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正交异性双材料界面裂纹尖端应力场*

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摘要: 通过构造新的应力函数,利用复合材料断裂复变方法,对正交异性双材料界面裂纹进行了研究. 在特征方程组的判别式都大于零的情形下,推出了Ⅰ型界面裂纹尖端的应力场、位移场的理论公式,其结果没有振荡奇异性及裂纹面没有相互嵌入现象.

关 键 词: 正交异性; 界面裂纹; 裂纹尖端; 应力场

中图分类号: O346.1;O174.5 **文献标识码:** A

引 言

文献[1-4]研究了复合材料界面裂纹尖端应力场. 文献[1]给出了各向同性双材料界面裂纹的应力场经典解. 在该文所构造的应力函数中的特征根 λ 只有是实数时才满足控制方程,但文中求出的 λ 是复数,事实上,当 λ 是复数时其应力函数是不满足原控制方程的. 正是由于这个复数特征根,致使应力场存在振荡奇异性及裂纹面有相互嵌入现象. 文献[2]也有类似的问题. 本文通过巧妙构造应力函数,结合复变函数理论,推出了满足控制方程且特征根为实数的应力函数,在特征方程组的判别式 $\Delta_1 > 0$ 和 $\Delta_2 > 0$ 的情形下,得到了正交异性双材料Ⅰ型界面裂纹尖端应力场、位移场的理论解,从而解决了该情况下界面裂纹尖端应力场振荡奇异性问题.

1 力学模型

如图1所示,$x \leqslant 0, y = 0$ 为界面裂纹,$x > 0, y = 0$ 为材料粘接界面. $y > 0$ 部分为第1种正交异性复合材料,其材料工程常数为 E_{11}、E_{12}、ν_{11}、ν_{12} 和 μ_1,而 $y < 0$ 为第2种正交异性复合材料,其材料工程常数为 E_{21}、E_{22}、ν_{21}、ν_{22} 和 μ_2.

由弹性力学可知,控制方程为

$$(b_{22})_j \frac{\partial^4 U_j}{\partial x^4} + [2(b_{12})_j + (b_{66})_j] \frac{\partial^4 U_j}{\partial x^2 \partial y^2} + (b_{11})_j \frac{\partial^4 U_j}{\partial y^4} = 0, \qquad j = 1,2, \tag{1}$$

而边界条件为

$$(\sigma_\theta)_1 = (\sigma_\theta)_2 = 0,\ (\tau_{r\theta})_1 = (\tau_{r\theta})_2 = 0, \qquad \theta = \pm\pi, \tag{2}$$

$$(\sigma_\theta)_1 = (\sigma_\theta)_2,\ (\tau_{r\theta})_1 = (\tau_{r\theta})_2, \qquad \theta = 0, \tag{3}$$

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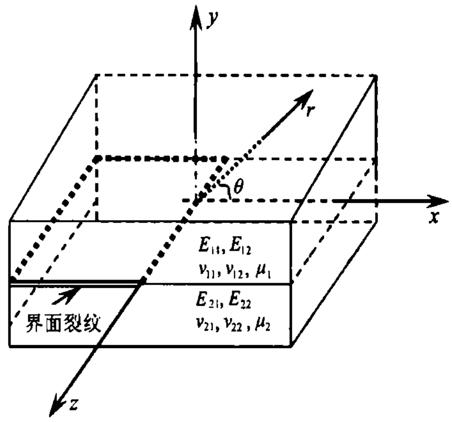

