \{t \in T, t \geqslant 0\}$.$t_k$ 表示脉冲时刻,满足 $0 = t_0 < t_1 < \cdots < t_k < \cdots, \lim_{k\to\infty} t_k = +\infty$.$\boldsymbol{z}(t) = (z_1(t), z_2(t), \cdots, z_n(t))^{\mathrm{T}} \in C^n$ 表示 t 时刻 n 个神经元的状态向量.$\boldsymbol{f}(\boldsymbol{z}(t)) = (f_1(z_1(t)), f_2(z_2(t)), \cdots, f_n(z_n(t)))^{\mathrm{T}} \in C^n$ 和 $\boldsymbol{f}(\boldsymbol{z}(t-\tau)) = (f_1(z_1(t-\tau_1)), f_2(z_2(t-\tau_2)), \cdots, f_n(z_n(t-\tau_n)))^{\mathrm{T}} \in C^n$ 表示神经元激励函数.τ 表示离散时滞,满足如果 $t \in T$,则 $t-\tau \in T, \tau > 0$.$\boldsymbol{C} = \mathrm{diag}(c_1, c_2, \cdots, c_n) \in R^{n\times n}, c_j > 0$ $(j = 1, 2, \cdots, n)$ 表示自反馈连接权矩阵.$\boldsymbol{D} = (d_{pq})_{n\times n} \in C^{n\times n}$ 和 $\boldsymbol{E} = (e_{pq})_{n\times n} \in C^{n\times n}$ 分别表示连接权矩阵和离散时滞连接权矩阵.$\boldsymbol{H} = (h_1, h_2, \cdots, h_n)^{\mathrm{T}} \in C^n$ 表示外部输入常向量.$\boldsymbol{J}_k(\cdot)$ 表示脉冲函数.

模型(1)的初始条件为

$$z_i(s) = \phi_i(s), \qquad s \in [-\tau, 0]_T,\ i = 1, 2, \cdots, n,$$

其中,$\phi_i(s)$ 在 $[-\tau, 0]_T$ 内有界且连续.

本文给出如下假设:

(H1) 令 $\mathrm{Re}(z)$ 和 $\mathrm{Im}(z)$ 分别表示复值 z 的实部和虚部,那么激励函数 $f_j(z)$ 可以表示为

$$f_j(z) = f_j^{\mathrm{R}}(\mathrm{Re}(z)) + \mathrm{i} f_j^{\mathrm{I}}(\mathrm{Im}(z)), \qquad j = 1, 2, \cdots, n,$$

其中 $f_j^{\mathrm{R}}(\cdot), f_j^{\mathrm{I}}(\cdot): \mathbf{R} \to \mathbf{R}$,且满足对任意的 $w_1, w_2 \in \mathbf{R}, w_1 \neq w_2$,有

$$\xi_j^{\mathrm{R}-} \leqslant \frac{f_j^{\mathrm{R}}(w_1) - f_j^{\mathrm{R}}(w_2)}{w_1 - w_2} \leqslant \xi_j^{\mathrm{R}+},$$

$$\xi_j^{\mathrm{I}-} \leqslant \frac{f_j^{\mathrm{I}}(w_1) - f_j^{\mathrm{I}}(w_2)}{w_1 - w_2} \leqslant \xi_j^{\mathrm{I}+}.$$

(H2) 如果常数复值向量 $\hat{z}$ 满足 $-\boldsymbol{C}\hat{z} + (\boldsymbol{D} + \boldsymbol{E})\boldsymbol{f}(\hat{z}) + \boldsymbol{H} = \boldsymbol{0}$,则它满足 $\boldsymbol{J}_k(\hat{z},\hat{z}) = \boldsymbol{0}$.

(H3) 时间标度 T 的粒度函数 $\mu(t): T \to \mathbf{R}_+$ 是有界的,即 $\mu(t) \leqslant \bar{\mu} < +\infty$.

定义 1[20] 令 T 是一个时间标度,定义前跃算子和后跃算子分别为

$$\sigma(t) = \inf\{s \in T: s > t\},$$

$$\rho(t) = \sup\{s \in T: s < t\},$$

时间标度的粒度函数 $\mu(t): T \to \mathbf{R}_+$,

$$\mu(t) = \sigma(t) - t.$$

在以上定义中,对于 $t \in T$,如果 $\sigma(t) = t$,即 $\mu(t) = 0$,则称 t 是右稠密的;如果 $\sigma(t) > t$,即 $\mu(t) > 0$,则称 t 是右稀疏的;如果 $\rho(t) = t$,则称 t 是左稠密的;如果 $\rho(t) > t$,则称 t 是左稀疏的.

定义 2[20] 如果函数 $f: T \to \mathbf{R}$ 在 T 的右稠密点连续并且在 T 的左稠密点存在左极限,则称 f 为 rd-连续的.rd-连续函数的集合被记为

$$C_{\mathrm{rd}} = C_{\mathrm{rd}}(T) = C_{\mathrm{rd}}(T, \mathbf{R}).$$

定义 3[20] 函数 $f: T \to \mathbf{R}$ 的 Δ 导数为 $f^{\Delta}(t)$,对于任意 $\varepsilon > 0$,存在 t 的一个邻域 U 使得

$$\left|[f(\sigma(t)) - f(s)] - f^{\Delta}(t)[\sigma(t) - s]\right| \leqslant |\sigma(t) - s|, \qquad s \in U.$$

定义

$$f^{\Delta}(t) = \begin{cases} \lim\limits_{s \to t^+} \dfrac{f(s) - f(t)}{s - t}, & \sigma(t) = t, \\ \dfrac{f(\sigma(t)) - f(t)}{\sigma(t) - t}, & \sigma(t) > t. \end{cases}$$

定义 4[20] 函数 f 是一个 rd-连续函数,如果 $F^{\Delta}(t) = f(t)$,Δ 积分定义如下:

$$\int_a^t f(s)\,\mathrm{d}s = F(t) - F(a).$$

容易验证以下的式子成立:

$$\int_t^{\sigma(t)} f(s)\,\mathrm{d}s = \mu(t) f(t).$$

定义 5[21] 设 $\hat{z}$ 为模型(1)在时间标度上的一个平衡点,$\boldsymbol{z}(t) = (z_1(t), z_2(t), \cdots, z_n(t))$ 为模型(1)的任意解,如果存在常数 $M > 0$, 使得

$$\|\boldsymbol{z}(t) - \hat{\boldsymbol{z}}\| \leqslant M \sup_{s \in [-\tau, 0]_T} \|\boldsymbol{\phi}(s)\|$$

成立,那么就说平衡点 $\hat{z}$ 是全局稳定的,其中

$$\|\boldsymbol{z}(t) - \hat{\boldsymbol{z}}\| = \left(\sum_{j=1}^{n} |\mathrm{Re}(z_j(t)) - \mathrm{Re}(\hat{z}_j)|^2 + \sum_{j=1}^{n} |\mathrm{Im}(z_j(t)) - \mathrm{Im}(\hat{z}_j)|^2\right)^{1/2},$$

$$\|\boldsymbol{\phi}(s)\| = \left(\sum_{j=1}^{n} |\mathrm{Re}(\boldsymbol{\phi}(s))|^2 + \sum_{j=1}^{n} |\mathrm{Im}(\boldsymbol{\phi}(s))|^2\right)^{1/2}.$$

引理 1[21] 如果 $\boldsymbol{\psi}(z): C^n \to C^n$ 是一个连续映射且满足下述条件:

1) $\boldsymbol{\psi}(z)$ 在 C^n 上为单射,

2) $\lim_{\|z\|\to\infty} \|\boldsymbol{\psi}(z)\| = \infty$,

则 $\boldsymbol{\psi}(z)$ 在 C^n 上为自同胚映射.

