应用数学和力学编委会编
重 庆 出 版 社 出 版

文章编号: 1000_0887(2003) 09_0919_10

一类混杂动态系统的能控性(Ⅰ)——基本结果*

谢广明, 王 龙, 叶庆凯

(北京大学 力学与工程科学系, 北京 100871)

(我刊编委叶庆凯来稿)

摘要: 首次将时滞现象引入到线性切换系统的模型中, 研究含有时滞线性切换系统的能控性及其判定条件• 全部工作由三部分组成, 第Ⅰ部分首先, 提出含时滞的线性切换系统的数学模型, 并介绍切换系统的基本概念——切换序列• 其次, 引入列空间、循环不变子空间和广义循环不变子空间等基本几何概念, 给出一些有关概念的基本性质, 特别是分离引理• 然后以一个基本引理的形式揭式某一积分方程的解集与广义循环不变子空间之间的联系, 这个引理将在能控性的判定中起关键作用• 这些概念和引理都将作为以后展开能控性分析所必需的研究工具•

关 键 词: 混杂动态系统; 线性切换系统; 时滞; 能控性; 广义循环不变子空间; 切换序列; 切换路径

中图分类号: TP13; TP273; O317 文献标识码: A

引 言

混杂系统既含有连续时间动态, 又含有离散事件动态• 这两种动态相互作用和影响, 使系统呈现出更为复杂的动态行为• 线性切换系统是一类重要的混杂系统, 一般包括一族线性定常系统和一个描述在它们之间如何切换的切换规则• 近年来, 因其在理论上与应用中的重要价值, 正激起人们越来越大的研究兴趣[1~21]•

目前关于切换系统的研究主要集中于系统的能控性和稳定性方面• 关于切换系统的稳定性和镇定问题, 文[1]给出比较系统的综述• 关于能控性方面, 文[2]首先研究了周期型切换系统的能控性和能观性, 给出了单周期能控的充要判据• 文[3]提出了周期型切换系统的多周期能控性和多周期能观性, 给出了多周期能控能观的充要判据• 文[4]给出了非周期型系统能控性的一个充分条件和一个必要条件• 文[5]进一步给出了非周期型系统能控性的一个充要条件• 文[6]研究了一类 2 阶线性切换系统的可达性•

时滞现象是实际系统一个比较普遍的现象, 特别在经济系统, 生物系统, 社会系统等复杂

* 收稿日期: 2002_01_29; 修订日期: 2003_03_25
基金项目: 国家杰出青年科学基金资助项目(69925307); 国家重点基础研究与发展计划基金资助项目(2002CB312200); 中国博士后基金资助项目
作者简介: 谢广明(1972—), 男, 北京人, 博士(后), 研究方向为混杂切换系统、广义系统、时滞系统和网络控制系统(E_mail: xiegming@mech.pku.edu.cn)•

系统之中• 对时滞现象的研究是控制理论中一个重要的研究方向• 关于含时滞的线性定常系统的能控性研究可参考[22] [23]•

能控性是现代控制理论中一个重要的基本概念,在系统分析和综合中占有重要地位• 截至目前关于切换系统能控性的研究,尚未涉及到时滞现象• 基此,本文将时滞引入到线性切换系统能控性的研究中• 这里首先给出明确的问题描述,给出含时滞的线性切换系统的数学模型,并给出切换系统的一个基本概念——切换序列• 然后给出一些基本概念和重要引理,包括列空间,循环不变子空间,广义循环不变子空间及其性质等,作为进一步分析系统能控性的准备•

1 问题描述

在本节,我们给出含单时滞的线性切换系统的系统描述,并定义切换系统的基本概念——切换序列• 输入函数含单时线性切换系统可描述如下:

$$\dot{\boldsymbol{x}}(t) = \boldsymbol{A}_{r(t)}\boldsymbol{x}(t) + \boldsymbol{B}_{r(t)}\boldsymbol{u}(t) + \boldsymbol{D}_{r(t)}\boldsymbol{u}(t - \tau), \tag{1}$$

其中, $\boldsymbol{x}(t) \in \mathscr{R}^n$ 为状态向量, $\boldsymbol{u}(t) \in \mathscr{R}^m$ 为输入函数, 分段定常函数 $r(t): \mathscr{R} \to \{1, 2, \cdots, N\}$ 为切换路径, $\{\boldsymbol{A}_i, \boldsymbol{B}_i, \boldsymbol{D}_i \mid i = 1, \cdots, N\}$ 为一族系统实现,或称切换模式• 进一步, $r(t) = i$ 表示在时刻 t 系统以模式 $(\boldsymbol{A}_i, \boldsymbol{B}_i, \boldsymbol{D}_i)$ 进行演变• $\tau > 0$ 为固定长度的时间滞后•