图 1 正交异性双材料界面裂纹模型及裂尖坐标系

$$(u_r)_1 = (u_r)_2,\ (u_\theta)_1 = (u_\theta)_2,\qquad \theta = 0, \tag{4}$$

其中 $U_j(j = 1,2)$ 是应力函数，如图 1 所示，x 和 y 为直角坐标，r 和 θ 为从裂纹边缘起度量的极坐标. 由弹性力学知,应力分量为

$$\begin{cases}(\sigma_r)_j = \dfrac{1}{r^2}\dfrac{\partial^2 U_j}{\partial\theta^2} + \dfrac{1}{r}\dfrac{\partial U_j}{\partial r},\\ (\sigma_\theta)_j = \dfrac{\partial^2 U_j}{\partial r^2},\\ (\tau_{r\theta})_j = -\dfrac{1}{r}\dfrac{\partial^2 U_j}{\partial r\partial\theta} + \dfrac{1}{r^2}\dfrac{\partial U_j}{\partial\theta},\\ \qquad j = 1,2,\end{cases} \tag{5}$$

应变分量为

$$\begin{cases}(\varepsilon_r)_j = (b_{11})_j(\sigma_r)_j + (b_{12})_j(\sigma_\theta)_j,\\ (\varepsilon_\theta)_j = (b_{21})_j(\sigma_r)_j + (b_{22})_j(\sigma_\theta)_j, \qquad j = 1,2,\\ (\gamma_{r\theta})_j = (b_{66})_j(\tau_{r\theta})_j,\end{cases} \tag{6}$$

$$\begin{cases}(\varepsilon_r)_j = \dfrac{\partial(u_r)_j}{\partial r},\ (\varepsilon_\theta)_j = \dfrac{1}{r}\dfrac{\partial(u_\theta)_j}{\partial\theta} + \dfrac{(u_r)_j}{r},\\ (\gamma_{r\theta})_j = \dfrac{1}{r}\dfrac{\partial(u_r)_j}{\partial\theta} + \dfrac{\partial(u_\theta)_j}{\partial r} - \dfrac{(u_\theta)_j}{r},\end{cases} \qquad j = 1,2, \tag{7}$$

而常数 $(b_{11})_j$、$(b_{12})_j$、$(b_{22})_j$、$(b_{66})_j$ 分别为

$$\begin{cases}(b_{11})_j = \dfrac{1}{E_{j1}},\ (b_{12})_j = -\dfrac{\nu_{j1}}{E_{j1}} = -\dfrac{\nu_{j2}}{E_{j2}} = (b_{21})_j,\\ (b_{22})_j = \dfrac{1}{E_{j2}},\ (b_{66})_j = \dfrac{1}{\mu_j},\end{cases} \qquad j = 1,2. \tag{8}$$

2 应力函数

令 s_{jk} 满足下列四次方程:

$$(b_{11})_j s_j^4 + [2(b_{12})_j + (b_{66})_j] s_j^2 + (b_{22})_j = 0, \qquad j = 1,2. \tag{9}$$

定义

$$\Delta_j = \left[\frac{2(b_{12})_j + (b_{66})_j}{(b_{11})_j}\right]^2 - 4\frac{(b_{22})_j}{(b_{11})_j}, \qquad j = 1,2. \tag{10}$$

本文仅讨论:当 $\Delta_1 > 0$ 和 $\Delta_2 > 0$ 的情形. 则方程(9)有如下的根:

$$s_{jk} = \mathrm{i}\beta_{jk},\ s_{j(k+2)} = \bar{s}_{jk},\ \beta_{j2} > \beta_{j1} > 0, \qquad j = 1,2,;k = 1,2, \tag{11}$$

$$\begin{cases}\beta_{j1}^2 = \dfrac{2(b_{12})_j + (b_{66})_j}{2(b_{11})_j} - \sqrt{\left(\dfrac{2(b_{12})_j + (b_{66})_j}{2(b_{11})_j}\right)^2 - \dfrac{(b_{22})_j}{(b_{11})_j}},\\ \beta_{j2}^2 = \dfrac{2(b_{12})_j + (b_{66})_j}{2(b_{11})_j} + \sqrt{\left(\dfrac{2(b_{12})_j + (b_{66})_j}{2(b_{11})_j}\right)^2 - \dfrac{(b_{22})_j}{(b_{11})_j}},\\ \qquad \beta_{j2} > \beta_{j1} > 0\ (j = 1,2).\end{cases} \tag{12}$$