引理 2[20] 如果 f 和 g 是时间标度 T 的两个 Δ 可微函数,则

$$(fg)^{\Delta}(t) = f^{\Delta}(t)g(t) + f(\sigma(t))g^{\Delta}(t) = g^{\Delta}(t)f(t) + g(\sigma(t))f^{\Delta}(t).$$

引理 3[21] Hermite 矩阵

$$\boldsymbol{S} = \begin{pmatrix} \boldsymbol{S}_{11} & \boldsymbol{S}_{12} \\ \boldsymbol{S}_{21} & \boldsymbol{S}_{22} \end{pmatrix} < \boldsymbol{0},$$

其中 $\boldsymbol{S}_{11}^* = \boldsymbol{S}_{11}, \boldsymbol{S}_{12}^* = \boldsymbol{S}_{21}, \boldsymbol{S}_{22}^* = \boldsymbol{S}_{22}$, 等价于

1) $\boldsymbol{S}_{22} < \boldsymbol{0}$, $\boldsymbol{S}_{11} - \boldsymbol{S}_{12}\boldsymbol{S}_{22}^{-1}\boldsymbol{S}_{21} < \boldsymbol{0}$,

2) $\boldsymbol{S}_{11} < \boldsymbol{0}$, $\boldsymbol{S}_{22} - \boldsymbol{S}_{21}\boldsymbol{S}_{11}^{-1}\boldsymbol{S}_{12} < \boldsymbol{0}$.

引理 4[21] 如果 $\boldsymbol{P} \in C^{n\times n}$ 是一个正定 Hermite 矩阵,则对 $\forall \boldsymbol{a}, \boldsymbol{b} \in C^n$, 有

$$\boldsymbol{a}^*\boldsymbol{b} + \boldsymbol{b}^*\boldsymbol{a} \leqslant \boldsymbol{a}^*\boldsymbol{P}\boldsymbol{a} + \boldsymbol{b}^*\boldsymbol{P}^{-1}\boldsymbol{b}.$$

引理 5[21] 对任意的常数矩阵 $\boldsymbol{W} \in C^{n\times n}, \boldsymbol{W} > \boldsymbol{0}$ 和标量函数 $\boldsymbol{\omega}: [a,b] \to C^n, a < b$ 有

$$\left(\int_a^b \boldsymbol{\omega}(s)\,\mathrm{d}s\right)^* \boldsymbol{W} \left(\int_a^b \boldsymbol{\omega}(s)\,\mathrm{d}s\right) \leqslant (b-a)\int_a^b \boldsymbol{\omega}(s)^* \boldsymbol{W}\boldsymbol{\omega}(s)\,\mathrm{d}s.$$

注 1 本文中“T”和“ * ”分别表示矩阵的转置和矩阵的共轭转置.对于矩阵 $\boldsymbol{A} \geqslant \boldsymbol{B}$ ($\boldsymbol{A} > \boldsymbol{B}$) 表示 $\boldsymbol{A} - \boldsymbol{B}$ 是半正定的(正定的).$\lambda_{\max}(\boldsymbol{A})$ 和 $\lambda_{\min}(\boldsymbol{A})$ 分别表示矩阵 $\boldsymbol{A}$ 的最大特征值和最小特征值.

2 主要结果

首先,在假设(H1)成立的条件下,我们将模型(1)分成实部和虚部两个部分,表示如下:

$$\begin{cases} \boldsymbol{x}^{\Delta}(t) = -\boldsymbol{C}\boldsymbol{x}(t) + \boldsymbol{D}^{\mathrm{R}}\boldsymbol{f}^{\mathrm{R}}(\boldsymbol{x}(t)) - \boldsymbol{D}^{\mathrm{I}}\boldsymbol{f}^{\mathrm{I}}(\boldsymbol{y}(t)) + \boldsymbol{E}^{\mathrm{R}}\boldsymbol{f}^{\mathrm{R}}(\boldsymbol{x}(t-\tau)) - \\ \qquad \boldsymbol{E}^{\mathrm{I}}\boldsymbol{f}^{\mathrm{I}}(\boldsymbol{y}(t-\tau)) + \boldsymbol{H}^{\mathrm{R}}, \qquad t \neq t_k,\ t \in T_0^+, \\ \Delta\boldsymbol{x}(t_k) = \boldsymbol{x}(t_k) - \boldsymbol{x}(t_k^-) = \boldsymbol{J}_k^{\mathrm{R}}(\boldsymbol{x}(t_k^-), \boldsymbol{x}_{t_k^-}), \qquad t = t_k,\ k \in \mathbf{Z}_+, \end{cases} \tag{2}$$

$$\begin{cases} \boldsymbol{y}^{\Delta}(t) = -\boldsymbol{C}\boldsymbol{y}(t) + \boldsymbol{D}^{\mathrm{I}}\boldsymbol{f}^{\mathrm{R}}(\boldsymbol{x}(t)) + \boldsymbol{D}^{\mathrm{R}}\boldsymbol{f}^{\mathrm{I}}(\boldsymbol{y}(t)) + \boldsymbol{E}^{\mathrm{I}}\boldsymbol{f}^{\mathrm{R}}(\boldsymbol{x}(t-\tau)) + \\ \qquad \boldsymbol{E}^{\mathrm{R}}\boldsymbol{f}^{\mathrm{I}}(\boldsymbol{y}(t-\tau)) + \boldsymbol{H}^{\mathrm{I}}, \qquad t \neq t_k,\ t \in T_0^+, \\ \Delta\boldsymbol{y}(t_k) = \boldsymbol{y}(t_k) - \boldsymbol{y}(t_k^-) = \boldsymbol{J}_k^{\mathrm{I}}(\boldsymbol{y}(t_k^-), \boldsymbol{y}_{t_k^-}), \qquad t = t_k,\ k \in \mathbf{Z}_+, \end{cases} \tag{3}$$

其中

$$\boldsymbol{x}(t) = \mathrm{Re}(\boldsymbol{z}(t)),\ \boldsymbol{y}(t) = \mathrm{Im}(\boldsymbol{z}(t)),$$
$$\boldsymbol{f}^{\mathrm{R}}(\cdot) = \mathrm{Re}(\boldsymbol{f}(\cdot)),\ \boldsymbol{f}^{\mathrm{I}}(\cdot) = \mathrm{Im}(\boldsymbol{f}(\cdot)),$$
$$\boldsymbol{D}^{\mathrm{R}} = \mathrm{Re}(\boldsymbol{D}),\ \boldsymbol{D}^{\mathrm{I}} = \mathrm{Im}(\boldsymbol{D}),\ \boldsymbol{E}^{\mathrm{R}} = \mathrm{Re}(\boldsymbol{E}),\ \boldsymbol{E}^{\mathrm{I}} = \mathrm{Im}(\boldsymbol{E}),$$
$$\boldsymbol{H}^{\mathrm{R}} = \mathrm{Re}(\boldsymbol{H}),\ \boldsymbol{H}^{\mathrm{I}} = \mathrm{Im}(\boldsymbol{H}),\ \boldsymbol{J}_k^{\mathrm{R}}(\cdot) = \mathrm{Re}(\boldsymbol{J}_k(\cdot)),\ \boldsymbol{J}_k^{\mathrm{I}}(\cdot) = \mathrm{Im}(\boldsymbol{J}_k(\cdot)).$$

定理 1 在假设(H1)成立的条件下,若存在矩阵 $\boldsymbol{U}_1, \boldsymbol{U}_2 \in C^{n\times n}$ 和正定对角矩阵 $\boldsymbol{R} \in R^{n\times n}$ 使得

$$\begin{pmatrix} \boldsymbol{\Pi}_{11} & \boldsymbol{\Pi}_{12} \\ * & \boldsymbol{\Pi}_{22} \end{pmatrix} < \boldsymbol{0} \tag{4}$$

成立,其中

$$\boldsymbol{\Pi}_{11}=\boldsymbol{\Gamma}^{*}\boldsymbol{R}\boldsymbol{\Gamma}-\boldsymbol{U}_1\boldsymbol{C}-\boldsymbol{C}\boldsymbol{U}_1^{*},\boldsymbol{\Pi}_{12}=\boldsymbol{U}_1(\boldsymbol{D}+\boldsymbol{E})-\boldsymbol{C}\boldsymbol{U}_2^{*},$$
$$\boldsymbol{\Pi}_{22}=\boldsymbol{U}_2(\boldsymbol{D}+\boldsymbol{E})+(\boldsymbol{D}^{*}+\boldsymbol{E}^{*})\boldsymbol{U}_2^{*}-\boldsymbol{R},$$

则神经网络(1)存在唯一的平衡点.