对系统(1),切换序列用来描述系统在某一时刻以哪一个模式进行演变•

定义 1(切换序列) 对系统(1),一个切换序列为一个有限集合

$$\pi \overset{\text{def}}{=} \{(i_1, h_1), \cdots, (i_M, h_M)\} \tag{2}$$

其中 M 为切换序列的长度,记为 $L(\pi)$• $i_m \in \{1, \cdots, M\}$ 表示为切换模式的序号, h_m 表示为切换模式持续的时间 $(m = 1, \cdots, M)$• 一般地,我们可将切换序列简写为 $\pi = \{(i_m, h_m)\}_{m=1}^{M}$•

注 1 为避免不必要的复杂性,我们假设所有切换序列中持续时间满足 $h_m > \tau$•

给定初始时刻 t_0 和切换序列 $\pi = \{(i_m, h_m)\}_{m=1}^{M}$, 我们可以确定一个相应的切换路径 $r(t)$ 如下:

$$r(t) = i_m \qquad \left(\text{当 } t \in \left[t_0 + \sum_{l=1}^{m-1} h_l,\ t_0 + \sum_{l=1}^{m} h_l\right),\ m = 1, \cdots, M\right) \cdot \tag{3}$$

注 2 当切换路径为 $r(t)$ 周期函数时,我们得到了"周期型切换系统"• 不失一般性,当讨论周期型系统时,我们就选择切换序列 $\pi = \{(1, h_1), \cdots, (n, h_n), (N, h_N)\}$ 作为周期型系统的周期•

2 数学准备

在本节,我们引入一些基本定义和引理,作为讨论系统能控性的基本工具•

定义 2(列空间)[24] 给定矩阵 $\boldsymbol{B} \in \mathscr{R}^{n\times m}$,如下定义的线性子空间

$$R(\boldsymbol{B}) \overset{\text{def}}{=} \{\boldsymbol{B}y \mid y \in \mathscr{R}^m\} \tag{4}$$

称为矩阵 B 的列空间•

定义 3(循环不变子空间)[24] 给定矩阵 $\boldsymbol{A} \in \mathscr{R}^{n\times n}$ 和线性子空间 $\mathscr{W} \in \mathscr{R}^n$,如下定义的线性子空间

$$\langle \boldsymbol{A} \mid \mathscr{W} \rangle \overset{\text{def}}{=} \sum_{i=0}^{n-1} \boldsymbol{A}^i \mathscr{W} \tag{5}$$

称为矩阵 $\boldsymbol{A}$ 在 $\mathscr{W}$上的循环不变子空间•

定义 4(广义循环不变子空间) 给定矩阵 $\boldsymbol{A}_1, \cdots, \boldsymbol{A}_N \in \mathscr{R}^{n\times n}$ 和 $\boldsymbol{B}_1, \cdots, \boldsymbol{B}_N \in \mathscr{R}^{n\times m}$, 广义循环不变子空间 $\langle \boldsymbol{A}_1 \mid \boldsymbol{B}_1 + \cdots + \boldsymbol{A}_N \mid \boldsymbol{B}_N \rangle$ 是如下定义的一个线性子空间:

$$\langle \boldsymbol{A}_1 \mid \boldsymbol{B}_1 + \cdots + \boldsymbol{A}_N \mid \boldsymbol{B}_N \rangle \overset{\text{def}}{=} \sum_{i=0}^{\infty} \mathscr{R}(\boldsymbol{A}_1^i \boldsymbol{B}_1 + \cdots + \boldsymbol{A}_N^i \boldsymbol{B}_N) \bullet \tag{6}$$

特别地,

$$\langle \boldsymbol{A}_1 \mid \boldsymbol{B}_1 \rangle \overset{\text{def}}{=} \sum_{i=0}^{\infty} R(\boldsymbol{A}_1^i \boldsymbol{B}_1) \bullet \tag{7}$$

引理 1 给定矩阵 $\boldsymbol{A}_1, \cdots, \boldsymbol{A}_N \in \mathscr{R}^{n\times n}$ 和 $\boldsymbol{B}_1, \cdots, \boldsymbol{B}_N \in \mathscr{R}^{n\times m}$,

$$\langle \boldsymbol{A}_1 \mid \boldsymbol{B}_1 + \cdots + \boldsymbol{A}_N \mid \boldsymbol{B}_N \rangle = \sum_{i=0}^{Nn-1} R(\boldsymbol{A}_1^i \boldsymbol{B}_1 + \cdots + \boldsymbol{A}_N^i \boldsymbol{B}_N) \bullet \tag{8}$$

特别地,

$$\langle \boldsymbol{A}_1 \mid \boldsymbol{B}_1 \rangle = \sum_{i=0}^{n-1} R(\boldsymbol{A}_1^i \boldsymbol{B}_1) \bullet \tag{9}$$