由界面边界条件(2)~(4),选取特殊应力函数如下:

$$U_j(r,\theta)=\sum_{k=1}^{2}\frac{1}{2}\left[\frac{1}{(\lambda_1+2)(\lambda_1+1)}\left(\frac{z_{jk}^{\lambda_1+2}+\bar{z}_{jk}^{\lambda_1+2}}{2}a_{jk}+\frac{z_{jk}^{\lambda_1+2}-\bar{z}_{jk}^{\lambda_1+2}}{2\mathrm{i}}b_{jk}\right)+\right.$$
$$\left.\frac{1}{(\lambda_2+2)(\lambda_2+1)}\left(\frac{z_{jk}^{\lambda_2+2}+\bar{z}_{jk}^{\lambda_2+2}}{2}c_{jk}+\frac{z_{jk}^{\lambda_2+2}-\bar{z}_{jk}^{\lambda_2+2}}{2\mathrm{i}}d_{jk}\right)\right],\quad j=1,2, \tag{13}$$

其中

$$\lambda_1=-\frac{1}{2}+\varepsilon_1,\ \lambda_2=-\frac{1}{2}+\varepsilon_2,$$
$$\begin{cases}z_{jk}=x+s_{jk}y=r(\cos\theta+s_{jk}\sin\theta),\\ \bar{z}_{jk}=x+\bar{s}_{jk}y=r(\cos\theta+\bar{s}_{jk}\sin\theta),\end{cases}\quad j=1,2;k=1,2. \tag{14}$$

显见式(13)给出的 $U_j(r,\theta)$ 满足偏微分方程(1),将式(13)、(14)代入式(5) ~ (7)得到$(\sigma_\theta)_j$、$(\sigma_r)_j$、$(\tau_{r\theta})_j$、$(u_r)_j$ 和$(u_\theta)_j(j=1,2)$的表达式,并将它们代入边界条件(2) ~ (4)得到两组分别关于$(a_{11},a_{12},b_{11},b_{12},a_{21},a_{22},b_{21},b_{22})$和$(c_{11},c_{12},d_{11},d_{12},c_{21},c_{22},d_{21},d_{22})$的八阶齐次线性方程组. 对两组方程的系数行列式分别进行适当的初等变换后,可得系数矩阵的行列式均为

$$\sin^4\lambda\pi\cdot\{\lambda[c_1^2+c_2c_4\cot^2\lambda\pi]+c_1c_3\}, \tag{15}$$

其中

$$\begin{cases}c_1=\dfrac{\beta_{11}\beta_{12}-\nu_{11}}{E_{11}}-\dfrac{\beta_{21}\beta_{22}-\nu_{21}}{E_{21}},\ c_2=\dfrac{\beta_{11}+\beta_{12}}{E_{11}}+\dfrac{\beta_{21}+\beta_{22}}{E_{21}},\\ c_3=\left(\dfrac{2(\nu_{11}+1)}{E_{11}}-\dfrac{1}{\mu_1}\right)-\left(\dfrac{2(\nu_{21}+1)}{E_{21}}-\dfrac{1}{\mu_2}\right),\\ c_4=\dfrac{\beta_{11}+\beta_{12}}{E_{11}}\beta_{11}\beta_{12}+\dfrac{\beta_{21}+\beta_{22}}{E_{21}}\beta_{21}\beta_{22}.\end{cases} \tag{16}$$

为使该齐次线性方程组有一组非零解,则其系数行列式必须为 0.

若 $\sin\lambda\pi=0$,则 $\lambda=n(n=0,1,2,\cdots)$. 因与复合材料工程常数无关,舍去此 λ.