注 2 “*”表示矩阵块的共轭转置.

证明 若模型(1)存在唯一的平衡点,即映射 $\boldsymbol{\psi}(z):C^n\to C^n$,

$$\boldsymbol{\psi}(z)=-\boldsymbol{C}z+(\boldsymbol{D}+\boldsymbol{E})\boldsymbol{f}(z)+\boldsymbol{H} \tag{5}$$

为自同胚映射.

下面分两步证明.

第一步:证明 $\boldsymbol{\psi}(z)$ 在 C^n 上为单射.

假设存在 $z,z'\in C^n,z\neq z'$,使得 $\boldsymbol{\psi}(z)=\boldsymbol{\psi}(z')$, 则有

$$\boldsymbol{0}=-\boldsymbol{C}(z-z')+(\boldsymbol{D}+\boldsymbol{E})(\boldsymbol{f}(z)-\boldsymbol{f}(z')). \tag{6}$$

式(6)两边同时乘以 $(z-z')^{*}\boldsymbol{U}_1+(\boldsymbol{f}(z)-\boldsymbol{f}(z'))^{*}\boldsymbol{U}_2$, 可得

$$\begin{aligned}0=&-(z-z')^{*}\boldsymbol{U}_1\boldsymbol{C}(z-z')+(z-z')^{*}\boldsymbol{U}_1(\boldsymbol{D}+\boldsymbol{E})(\boldsymbol{f}(z)-\boldsymbol{f}(z'))-\\&(\boldsymbol{f}(z)-\boldsymbol{f}(z'))^{*}\boldsymbol{U}_2\boldsymbol{C}(z-z')+\\&(\boldsymbol{f}(z)-\boldsymbol{f}(z'))^{*}\boldsymbol{U}_2(\boldsymbol{D}+\boldsymbol{E})(\boldsymbol{f}(z)-\boldsymbol{f}(z')),\end{aligned} \tag{7}$$

且式(7)的共轭转置为

$$\begin{aligned}0=&-(z-z')^{*}\boldsymbol{C}\boldsymbol{U}_1^{*}(z-z')+(\boldsymbol{f}(z)-\boldsymbol{f}(z'))^{*}(\boldsymbol{D}^{*}+\boldsymbol{E}^{*})\boldsymbol{U}_1^{*}(z-z')-\\&(z-z')^{*}\boldsymbol{C}\boldsymbol{U}_2^{*}(\boldsymbol{f}(z)-\boldsymbol{f}(z'))+\\&(\boldsymbol{f}(z)-\boldsymbol{f}(z'))^{*}(\boldsymbol{D}^{*}+\boldsymbol{E}^{*})\boldsymbol{U}_2^{*}(\boldsymbol{f}(z)-\boldsymbol{f}(z')).\end{aligned} \tag{8}$$

将式(7)、(8)相加,由引理 4 可得

$$\begin{aligned}0=&-(z-z')^{*}(\boldsymbol{U}_1\boldsymbol{C}+\boldsymbol{C}\boldsymbol{U}_1^{*})(z-z')+\\&(z-z')^{*}[\boldsymbol{U}_1(\boldsymbol{D}+\boldsymbol{E})-\boldsymbol{C}\boldsymbol{U}_2^{*}](\boldsymbol{f}(z)-\boldsymbol{f}(z'))+\\&(\boldsymbol{f}(z)-\boldsymbol{f}(z'))^{*}[(\boldsymbol{D}^{*}+\boldsymbol{E}^{*})\boldsymbol{U}_1^{*}-\boldsymbol{U}_2\boldsymbol{C}](z-z')+\\&(\boldsymbol{f}(z)-\boldsymbol{f}(z'))^{*}[\boldsymbol{U}_2(\boldsymbol{D}+\boldsymbol{E})+(\boldsymbol{D}^{*}+\boldsymbol{E}^{*})\boldsymbol{U}_2^{*}](\boldsymbol{f}(z)-\boldsymbol{f}(z'))\leqslant\\&-(z-z')^{*}(\boldsymbol{U}_1\boldsymbol{C}+\boldsymbol{C}\boldsymbol{U}_1^{*})(z-z')+\\&(z-z')^{*}[\boldsymbol{U}_1(\boldsymbol{D}+\boldsymbol{E})-\boldsymbol{C}\boldsymbol{U}_2^{*}]\boldsymbol{\Sigma}_1^{-1}[(\boldsymbol{D}^{*}+\boldsymbol{E}^{*})\boldsymbol{U}_1^{*}-\boldsymbol{U}_2\boldsymbol{C}](z-z')+\\&(\boldsymbol{f}(z)-\boldsymbol{f}(z'))^{*}\boldsymbol{\Sigma}_1(\boldsymbol{f}(z)-\boldsymbol{f}(z'))+\\&(\boldsymbol{f}(z)-\boldsymbol{f}(z'))^{*}[\boldsymbol{U}_2(\boldsymbol{D}+\boldsymbol{E})+(\boldsymbol{D}^{*}+\boldsymbol{E}^{*})\boldsymbol{U}_2^{*})](\boldsymbol{f}(z)-\boldsymbol{f}(z'))=\\&(z-z')^{*}\boldsymbol{\Sigma}_2(z-z')+(\boldsymbol{f}(z)-\boldsymbol{f}(z'))^{*}\boldsymbol{R}(\boldsymbol{f}(z)-\boldsymbol{f}(z')),\end{aligned} \tag{9}$$

其中

$$\boldsymbol{\Sigma}_1=\boldsymbol{R}-\boldsymbol{U}_2(\boldsymbol{D}+\boldsymbol{E})-(\boldsymbol{D}^{*}+\boldsymbol{E}^{*})\boldsymbol{U}_2^{*},$$
$$\boldsymbol{\Sigma}_2=-\boldsymbol{U}_1\boldsymbol{C}-\boldsymbol{C}\boldsymbol{U}_1^{*}+[\boldsymbol{U}_1(\boldsymbol{D}+\boldsymbol{E})-\boldsymbol{C}\boldsymbol{U}_2^{*}]\boldsymbol{\Sigma}_1^{-1}[(\boldsymbol{D}^{*}+\boldsymbol{E}^{*})\boldsymbol{U}_1^{*}-\boldsymbol{U}_2\boldsymbol{C}].$$

显然 $\boldsymbol{\Sigma}_1=\boldsymbol{\Sigma}_1^{*}$,所以由引理 3 和式(4) 知 $\boldsymbol{\Sigma}_1$ 是正定 Hermite 矩阵.