证明 对 $m = 1, \cdots, N$, 设

$$\boldsymbol{A}_m^n = \sum_{i=0}^{n-1} \alpha_{m,i} \boldsymbol{A}_m^i \bullet \tag{10}$$

令

$$f_m(\boldsymbol{x}) = \boldsymbol{x}^n - \sum_{i=0}^{n-1} \alpha_{m,i} \boldsymbol{x}^i,\ f(\boldsymbol{x}) = \prod_{m=1}^{N} f_m(\boldsymbol{x}) \bullet$$

设

$$f(\boldsymbol{x}) = \boldsymbol{x}^{Nn} - \sum_{i=1}^{Nn-1} \alpha_i \boldsymbol{x}^i,$$

显然, 对 $m = 1, \cdots, N$, 有

$$A_m^{Nn} = \sum_{i=0}^{Nn-1} \alpha_i \boldsymbol{A}_m^i \bullet \tag{11}$$

于是, 对任何 $m > Nn - 1$, 都有

$$R(\boldsymbol{A}_1^m B_1 + \cdots + \boldsymbol{A}_N^m \boldsymbol{B}_N) \subseteq \sum_{i=0}^{Nn-1} R(\boldsymbol{A}_1^i \boldsymbol{B}_1 + \cdots + \boldsymbol{A}_N^i \boldsymbol{B}_N) \bullet \tag{12}$$

因此, (8) 成立• (Q. E. D)

注 3 子空间 $\langle \boldsymbol{A}_1 \mid \boldsymbol{B}_1 + \cdots + \boldsymbol{A}_N \mid \boldsymbol{B}_N \rangle$ 和 $\langle \boldsymbol{A}_1 \mid \boldsymbol{B}_1 \rangle + \cdots + \langle \boldsymbol{A}_N \mid \boldsymbol{B}_N \rangle$ 是有相当区别的• 显然有 $\langle \boldsymbol{A}_1 \mid \boldsymbol{B}_1 + \cdots + \boldsymbol{A}_N \mid \boldsymbol{B}_N \rangle \subseteq \langle \boldsymbol{A}_1 \mid \boldsymbol{B}_1 \rangle + \cdots + \langle \boldsymbol{A}_N \mid \boldsymbol{B}_N \rangle$•

引理 2 给定矩阵 $\boldsymbol{A} \in \mathscr{R}^{n\times n}$, $\boldsymbol{B} \in \mathscr{R}^{n\times m}$, 对任意可逆矩阵 $\boldsymbol{P} \in \mathscr{R}^{m\times m}$, 都有

$$\langle \boldsymbol{A} \mid \boldsymbol{BP} \rangle = \langle \boldsymbol{A} \mid \boldsymbol{B} \rangle \bullet \tag{13}$$

证明 对任意可逆矩阵 $\boldsymbol{P} \in \mathscr{R}^{m\times m}$, 有

$$R(\boldsymbol{BP}) = \left\{ \boldsymbol{BPy} \mid \boldsymbol{y} \in \mathscr{R}^m \right\} = \left\{ \boldsymbol{Bz} \mid \boldsymbol{z} = \boldsymbol{Py}, \boldsymbol{y} \in \mathscr{R}^m \right\},$$

于是

$$R(\boldsymbol{BP}) \subseteq R(\boldsymbol{B}) \bullet$$

另一方面,

$$R(\boldsymbol{B}) = R[(\boldsymbol{BP})\boldsymbol{P}^{-1}] \subseteq R(\boldsymbol{BP}),$$

于是

$$R(\boldsymbol{BP}) = R(\boldsymbol{B})\cdot$$

因此,

$$\langle \boldsymbol{A} \mid \boldsymbol{BP}\rangle = R(\boldsymbol{BP}) + R(\boldsymbol{ABP}) + \cdots + R(\boldsymbol{A}^{n-1}\boldsymbol{BP}) = R(\boldsymbol{B}) + R(\boldsymbol{AB}) + \cdots + R(\boldsymbol{A}^{n-1}\boldsymbol{B}) = \langle \boldsymbol{A} \mid \boldsymbol{B}\rangle\cdot \qquad \text{(Q. E. D)}$$

引理 3 给定矩阵 $\boldsymbol{A} \in \mathscr{R}^{n\times n}$, $\boldsymbol{B} \in \mathscr{R}^{n\times m}$, 对任意常数 $h \in \mathscr{R}$, 有

$$\exp(\boldsymbol{A}h)\langle \boldsymbol{A} \mid \boldsymbol{B}\rangle = \langle \boldsymbol{A} \mid \boldsymbol{B}\rangle, \tag{14}$$

$$\langle \boldsymbol{A} \mid \exp(\boldsymbol{A}h)\boldsymbol{B}\rangle = \langle \boldsymbol{A} \mid \boldsymbol{B}\rangle\cdot \tag{15}$$