若 $\sin\lambda\pi\neq0$, 则有

$$\lambda[c_1^2+c_2c_4\cot^2\lambda\pi]+c_1c_3=0. \tag{17}$$

设 $\lambda=-1/2+\varepsilon$,当 $|\varepsilon|\ll1$ 时,略去 ε 的 3 阶及其以上阶小量,式(17)又化为

$$c_2c_4\pi^2\varepsilon^2-2c_1^2\varepsilon+c_1^2-2c_1c_3=0. \tag{18}$$

求解方程(18),当 $\Delta=4c_1^4-4c_1^2c_2c_4\pi^2+8c_1c_2c_3c_4\pi^2>0$ 时,可得

$$\begin{cases}\varepsilon_1=\dfrac{c_1^2}{\pi^2c_2c_4}+\dfrac{\sqrt{c_1^4-c_1^2c_2c_4\pi^2+2c_1c_2c_3c_4\pi^2}}{\pi^2c_2c_4},\\ \varepsilon_2=\dfrac{c_1^2}{\pi^2c_2c_4}-\dfrac{\sqrt{c_1^4-c_1^2c_2c_4\pi^2+2c_1c_2c_3c_4\pi^2}}{\pi^2c_2c_4},\end{cases} \tag{19}$$

即 $$\lambda_1=-\frac{1}{2}+\varepsilon_1,\ \lambda_2=-\frac{1}{2}+\varepsilon_2.$$

当 λ 满足式(19)时,齐次线性方程组的未知数 a_{jk}、$b_{jk}(j=1,2;k=1,2)$线性相关,它们皆可以由 b 表出. 同样齐次线性方程组的未知数 c_{jk}、$d_{jk}(j=1,2;k=1,2)$线性相关,它们皆可以由 d 表出.

记

$$\begin{cases}\beta_{*1}=\beta_{12},\ \beta_{*2}=\beta_{11},\ \beta_{1*}=\beta_{22},\ \beta_{2*}=\beta_{21},\ f_{*1}=f_{12},\ f_{*2}=f_{11},\\ \beta_{22}f_{11}-\beta_{21}f_{12}=(\beta_{22}-\beta_{21})\left(\dfrac{\beta_{11}\beta_{12}-\nu_{11}}{E_{11}}-\dfrac{\beta_{21}\beta_{22}-\nu_{21}}{E_{21}}\right),\\ f_{11}-f_{12}=(\beta_{21}-\beta_{22})\left(\dfrac{\beta_{11}+\beta_{12}}{E_{11}}+\dfrac{\beta_{21}+\beta_{22}}{E_{21}}\right),\end{cases}\tag{20}$$

则

$$a_{jk}=A_{jk}b,\ b_{jk}=B_{jk}b,\ c_{jk}=C_{jk}d,\ d_{jk}=D_{jk}d,\tag{21}$$

其中

$$\begin{cases}A_{1k}=(-1)^{k+1}\dfrac{(\beta_{22}-\beta_{21})}{(\beta_{12}-\beta_{11})}[(\beta_{22}f_{11}-\beta_{21}f_{12})+\beta_{*k}(f_{11}-f_{12})]\tan\pi\varepsilon_1,\\ B_{1k}=(-1)^{k}\dfrac{(\beta_{22}-\beta_{21})}{(\beta_{12}-\beta_{11})}[(\beta_{22}f_{11}-\beta_{21}f_{12})-\beta_{*k}(f_{11}-f_{12})]\tan^2\pi\varepsilon_1,\\ A_{2k}=(-1)^{k}(\beta_{22}-\beta_{21})f_{*k}\tan\pi\varepsilon_1,\\ B_{2k}=(-1)^{k}[(\beta_{22}f_{11}-\beta_{21}f_{12})+\beta_{k*}(f_{11}-f_{12})\tan^2\pi\varepsilon_1],\\ C_{1k}=(-1)^{k+1}\dfrac{(\beta_{22}-\beta_{21})}{(\beta_{12}-\beta_{11})}[(\beta_{22}f_{11}-\beta_{21}f_{12})+\beta_{*k}(f_{11}-f_{12})]\tan\pi\varepsilon_2,\\ D_{1k}=(-1)^{k}\dfrac{(\beta_{22}-\beta_{21})}{(\beta_{12}-\beta_{11})}[(\beta_{22}f_{11}-\beta_{21}f_{12})-\beta_{*k}(f_{11}-f_{12})]\tan^2\pi\varepsilon_2,\\ C_{2k}=(-1)^{k}(\beta_{22}-\beta_{21})f_{*k}\tan\pi\varepsilon_2,\\ D_{2k}=(-1)^{k}[(\beta_{22}f_{11}-\beta_{21}f_{12})+\beta_{k*}(f_{11}-f_{12})\tan^2\pi\varepsilon_2],\qquad k=1,2.\end{cases}\tag{22}$$