由假设(H1)和式(2)、(3)知

$$\xi_j^{R-}\leqslant\frac{f_j^{R}(x_j)-f_j^{R}(x_j')}{x_j-x_j'}\leqslant\xi_j^{R+},\qquad j=1,2,\cdots,n, \tag{10}$$

$$\xi_j^{I-}\leqslant\frac{f_j^{I}(y_j)-f_j^{I}(y_j')}{y_j-y_j'}\leqslant\xi_j^{I+},\qquad j=1,2,\cdots,n. \tag{11}$$

从而

$$|f_j^{\mathrm{R}}(x_j)-f_j^{\mathrm{R}}(x_j')|\leqslant \xi_j^{\mathrm{R}}|x_j-x_j'|,\qquad j=1,2,\cdots,n,\tag{12}$$

$$|f_j^{\mathrm{I}}(y_j)-f_j^{\mathrm{I}}(y_j')|\leqslant \xi_j^{\mathrm{I}}|y_j-y_j'|,\qquad j=1,2,\cdots,n,\tag{13}$$

其中

$$\xi_j^{\mathrm{R}}=\max\{|\xi_j^{\mathrm{R}-}|,|\xi_j^{\mathrm{R}+}|\},\ \xi_j^{\mathrm{I}}=\max\{|\xi_j^{\mathrm{I}-}|,|\xi_j^{\mathrm{I}+}|\}.$$

于是有

$$r_j(f_j(z_j)-f_j(z_j'))^*(f_j(z_j)-f_j(z_j'))\leqslant r_j\xi_j^2(z_j-z_j')^*(z_j-z_j'),\quad j=1,2,\cdots,n,\tag{14}$$

其中 $\xi_j=\max\{\xi_j^{\mathrm{R}},\xi_j^{\mathrm{I}}\}$，$r_j$ 为常数且 $r_j>0$，所以

$$(\boldsymbol{f}(z)-\boldsymbol{f}(z'))^*\boldsymbol{R}(\boldsymbol{f}(z)-\boldsymbol{f}(z'))\leqslant(z-z')^*\boldsymbol{\Gamma}^*\boldsymbol{R}\boldsymbol{\Gamma}(z-z'),\tag{15}$$

其中 $\boldsymbol{\Gamma}=(\xi_1,\xi_2,\cdots,\xi_n)$.那么，由式(9)和式(15)得

$$(z-z')^*\boldsymbol{\Sigma}_3(z-z')\geqslant 0,\tag{16}$$

其中

$$\boldsymbol{\Sigma}_3=\boldsymbol{\Gamma}^*\boldsymbol{R}\boldsymbol{\Gamma}+\boldsymbol{\Sigma}_2.$$

由引理 3 和式(4)可知 $\boldsymbol{\Sigma}_3<\boldsymbol{0}$，所以 $z-z'=\boldsymbol{0}$，故 $\boldsymbol{\psi}(z)$ 在 C^n 上为单射.

第二步：证明 $\lim_{\|z\|\to\infty}\|\boldsymbol{\psi}(z)\|=\infty$.令

$$\tilde{\boldsymbol{\psi}}(z)=\boldsymbol{\psi}(z)-\boldsymbol{\psi}(\boldsymbol{0})=-\boldsymbol{C}z+(\boldsymbol{D}+\boldsymbol{E})(\boldsymbol{f}(z)-\boldsymbol{f}(\boldsymbol{0})),\tag{17}$$

则有

$$\begin{aligned}&[z^*\boldsymbol{U}_1+(\boldsymbol{f}(z)-\boldsymbol{f}(\boldsymbol{0}))^*\boldsymbol{U}_2]\tilde{\boldsymbol{\psi}}(z)+\\&\quad\tilde{\boldsymbol{\psi}}(z)^*[z^*\boldsymbol{U}_1+(\boldsymbol{f}(z)-\boldsymbol{f}(\boldsymbol{0}))^*\boldsymbol{U}_2]^*=\\&\quad -z^*(\boldsymbol{U}_1\boldsymbol{C}+\boldsymbol{C}\boldsymbol{U}_1^*)z+z^*[\boldsymbol{U}_1(\boldsymbol{D}+\boldsymbol{E})-\boldsymbol{C}\boldsymbol{U}_2^*](\boldsymbol{f}(z)-\boldsymbol{f}(\boldsymbol{0}))+\\&\quad(\boldsymbol{f}(z)-\boldsymbol{f}(\boldsymbol{0}))^*[(\boldsymbol{D}^*+\boldsymbol{E}^*)\boldsymbol{U}_1^*-\boldsymbol{U}_2\boldsymbol{C}]z+\\&\quad(\boldsymbol{f}(z)-\boldsymbol{f}(\boldsymbol{0}))^*[\boldsymbol{U}_2(\boldsymbol{D}+\boldsymbol{E})+(\boldsymbol{D}^*+\boldsymbol{E}^*)\boldsymbol{U}_2^*](\boldsymbol{f}(z)-\boldsymbol{f}(\boldsymbol{0}))\leqslant\\&\quad z^*\boldsymbol{\Sigma}_2z+(\boldsymbol{f}(z)-\boldsymbol{f}(\boldsymbol{0}))^*\boldsymbol{R}(\boldsymbol{f}(z)-\boldsymbol{f}(\boldsymbol{0}))\leqslant\\&\quad z^*\boldsymbol{\Sigma}_3z\leqslant-\lambda_{\min}(-\boldsymbol{\Sigma}_3)\|z\|^2.\end{aligned}\tag{18}$$

根据式(18)，可得

$$\begin{aligned}\lambda_{\min}(-\boldsymbol{\Sigma}_3)\|z\|^2&\leqslant 2\|z^*\boldsymbol{U}_1+(\boldsymbol{f}(z)-\boldsymbol{f}(\boldsymbol{0}))^*\boldsymbol{U}_2\|\,\|\tilde{\boldsymbol{\psi}}(z)\|\leqslant\\&\quad 2(\|\boldsymbol{U}_1\|+\|\boldsymbol{\Gamma}^*\boldsymbol{U}_2\|)\|z\|\,\|\tilde{\boldsymbol{\psi}}(z)\|.\end{aligned}\tag{19}$$

于是，当 $z\neq\boldsymbol{0}$ 时，有

$$\|\tilde{\boldsymbol{\psi}}(z)\|\geqslant\frac{\lambda_{\min}(-\boldsymbol{\Sigma}_3)\|z\|}{2(\|\boldsymbol{U}_1\|+\|\boldsymbol{\Gamma}^*\boldsymbol{U}_2\|)}.\tag{20}$$

因此，当 $\|z\|\to\infty$ 时，$\|\tilde{\boldsymbol{\psi}}(z)\|\to\infty$.即 $\|z\|\to\infty$ 时，$\|\boldsymbol{\psi}(z)\|\to\infty$.

综上所述，由引理 1 可得 $\boldsymbol{\psi}(z)$ 在 C^n 上是自同胚映射，故模型(1)存在唯一的平衡点.证毕.