将式(13)给出的应力函数改写成如下形式：

$$U_j(x,y)=2\mathrm{Re}[U_{j1}(z_{j1})+U_{j2}(z_{j2})],\qquad j=1,2.\tag{23}$$

记

$$\begin{cases}\dfrac{\mathrm{d}U_{jk}}{\mathrm{d}z_{jk}}=\Phi_{jk}(z_{jk}),\ \Phi'_{jk}(z_{jk})=\dfrac{\mathrm{d}\Phi_{jk}(z_{jk})}{\mathrm{d}z_{jk}},\qquad j=1,2;k=1,2;\\ \Phi'_{jk}(z_{jk})=\dfrac{1}{4}(A_{jk}-\mathrm{i}B_{jk})bz_{jk}^{-1/2+\varepsilon_1}+\dfrac{1}{4}(C_{jk}-\mathrm{i}D_{jk})dz_{jk}^{-1/2+\varepsilon_2}.\end{cases}\tag{24}$$

3 Ⅰ型界面裂纹尖端的应力场和位移场

如图2所示，受对称载荷 σ 作用，Ⅰ型界面裂纹的边界条件除式(2)～(4)外，尚须补充

$$(\sigma_y)_j=\sigma,\ (\tau_{xy})_j=0,\qquad |y|\to\infty,\tag{25}$$

$$(\tau_{xy})_j=0,\qquad |x|\to\infty.\tag{26}$$

该裂纹长度为 $2a$，通过坐标变换 $z=z^*+a$ 并结合复变函数级数展开理论及边界条件(25)、(26)，式(24)在新坐标系下的表达式为

$$\Phi'_{jk}(z_{jk})=\frac{a_{jk}-\mathrm{i}b_{jk}}{4}U_{jk}(z_{jk})+\frac{c_{jk}-\mathrm{i}d_{jk}}{4}V_{jk}(z_{jk}),\qquad j=1,2;k=1,2.\tag{27}$$

其中

$$U_{jk}(z_{jk})=\frac{z_{jk}^{1-2\varepsilon_1}}{(z_{jk}^2-a^2)^{1/2-\varepsilon_1}}\sigma,\ V_{jk}(z_{jk})=\frac{z_{jk}^{1-2\varepsilon_2}}{(z_{jk}^2-a^2)^{1/2-\varepsilon_2}}\sigma.$$

由式(25)、(26)，当 $|z_{jk}|\to\infty$ 时，可得方程组

$$\begin{cases}(\sigma_y)_j=(A_{j1}+A_{j2})\sigma b+(C_{j1}+C_{j2})\sigma d=\sigma,\\ (\tau_{xy})_j=-(\beta_{j1}B_{j1}+\beta_{j2}B_{j2})\sigma b-(\beta_{j1}D_{j1}+\beta_{j2}D_{j2})\sigma d=0.\end{cases}\tag{28}$$

求解方程组,有

$$\begin{cases} b = \dfrac{2}{(\beta_{22}-\beta_{21})(f_{11}-f_{12})(\tan\pi\varepsilon_1-\tan\pi\varepsilon_2)}, \\ d = \dfrac{-2}{(\beta_{22}-\beta_{21})(f_{11}-f_{12})(\tan\pi\varepsilon_1-\tan\pi\varepsilon_2)}. \end{cases} \tag{29}$$

定义Ⅰ型界面裂纹应力强度因子

$$\begin{cases} K_{\mathrm{I}1} = \lim\limits_{z_{jk}\to a}[2(z_{jk} - a)]^{1/2-\varepsilon_1} U_{jk}(z_{jk}) = a^{1/2-\varepsilon_1}\sigma, \\ K_{\mathrm{I}2} = \lim\limits_{z_{jk}\to a}[2(z_{jk} - a)]^{1/2-\varepsilon_2} V_{jk}(z_{jk}) = a^{1/2-\varepsilon_2}\sigma, \end{cases} \tag{30}$$

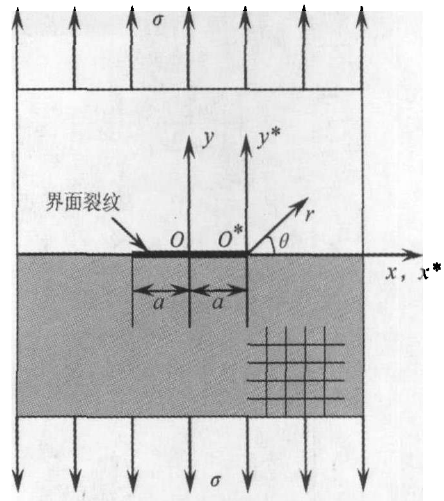

图 2 正交异性双材料Ⅰ型界面裂纹模型

则正交异性双材料Ⅰ型界面裂纹尖端附近的应力场为

$$\begin{cases} (\sigma_x)_j = \dfrac{1}{(\beta_{22}-\beta_{21})(f_{11}-f_{12})(\tan\pi\varepsilon_1-\tan\pi\varepsilon_2)} \times \\ \quad \Big\{-\dfrac{K_{\mathrm{I}1}}{2^{1/2-\varepsilon_1}}\{[\beta_{j1}^2 A_{j1}\mathrm{Re}(z_{j1}-a)^{-1/2+\varepsilon_1} + \beta_{j2}^2 A_{j2}\mathrm{Re}(z_{j2}-a)^{-1/2+\varepsilon_1}] + \\ \quad [\beta_{j1}^2 B_{j1}\mathrm{Im}(z_{j1}-a)^{-1/2+\varepsilon_1} + \beta_{j2}^2 B_{j2}\mathrm{Im}(z_{j2}-a)^{-1/2+\varepsilon_1}]\} + \\ \quad \dfrac{K_{\mathrm{I}2}}{2^{1/2-\varepsilon_2}}\{[\beta_{j1}^2 C_{j1}\mathrm{Re}(z_{j1}-a)^{-1/2+\varepsilon_2} + \beta_{j2}^2 C_{j2}\mathrm{Re}(z_{j2}-a)^{-1/2+\varepsilon_2}] + \\ \quad [\beta_{j1}^2 D_{j1}\mathrm{Im}(z_{j1}-a)^{-1/2+\varepsilon_2} + \beta_{j2}^2 D_{j2}\mathrm{Im}(z_{j2}-a)^{-1/2+\varepsilon_2}]\}\Big\}, \\ (\sigma_y)_j = \dfrac{1}{(\beta_{22}-\beta_{21})(f_{11}-f_{12})(\tan\pi\varepsilon_1-\tan\pi\varepsilon_2)} \times \\ \quad \Big\{\dfrac{K_{\mathrm{I}1}}{2^{1/2-\varepsilon_1}}[A_{j1}\mathrm{Re}(z_{j1}-a)^{-1/2+\varepsilon_1} + A_{j2}\mathrm{Re}(z_{j2}-a)^{-1/2+\varepsilon_1} + \\ \quad B_{j1}\mathrm{Im}(z_{j1}-a)^{-1/2+\varepsilon_1} + B_{j2}\mathrm{Im}(z_{j2}-a)^{-1/2+\varepsilon_1}] - \\ \quad \dfrac{K_{\mathrm{I}2}}{2^{1/2-\varepsilon_2}}[C_{j1}\mathrm{Re}(z_{j1}-a)^{-1/2+\varepsilon_2} + C_{j2}\mathrm{Re}(z_{j2}-a)^{-1/2+\varepsilon_2} + \\ \quad D_{j1}\mathrm{Im}(z_{j1}-a)^{-1/2+\varepsilon_2} + D_{j2}\mathrm{Im}(z_{j2}-a)^{-1/2+\varepsilon_2}]\Big\}, \\ (\tau_{xy})_j = \dfrac{1}{(\beta_{22}-\beta_{21})(f_{11}-f_{12})(\tan\pi\varepsilon_1-\tan\pi\varepsilon_2)} \times \\ \quad \Big\{\dfrac{K_{\mathrm{I}1}}{2^{1/2-\varepsilon_1}}\{-[\beta_{j1}B_{j1}\mathrm{Re}(z_{j1}-a)^{-1/2+\varepsilon_1} + \beta_{j2}B_{j2}\mathrm{Re}(z_{j2}-a)^{-1/2+\varepsilon_1}] + \\ \quad [\beta_{j1}A_{j1}\mathrm{Im}(z_{j1}-a)^{-1/2+\varepsilon_1} + \beta_{j2}A_{j2}\mathrm{Im}(z_{j2}-a)^{-1/2+\varepsilon_1}]\} + \\ \quad \dfrac{K_{\mathrm{I}2}}{2^{1/2-\varepsilon_2}}\{[\beta_{j1}D_{j1}\mathrm{Re}(z_{j1}-a)^{-1/2+\varepsilon_2} + \beta_{j2}D_{j2}\mathrm{Re}(z_{j2}-a)^{-1/2+\varepsilon_2}] - \\ \quad [\beta_{j1}C_{j1}\mathrm{Im}(z_{j1}-a)^{-1/2+\varepsilon_2} + \beta_{j2}C_{j2}\mathrm{Im}(z_{j2}-a)^{-1/2+\varepsilon_2}]\}\Big\}, \end{cases} \tag{31}$$