定理 2 如果存在正定 Hermite 矩阵 $\boldsymbol{P}_1,\boldsymbol{P}_2,\boldsymbol{P}_3\in C^{n\times n}$，正定对角矩阵 $\boldsymbol{R}_1,\boldsymbol{R}_2\in R^{n\times n}$ 和复值矩阵 $\boldsymbol{Q}_1,\boldsymbol{Q}_2,\boldsymbol{Q}_3\in C^{n\times n}$，使得不等式

$$\begin{pmatrix}\boldsymbol{P}_1 & (\boldsymbol{I}+\boldsymbol{N}_k)\boldsymbol{P}_1\\ * & \boldsymbol{P}_1\end{pmatrix}>\boldsymbol{0},\tag{21}$$

$$\begin{pmatrix} \boldsymbol{\Omega}_{11} & \boldsymbol{\Omega}_{12} & \frac{1}{\tau}\boldsymbol{P}_2 - \boldsymbol{C}\boldsymbol{Q}_3 & \boldsymbol{Q}_2^*\boldsymbol{D} & \boldsymbol{Q}_2^*\boldsymbol{E} \\ * & \boldsymbol{\Omega}_{22} & -\boldsymbol{Q}_3 & \boldsymbol{Q}_1^*\boldsymbol{D} & \boldsymbol{Q}_1^*\boldsymbol{E} \\ * & * & \boldsymbol{\Omega}_{33} & \boldsymbol{Q}_3^*\boldsymbol{D} & \boldsymbol{Q}_3^*\boldsymbol{E} \\ * & * & * & -\boldsymbol{R}_1 & \boldsymbol{0} \\ * & * & * & * & -\boldsymbol{R}_2 \end{pmatrix} < \boldsymbol{0} \tag{22}$$

成立，其中

$$\boldsymbol{\Omega}_{11} = \boldsymbol{P}_3 + \boldsymbol{\Gamma}^*\boldsymbol{R}_1\boldsymbol{\Gamma} - (1/\tau)\boldsymbol{P}_2 - \boldsymbol{Q}_2^*\boldsymbol{C} - \boldsymbol{C}\boldsymbol{Q}_2,$$

$$\boldsymbol{\Omega}_{12} = \boldsymbol{P}_1 - \boldsymbol{Q}_2^* - \boldsymbol{C}\boldsymbol{Q}_1,\ \boldsymbol{\Omega}_{22} = \bar{\mu}\boldsymbol{P}_1 + \tau\boldsymbol{P}_2 - \boldsymbol{Q}_1^* - \boldsymbol{Q}_1,$$

$$\boldsymbol{\Omega}_{33} = -(1/\tau)\boldsymbol{P}_2 - \boldsymbol{P}_3 - \boldsymbol{\Gamma}^*\boldsymbol{R}_2\boldsymbol{\Gamma},$$

则神经网络(1)的平衡点是全局稳定的。

证明 在定理 1 成立的条件下，系统(1)有唯一的平衡点 $\hat{z} = (\hat{z}_1, \hat{z}_2, \cdots, \hat{z}_n)^{\mathrm{T}} \in C^n$，令

$$w_i(t) = z_i(t) - \hat{z}_i,\ f_i(w_i(t)) = f_i(z_i(t)) - f_i(\hat{z}_i), \qquad i = 1,2,\cdots,n,$$

则系统(1)的平衡点由 $\hat{z}$ 移动到原点，于是有

$$\begin{cases} \boldsymbol{w}^{\Delta}(t) = -\boldsymbol{C}\boldsymbol{w}(t) + \boldsymbol{D}\boldsymbol{f}(\boldsymbol{w}(t)) + \boldsymbol{E}\boldsymbol{f}(\boldsymbol{w}(t-\tau)), & t \neq t_k,\ t \in \boldsymbol{T}_0^+, \\ \Delta\boldsymbol{w}(t_k) = \boldsymbol{w}(t_k) - \boldsymbol{w}(t_k^-) = \boldsymbol{J}_k(\boldsymbol{w}(t_k^-), \boldsymbol{w}_{t_k^-}), & t = t_k,\ k \in \mathbf{Z}_+, \end{cases} \tag{23}$$

其中，令脉冲函数 $\boldsymbol{J}_k(\boldsymbol{z}(t_k^-), \boldsymbol{z}_{t_k^-}) = \boldsymbol{N}_k(\boldsymbol{z}(t_k^-) - \hat{\boldsymbol{z}}) = \boldsymbol{N}_k\boldsymbol{w}(t_k^-)$，显然 $\boldsymbol{J}_k(\hat{\boldsymbol{z}}, \hat{\boldsymbol{z}}) = \boldsymbol{0}$。

构造 Lyapunov-Krasovskii 泛函：

$$\boldsymbol{V}(t) = \boldsymbol{V}_1(t) + \boldsymbol{V}_2(t), \tag{24}$$

其中

$$\boldsymbol{V}_1(t) = \boldsymbol{w}^*(t)\boldsymbol{P}_1\boldsymbol{w}(t),$$

$$\boldsymbol{V}_2(t) = \int_0^{\tau}\int_{t-\theta}^{t}(\boldsymbol{w}^{\Delta}(s))^*\boldsymbol{P}_2\boldsymbol{w}^{\Delta}(s)\,\mathrm{d}s\mathrm{d}\theta + \int_{t-\tau}^{t}\boldsymbol{w}^*(s)\boldsymbol{P}_3\boldsymbol{w}(s)\,\mathrm{d}s.$$

当 $t \neq t_k$ 时，沿着模型(23)求导，由引理 2 和引理 5 可得

$$\begin{aligned} \boldsymbol{V}_1^{\Delta}(t) &= (\boldsymbol{w}^{\Delta}(t))^*\boldsymbol{P}_1\boldsymbol{w}(t) + \boldsymbol{w}^*(\sigma(t))\boldsymbol{P}_1\boldsymbol{w}^{\Delta}(t) = \\ &(\boldsymbol{w}^{\Delta}(t))^*\boldsymbol{P}_1\boldsymbol{w}(t) + (\boldsymbol{w}(t) + \mu(t)\boldsymbol{w}^{\Delta}(t))^*\boldsymbol{P}_1\boldsymbol{w}^{\Delta}(t) = \\ &(\boldsymbol{w}^{\Delta}(t))^*\boldsymbol{P}_1\boldsymbol{w}(t) + \boldsymbol{w}^*(t)\boldsymbol{P}_1\boldsymbol{w}^{\Delta}(t) + \mu(t)(\boldsymbol{w}^{\Delta}(t))^*\boldsymbol{P}_1\boldsymbol{w}^{\Delta}(t) \leqslant \\ &(\boldsymbol{w}^{\Delta}(t))^*\boldsymbol{P}_1\boldsymbol{w}(t) + \boldsymbol{w}^*(t)\boldsymbol{P}_1\boldsymbol{w}^{\Delta}(t) + \bar{\mu}(\boldsymbol{w}^{\Delta}(t))^*\boldsymbol{P}_1\boldsymbol{w}^{\Delta}(t), \end{aligned} \tag{25}$$

$$\begin{aligned} \boldsymbol{V}_2^{\Delta}(t) &= \tau(\boldsymbol{w}^{\Delta}(t))^*\boldsymbol{P}_2\boldsymbol{w}^{\Delta}(t) - \int_0^{\tau}(\boldsymbol{w}^{\Delta}(t-\theta))^*\boldsymbol{P}_2\boldsymbol{w}^{\Delta}(t-\theta)\,\mathrm{d}\theta + \\ &\boldsymbol{w}^*(t)\boldsymbol{P}_3\boldsymbol{w}(t) - \boldsymbol{w}^*(t-\tau)\boldsymbol{P}_3\boldsymbol{w}(t-\tau) = \\ &\tau(\boldsymbol{w}^{\Delta}(t))^*\boldsymbol{P}_2\boldsymbol{w}^{\Delta}(t) - \int_{t-\tau}^{t}(\boldsymbol{w}^{\Delta}(s))^*\boldsymbol{P}_2\boldsymbol{w}^{\Delta}(s)\,\mathrm{d}s + \\ &\boldsymbol{w}^*(t)\boldsymbol{P}_3\boldsymbol{w}(t) - \boldsymbol{w}^*(t-\tau)\boldsymbol{P}_3\boldsymbol{w}(t-\tau) \leqslant \\ &\tau(\boldsymbol{w}^{\Delta}(t))^*\boldsymbol{P}_2\boldsymbol{w}^{\Delta}(t) - \frac{1}{\tau}\int_{t-\tau}^{t}(\boldsymbol{w}^{\Delta}(s))^*\mathrm{d}s\,\boldsymbol{P}_2\int_{t-\tau}^{t}\boldsymbol{w}^{\Delta}(s)\,\mathrm{d}s + \\ &\boldsymbol{w}^*(t)\boldsymbol{P}_3\boldsymbol{w}(t) - \boldsymbol{w}^*(t-\tau)\boldsymbol{P}_3\boldsymbol{w}(t-\tau) = \\ &\tau(\boldsymbol{w}^{\Delta}(t))^*\boldsymbol{P}_2\boldsymbol{w}^{\Delta}(t) - \frac{1}{\tau}(\boldsymbol{w}^*(t) - \boldsymbol{w}^*(t-\tau))\boldsymbol{P}_2(\boldsymbol{w}(t) - \boldsymbol{w}(t-\tau)) + \end{aligned}$$