位移场为

$$
\begin{cases}
(u)_j = \dfrac{1}{(\beta_{22}-\beta_{21})(f_{11}-f_{12})(\tan\pi\varepsilon_1-\tan\pi\varepsilon_2)} \times \\
\left\{\dfrac{2}{1+2\varepsilon_1}\dfrac{K_{\mathrm{I}1}}{2^{1/2-\varepsilon_1}}\{[-(b_{11})_j\beta_{j1}^2+(b_{12})_j]A_{j1}\mathrm{Re}(z_{j1}-a)^{1/2+\varepsilon_1}+\right. \\
[-(b_{11})_j\beta_{j2}^2+(b_{12})_j]A_{j2}\mathrm{Re}(z_{j2}-a)^{1/2+\varepsilon_1}+ \\
[-(b_{11})_j\beta_{j1}^2+(b_{12})_j]B_{j1}\mathrm{Im}(z_{j1}-a)^{1/2+\varepsilon_1}+ \\
[-(b_{11})_j\beta_{j2}^2+(b_{12})_j]B_{j2}\mathrm{Im}(z_{j2}-a)^{1/2+\varepsilon_1}\}- \\
\dfrac{2}{1+2\varepsilon_2}\dfrac{K_{\mathrm{I}2}}{2^{1/2-\varepsilon_2}}\{[-(b_{11})_j\beta_{j1}^2+(b_{12})_j]C_{j1}\mathrm{Re}(z_{j1}-a)^{1/2+\varepsilon_2}+ \\
[-(b_{11})_j\beta_{j2}^2+(b_{12})_j]C_{j2}\mathrm{Re}(z_{j2}-a)^{1/2+\varepsilon_2}+ \\
[-(b_{11})_j\beta_{j1}^2+(b_{12})_j]D_{j1}\mathrm{Im}(z_{j1}-a)^{1/2+\varepsilon_2}+ \\
\left.[-(b_{11})_j\beta_{j2}^2+(b_{12})_j]D_{j2}\mathrm{Im}(z_{j2}-a)^{1/2+\varepsilon_2}\}\right\}, \\
(v)_j = \dfrac{1}{(\beta_{22}-\beta_{21})(f_{11}-f_{12})(\tan\pi\varepsilon_1-\tan\pi\varepsilon_2)} \times \\
\left\{\dfrac{2}{1+2\varepsilon_1}\dfrac{K_{\mathrm{I}1}}{2^{1/2-\varepsilon_1}}\left\{\left[-(b_{12})_j\beta_{j1}+(b_{22})_j\dfrac{1}{\beta_{j1}}\right]A_{j1}\mathrm{Im}(z_{j1}-a)^{1/2+\varepsilon_1}+\right.\right. \\
\left[-(b_{12})_j\beta_{j2}+(b_{22})_j\dfrac{1}{\beta_{j2}}\right]A_{j2}\mathrm{Im}(z_{j2}-a)^{1/2+\varepsilon_1}+ \\
\left[(b_{12})_j\beta_{j1}-(b_{22})_j\dfrac{1}{\beta_{j1}}\right]B_{j1}\mathrm{Re}(z_{j1}-a)^{1/2+\varepsilon_1}+\left[(b_{12})_j\beta_{j2}-\right. \\
\left.\left.(b_{22})_j\dfrac{1}{\beta_{j2}}\right]B_{j2}\mathrm{Re}(z_{j2}-a)^{1/2+\varepsilon_1}\right\}-\dfrac{2}{1+2\varepsilon_2}\dfrac{K_{\mathrm{I}2}}{2^{1/2-\varepsilon_2}}\left\{\left[-(b_{12})_j\beta_{j1}+\right.\right. \\
\left.(b_{22})_j\dfrac{1}{\beta_{j1}}\right]C_{j1}\mathrm{Im}(z_{j1}-a)^{1/2+\varepsilon_2}+ \\
\left[-(b_{12})_j\beta_{j2}+(b_{22})_j\dfrac{1}{\beta_{j2}}\right]C_{j2}\mathrm{Im}(z_{j2}-a)^{1/2+\varepsilon_2}+ \\
\left[(b_{12})_j\beta_{j1}-(b_{22})_j\dfrac{1}{\beta_{j1}}\right]D_{j1}\mathrm{Re}(z_{j1}-a)^{1/2+\varepsilon_2}+ \\
\left.\left.\left[(b_{12})_j\beta_{j2}-(b_{22})_j\dfrac{1}{\beta_{j2}}\right]D_{j2}\mathrm{Re}(z_{j2}-a)^{1/2+\varepsilon_2}\right\}\right\}.
\end{cases}
\tag{32}
$$

4 结 论

本文采用建立在 z_j 平面上的复变函数方法，对正交异性双材料界面裂纹问题进行了研究，通过引入新的应力函数，求解得出了正交异性双材料 Ⅰ 型界面裂纹尖端的应力场和位移场．结果显示：1) 两种正交各向异性材料界面裂纹具有 $r^{-1/2+\varepsilon_1}$ 及 $r^{-1/2+\varepsilon_2}$ 的奇异性(其中 ε_1、ε_2 为实双材料参数)；2) 运用文中推导过程，当上下半平面材料参数相同时，可获得正交异性复合材料的应力场和位移场，其结果与文献[5]中结果相同；3) 正交异性双材料界面裂纹尖端的应力场没有振荡奇异性，裂纹面没有出现相互嵌入现象；4) 正交异性双材料Ⅱ型、Ⅰ＋Ⅱ型界面裂纹尖端应力场、位移场也可类似得到．

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Study of Stress Field Near Interface Crack Tip of Double Dissimilar Orthotropic Composite Materials

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Abstract: A study of double dissimilar orthotropic composite materials interfacial crack was made by constructing new stress functions and employing the method of composite material complex. In the case that the characteristic equations' discriminants are all more than zero, the theoretical formula of the stress field and the displacement field near the mode I interface crack tip, without oscillation and inter-embedding between the interfaces of the crack were derived.

Key words: orthotropic; interface crack; crack tip; stress field