$$
\begin{aligned}
&\boldsymbol{w}^*(t)\boldsymbol{P}_3\boldsymbol{w}(t)-\boldsymbol{w}^*(t-\tau)\boldsymbol{P}_3\boldsymbol{w}(t-\tau)=\\
&\tau(\boldsymbol{w}^{\Delta}(t))^*\boldsymbol{P}_2\boldsymbol{w}^{\Delta}(t)+\boldsymbol{w}^*(t)\left(\boldsymbol{P}_3-\frac{1}{\tau}\boldsymbol{P}_2\right)\boldsymbol{w}(t)+\\
&\frac{1}{\tau}\boldsymbol{w}^*(t)\boldsymbol{P}_2\boldsymbol{w}(t-\tau)+\frac{1}{\tau}\boldsymbol{w}^*(t-\tau)\boldsymbol{P}_2\boldsymbol{w}(t)-\\
&\boldsymbol{w}^*(t-\tau)\left(\frac{1}{\tau}\boldsymbol{P}_2+\boldsymbol{P}_3\right)\boldsymbol{w}(t-\tau).
\end{aligned}\tag{26}
$$

将式(25)和(26)相加,得

$$
\begin{aligned}
\boldsymbol{V}^{\Delta}(t)\leqslant&(\boldsymbol{w}^{\Delta}(t))^*\boldsymbol{P}_1\boldsymbol{w}(t)+\boldsymbol{w}^*(t)\boldsymbol{P}_1\boldsymbol{w}^{\Delta}(t)+\\
&(\boldsymbol{w}^{\Delta}(t))^*(\bar{\mu}\boldsymbol{P}_1+\tau\boldsymbol{P}_2)\boldsymbol{w}^{\Delta}(t)+\boldsymbol{w}^*(t)\left(\boldsymbol{P}_3-\frac{1}{\tau}\boldsymbol{P}_2\right)\boldsymbol{w}(t)+\\
&\frac{1}{\tau}\boldsymbol{w}^*(t)\boldsymbol{P}_2\boldsymbol{w}(t-\tau)+\frac{1}{\tau}\boldsymbol{w}^*(t-\tau)\boldsymbol{P}_2\boldsymbol{w}(t)-\\
&\boldsymbol{w}^*(t-\tau)\left(\frac{1}{\tau}\boldsymbol{P}_2+\boldsymbol{P}_3\right)\boldsymbol{w}(t-\tau).
\end{aligned}\tag{27}
$$

根据式(15)可知

$$
\boldsymbol{f}^*(\boldsymbol{w}(t))\boldsymbol{R}_1\boldsymbol{f}(\boldsymbol{w}(t))-\boldsymbol{w}^*(t)\boldsymbol{\Gamma}^*\boldsymbol{R}_1\boldsymbol{\Gamma}\boldsymbol{w}(t)\leqslant 0.\tag{28}
$$

同理,有

$$
\boldsymbol{f}^*(\boldsymbol{w}(t-\tau))\boldsymbol{R}_2\boldsymbol{f}(\boldsymbol{w}(t-\tau))-\boldsymbol{w}^*(t-\tau)\boldsymbol{\Gamma}^*\boldsymbol{R}_2\boldsymbol{\Gamma}\boldsymbol{w}(t-\tau)\leqslant 0.\tag{29}
$$

且由式(23)得

$$
\begin{aligned}
0=&[-\boldsymbol{w}^{\Delta}(t)-\boldsymbol{C}\boldsymbol{w}(t)+\boldsymbol{D}\boldsymbol{f}(\boldsymbol{w}(t))+\\
&\boldsymbol{E}\boldsymbol{f}(\boldsymbol{w}(t-\tau))]^*(\boldsymbol{Q}_1\boldsymbol{w}^{\Delta}(t)+\boldsymbol{Q}_2\boldsymbol{w}(t)+\boldsymbol{Q}_3\boldsymbol{w}(t-\tau))+\\
&(\boldsymbol{Q}_1\boldsymbol{w}^{\Delta}(t)+\boldsymbol{Q}_2\boldsymbol{w}(t)+\boldsymbol{Q}_3\boldsymbol{w}(t-\tau))^*[-\boldsymbol{w}^{\Delta}(t)-\\
&\boldsymbol{C}\boldsymbol{w}(t)+\boldsymbol{D}\boldsymbol{f}(\boldsymbol{w}(t))+\boldsymbol{E}\boldsymbol{f}(\boldsymbol{w}(t-\tau))].
\end{aligned}\tag{30}
$$

于是,将式(27)、(28)、(29)和(30)相加,可得

$$
\boldsymbol{V}^{\Delta}(t)\leqslant\boldsymbol{v}^*(t)\boldsymbol{\Omega}\boldsymbol{v}(t),\tag{31}
$$

其中

$$
\boldsymbol{v}(t)=(\boldsymbol{w}(t),\boldsymbol{w}^{\Delta}(t),\boldsymbol{w}(t-\tau),\boldsymbol{f}(\boldsymbol{w}(t)),\boldsymbol{f}(\boldsymbol{w}(t-\tau)))^*,
$$

$$
\boldsymbol{\Omega}=\begin{pmatrix}
\boldsymbol{\Omega}_{11} & \boldsymbol{\Omega}_{12} & \frac{1}{\tau}\boldsymbol{P}_2-\boldsymbol{C}\boldsymbol{Q}_3 & \boldsymbol{Q}_2^*\boldsymbol{D} & \boldsymbol{Q}_2^*\boldsymbol{E}\\
* & \boldsymbol{\Omega}_{22} & -\boldsymbol{Q}_3 & \boldsymbol{Q}_1^*\boldsymbol{D} & \boldsymbol{Q}_1^*\boldsymbol{E}\\
* & * & \boldsymbol{\Omega}_{33} & \boldsymbol{Q}_3^*\boldsymbol{D} & \boldsymbol{Q}_3^*\boldsymbol{E}\\
* & * & * & -\boldsymbol{R}_1 & \boldsymbol{0}\\
* & * & * & * & -\boldsymbol{R}_2
\end{pmatrix},
$$

$$
\boldsymbol{\Omega}_{11}=\boldsymbol{P}_3+\boldsymbol{\Gamma}^*\boldsymbol{R}_1\boldsymbol{\Gamma}-\frac{1}{\tau}\boldsymbol{P}_2-\boldsymbol{Q}_2^*\boldsymbol{C}-\boldsymbol{C}\boldsymbol{Q}_2,\ \boldsymbol{\Omega}_{12}=\boldsymbol{P}_1-\boldsymbol{Q}_2^*-\boldsymbol{C}\boldsymbol{Q}_1,
$$

$$
\boldsymbol{\Omega}_{22}=\bar{\mu}\boldsymbol{P}_1+\tau\boldsymbol{P}_2-\boldsymbol{Q}_1^*-\boldsymbol{Q}_1,\ \boldsymbol{\Omega}_{33}=-\frac{1}{\tau}\boldsymbol{P}_2-\boldsymbol{P}_3-\boldsymbol{\Gamma}^*\boldsymbol{R}_2\boldsymbol{\Gamma}.
$$

故由式(22)和(31)知

$$
\boldsymbol{V}^{\Delta}(t)\leqslant 0,\qquad t\neq t_k,\ t\in T_0^+,\ k\in\mathbf{Z}_+.\tag{32}
$$

当 $t = t_k$ 时，因为

$$\begin{pmatrix} \boldsymbol{P}_1 & (\boldsymbol{I}+\boldsymbol{N}_k)\boldsymbol{P}_1 \\ * & \boldsymbol{P}_1 \end{pmatrix} > \boldsymbol{0} \Leftrightarrow$$

$$\begin{pmatrix} \boldsymbol{I} & \boldsymbol{0} \\ * & \boldsymbol{P}_1^{-1} \end{pmatrix}\begin{pmatrix} \boldsymbol{P}_1 & (\boldsymbol{I}+\boldsymbol{N}_k)\boldsymbol{P}_1 \\ * & \boldsymbol{P}_1 \end{pmatrix}\begin{pmatrix} \boldsymbol{I} & \boldsymbol{0} \\ * & \boldsymbol{P}_1^{-1} \end{pmatrix} > \boldsymbol{0} \Leftrightarrow$$

$$\begin{pmatrix} \boldsymbol{P}_1 & \boldsymbol{I}+\boldsymbol{N}_k \\ * & \boldsymbol{P}_1^{-1} \end{pmatrix} > \boldsymbol{0} \Leftrightarrow \boldsymbol{P}_1 - (\boldsymbol{I}+\boldsymbol{N}_k)^* \boldsymbol{P}_1 (\boldsymbol{I}+\boldsymbol{N}_k) > \boldsymbol{0}, \tag{33}$$

所以

$$\begin{aligned}
\boldsymbol{V}(t_k) &= \boldsymbol{V}_1(t_k) + \boldsymbol{V}_2(t_k) = \\
&\boldsymbol{w}^*(t_k)\boldsymbol{P}_1\boldsymbol{w}(t_k) + \\
&\int_0^\tau\int_{t_k-\theta}^{t_k}(\boldsymbol{w}^\Delta(s))^*\boldsymbol{P}_2\boldsymbol{w}^\Delta(s)\,\mathrm{d}s\mathrm{d}\theta + \int_{t_k-\tau}^{t_k}\boldsymbol{w}^*(s)\boldsymbol{P}_3\boldsymbol{w}(s)\,\mathrm{d}s = \\
&(\boldsymbol{w}(t_k^-) + \boldsymbol{N}_k\boldsymbol{w}(t_k^-))^*\boldsymbol{P}_1(\boldsymbol{w}(t_k^-) + \boldsymbol{N}_k\boldsymbol{w}(t_k^-)) + \\
&\int_0^\tau\int_{t_k^--\theta}^{t_k^-}(\boldsymbol{w}^\Delta(s))^*\boldsymbol{P}_2\boldsymbol{w}^\Delta(s)\,\mathrm{d}s\mathrm{d}\theta + \int_{t_k^--\tau}^{t_k^-}\boldsymbol{w}^*(s)\boldsymbol{P}_3\boldsymbol{w}(s)\,\mathrm{d}s = \\
&\boldsymbol{w}^*(t_k^-)(\boldsymbol{I}+\boldsymbol{N}_k)^*\boldsymbol{P}_1(\boldsymbol{I}+\boldsymbol{N}_k)\boldsymbol{w}(t_k^-) + \\
&\int_0^\tau\int_{t_k^--\theta}^{t_k^-}(\boldsymbol{w}^\Delta(s))^*\boldsymbol{P}_2\boldsymbol{w}^\Delta(s)\,\mathrm{d}s\mathrm{d}\theta + \int_{t_k^--\tau}^{t_k^-}\boldsymbol{w}^*(s)\boldsymbol{P}_3\boldsymbol{w}(s)\,\mathrm{d}s \leqslant \\
&\boldsymbol{w}^*(t_k^-)\boldsymbol{P}_1\boldsymbol{w}(t_k^-) + \\
&\int_0^\tau\int_{t_k^--\theta}^{t_k^-}(\boldsymbol{w}^\Delta(s))^*\boldsymbol{P}_2\boldsymbol{w}^\Delta(s)\,\mathrm{d}s\mathrm{d}\theta + \int_{t_k^--\tau}^{t_k^-}\boldsymbol{w}^*(s)\boldsymbol{P}_3\boldsymbol{w}(s)\,\mathrm{d}s = \\
&\boldsymbol{V}_1(t_k^-) + \boldsymbol{V}_2(t_k^-) = \boldsymbol{V}(t_k^-),
\end{aligned} \tag{34}$$

即

$$\boldsymbol{V}(t_k) \leqslant \boldsymbol{V}(t_k^-), \qquad k \in \mathbf{Z}_+. \tag{35}$$

根据式(32)和(35)知，$\boldsymbol{V}(t)$ 在 $t \in T_0^+$ 上为单调不增函数，于是

$$\begin{aligned}
\boldsymbol{V}(t) &\leqslant \boldsymbol{V}(0) = \\
&\boldsymbol{w}^*(0)\boldsymbol{P}_1\boldsymbol{w}(0) + \int_0^\tau\int_{-\theta}^0(\boldsymbol{w}^\Delta(s))^*\boldsymbol{P}_2\boldsymbol{w}^\Delta(s)\,\mathrm{d}s\mathrm{d}\theta + \int_{-\tau}^0\boldsymbol{w}^*(s)\boldsymbol{P}_3\boldsymbol{w}(s)\,\mathrm{d}s \leqslant \\
&(\|\boldsymbol{P}_1\| + \tau^2\|\boldsymbol{P}_2\| + \tau\|\boldsymbol{P}_3\|)(\sup_{s\in[-\tau,0]_T}\|\boldsymbol{\phi}(s)\|)^2.
\end{aligned} \tag{36}$$

又因为

$$\boldsymbol{V}(t) \geqslant \boldsymbol{V}_1(t) = \boldsymbol{w}^*(t)\boldsymbol{P}_1\boldsymbol{w}(t) \geqslant \lambda_{\min}(\boldsymbol{P}_1)\|\boldsymbol{w}(t)\|^2, \tag{37}$$

所以

$$\|\boldsymbol{w}(t)\| \leqslant \frac{\sqrt{\|\boldsymbol{P}_1\| + \tau^2\|\boldsymbol{P}_2\| + \tau\|\boldsymbol{P}_3\|}}{\sqrt{\lambda_{\min}(\boldsymbol{P}_1)}}\sup_{s\in[-\tau,0]_T}\|\boldsymbol{\phi}(s)\|, \tag{38}$$

即

$$\|\boldsymbol{w}(t)\| \leqslant M\sup_{s\in[-\tau,0]_T}\|\boldsymbol{\phi}(s)\|, \tag{39}$$

其中

$$M = \frac{\sqrt{\|\boldsymbol{P}_1\| + \tau^2\|\boldsymbol{P}_2\| + \tau\|\boldsymbol{P}_3\|}}{\sqrt{\lambda_{\min}(\boldsymbol{P}_1)}}.$$

故由定义 5 知,神经网络(1)的平衡点是全局稳定的.证毕.

3 数值仿真例子

例 考虑模型(1)在时间标度 $T=\cup_{k=-1}^{+\infty}[2k,\ 2k+1]$ 上的 2 神经元脉冲复值神经网络,其中

$$\boldsymbol{C}=\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix},\ \boldsymbol{D}=\begin{pmatrix}2+\mathrm{i} & -3-\mathrm{i}\\ -2+\mathrm{i} & -1\end{pmatrix},\ \boldsymbol{E}=\begin{pmatrix}-2 & \mathrm{i}\\ -1+\mathrm{i} & -2\end{pmatrix},$$

$$\boldsymbol{H}=\begin{pmatrix}2+\mathrm{i}\\ 2+\mathrm{i}\end{pmatrix},\ \boldsymbol{N}_k=\begin{pmatrix}-1 & 0.5\mathrm{i}\\ 0.5\mathrm{i} & -0.5\end{pmatrix},\ \tau=0.11,$$

$$\begin{cases} f_j^{\mathrm{R}}(\mathrm{Re}(z_j))=\dfrac{1}{20}(|\mathrm{Re}(z_j)+1|+|\mathrm{Re}(z_j)-1|), \\ f_j^{\mathrm{I}}(\mathrm{Im}(z_j))=\dfrac{1}{10}(|\mathrm{Im}(z_j)+1|+|\mathrm{Im}(z_j)-1|), \end{cases} \qquad j=1,2,$$

$$\boldsymbol{\Gamma}=\mathrm{diag}(0.04,0.04),\ t_k=0.4k,\qquad k\in\mathbf{Z}_+.$$

由此定义,显然激励函数不是有界的.并且容易验证,时间标度 T 上的粒度函数 $\mu(t)$ 满足 $0\leqslant\mu(t)\leqslant 1$,所以 $\bar{\mu}=1$.运用 MATLAB 软件的 YALMIP Toolbox,求解线性矩阵不等式(4)、(21)和(22),可得

$$\boldsymbol{U}_1=10^5\times\begin{pmatrix}0.368\,5-1.913\,1\mathrm{i} & 1.105\,4-1.338\,5\mathrm{i}\\ -1.105\,4-2.075\,4\mathrm{i} & 0.368\,5+0.853\,5\mathrm{i}\end{pmatrix},$$

$$\boldsymbol{U}_2=10^5\times\begin{pmatrix}5.327\,0-1.105\,4\mathrm{i} & -2.774\,5+4.341\,5\mathrm{i}\\ 13.741\,4+29.687\,1\mathrm{i} & -1.105\,4-1.105\,4\mathrm{i}\end{pmatrix},$$

$$\boldsymbol{R}=10^5\times\begin{pmatrix}2.228\,6 & 0\\ 0 & 3.987\,4\end{pmatrix},$$

$$\boldsymbol{P}_1=\begin{pmatrix}2.508\,9 & -0.328\,8+0.144\,6\mathrm{i}\\ -0.328\,8-0.144\,6\mathrm{i} & 3.809\,9\end{pmatrix},$$

$$\boldsymbol{P}_2=\begin{pmatrix}0.018\,7 & -0.004\,8-0.000\,0\mathrm{i}\\ -0.004\,8+0.000\,0\mathrm{i} & 0.032\,8\end{pmatrix},$$

$$\boldsymbol{P}_3=\begin{pmatrix}0.748\,5 & -0.092\,6+0.044\,5\mathrm{i}\\ -0.092\,6-0.044\,5\mathrm{i} & 1.054\,1\end{pmatrix},$$

$$\boldsymbol{Q}_1=\begin{pmatrix}1.880\,8+0.076\,2\mathrm{i} & -0.194\,9-0.177\,3\mathrm{i}\\ -0.325\,4-0.036\,2\mathrm{i} & 2.989\,7-0.053\,7\mathrm{i}\end{pmatrix},$$

$$\boldsymbol{Q}_2=\begin{pmatrix}0.885\,2+0.087\,2\mathrm{i} & -0.092\,6-0.146\,0\mathrm{i}\\ -0.119\,0-0.103\,2\mathrm{i} & 1.258\,6-0.059\,7\mathrm{i}\end{pmatrix},$$

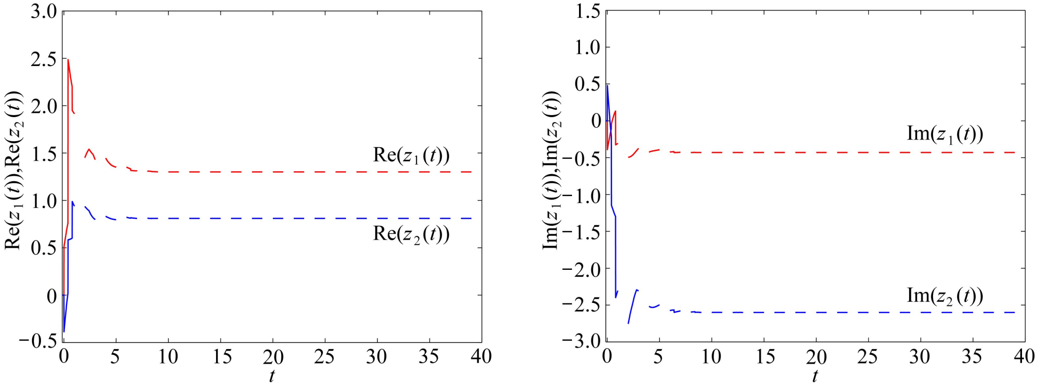
$$\boldsymbol{Q}_3=\begin{pmatrix}0.068\,9+0.001\,5\mathrm{i} & 0.070\,3-0.023\,6\mathrm{i}\\ -0.107\,4+0.021\,6\mathrm{i} & 0.104\,3-0.001\,5\mathrm{i}\end{pmatrix},$$

$$\boldsymbol{R}_1=\begin{pmatrix}267.631\,9 & 0\\ 0 & 274.414\,6\end{pmatrix},$$

$$\boldsymbol{R}_2=\begin{pmatrix}217.946\,1 & 0\\ 0 & 212.219\,0\end{pmatrix}.$$

由定理 1 和定理 2 可知,模型(1)存在唯一的平衡点,而且此平衡点是全局稳定的.如图 1

所示,状态变量的时间响应轨线图验证了平衡点的存在性、唯一性、全局稳定性,其中初始条件为 $z_1(t) = 0.5 - 0.4\mathrm{i}$, $z_2(t) = -0.4 + 0.5\mathrm{i}$.



(a) 实部的时间响应轨线

(a) Time responses of the real part

(b) 虚部的时间响应轨线

(b) Time responses of the imaginary part

图 1 状态变量的时间响应轨线

Fig. 1 Time responses of the state variables

4 结　　论

本文研究了时间标度上具有离散时滞的脉冲复值神经网络的稳定性问题,在不要求激励函数有界的条件下,给出了网络平衡点的存在性、唯一性和全局稳定性的复值 LMI 判据.获得的结果易于 MATLAB 软件的 YALMIP Toolbox 实现.数值仿真实例验证了获得结果的有效性.

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Global Stability of Impulsive Complex-Valued Neural Networks With Time Delay on Time Scales

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Abstract: The global stability of impulsive complex-valued neural networks with time delay on time scales was investigated. Based on the time scale calculus theory, both the continuous-time and discrete-time neural networks were described under the same framework. For the considered complex-valued neural networks, the activation functions need not be bounded. According to the homeomorphism mapping principle in the complex domain, a sufficient condition for the existence and uniqueness of the equilibrium point of the addressed complex-valued neural networks was proposed in complex-valued linear matrix inequality (LMI). Through the construction of appropriate Lyapunov-Krasovskii functionals, and with the free weighting matrix method and matrix inequality technique, a delay-dependent criterion for checking the global stability of the complex-valued neural networks was established in the complex-valued LMIs. Finally, a simulation example shows the effectiveness of the obtained results.

Key words: complex-valued neural network; time scale; time delay; impulsive; linear matrix inequality; global stability

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