证明 易证 $\exp(\boldsymbol{A}h)\langle \boldsymbol{A} \mid \boldsymbol{B}\rangle \subseteq \langle \boldsymbol{A} \mid \boldsymbol{B}\rangle$• 由 $\exp(\boldsymbol{A}h)$ 非奇异, 得

$$\dim(\exp(\boldsymbol{A}h)\langle \boldsymbol{A} \mid \boldsymbol{B}\rangle) = \dim(\langle \boldsymbol{A} \mid \boldsymbol{B}\rangle)\cdot$$

于是, (14) 成立• 对任意正整数 m, 有

$$R(\boldsymbol{A}^m\exp(\boldsymbol{A}h)\boldsymbol{B}) = R(\exp(\boldsymbol{A}h)\boldsymbol{A}^m\boldsymbol{B}) = \exp(\boldsymbol{A}h)R(\boldsymbol{A}^m\boldsymbol{B})\cdot$$

因此,

$$\langle \boldsymbol{A} \mid \exp(\boldsymbol{A}h)\boldsymbol{B}\rangle = \sum_{m=0}^{n-1} R(\boldsymbol{A}^m\exp(\boldsymbol{A}h)\boldsymbol{B}) = \exp(\boldsymbol{A}h)\sum_{m=0}^{n-1} R(\boldsymbol{A}^m\boldsymbol{B}) = \exp(\boldsymbol{A}h)\langle \boldsymbol{A} \mid \boldsymbol{B}\rangle = \langle \boldsymbol{A} \mid \boldsymbol{B}\rangle\cdot$$

(Q. E. D)

引理 4(分离引理) 给定矩阵 $\boldsymbol{A}_1, \boldsymbol{A}_2 \in \mathscr{R}^{n\times n}$; $\boldsymbol{B}_1, \boldsymbol{B}_2 \in \mathscr{R}^{n\times m}$, 有

$$\langle \boldsymbol{A}_1 \mid \boldsymbol{B}_1 + \boldsymbol{A}_2 \mid \boldsymbol{B}_2\rangle + \langle \boldsymbol{A}_2 \mid \boldsymbol{B}_2\rangle = \langle \boldsymbol{A}_1 \mid \boldsymbol{B}_1\rangle + \langle \boldsymbol{A}_2 \mid \boldsymbol{B}_2\rangle\cdot \tag{16}$$

证明 对任意正整数 m, 有

$$R(\boldsymbol{A}_1^m\boldsymbol{B}_1 + \boldsymbol{A}_2^m\boldsymbol{B}_2) + R(\boldsymbol{A}_2^m\boldsymbol{B}_2) = R([\boldsymbol{A}_1^m\boldsymbol{B}_1 + \boldsymbol{A}_2^m\boldsymbol{B}_2, \boldsymbol{A}_2^m\boldsymbol{B}_2]) = R\left([\boldsymbol{A}_1^m\boldsymbol{B}_1, \boldsymbol{A}_2^m\boldsymbol{B}_2]\begin{bmatrix}\boldsymbol{I} & \boldsymbol{0}\\ \boldsymbol{I} & \boldsymbol{I}\end{bmatrix}\right) = R([\boldsymbol{A}_1^m\boldsymbol{B}_1, \boldsymbol{A}_2^m\boldsymbol{B}_2])\cdot$$

因此,

$$\langle \boldsymbol{A}_1 \mid \boldsymbol{B}_1 + \boldsymbol{A}_2 \mid \boldsymbol{B}_2\rangle + \langle \boldsymbol{A}_2 \mid \boldsymbol{B}_2\rangle = \sum_{m=0}^{2n-1}(R(\boldsymbol{A}_1^m\boldsymbol{B}_1 + \boldsymbol{A}_2^m\boldsymbol{B}_2) + R(\boldsymbol{A}_2^m\boldsymbol{B}_2)) = \sum_{m=0}^{2n-1}(R(\boldsymbol{A}_1^m\boldsymbol{B}_1) + R(\boldsymbol{A}_2^m\boldsymbol{B}_2)) = \langle \boldsymbol{A}_1 \mid \boldsymbol{B}_1\rangle + \langle \boldsymbol{A}_2 \mid \boldsymbol{B}_2\rangle\cdot$$

(Q. E. D)

注 4 分离引理可以推广到多个的情形, 即

$$\langle \boldsymbol{A}_1 \mid \boldsymbol{B}_1 + \cdots + \boldsymbol{A}_N \mid \boldsymbol{B}_N + \boldsymbol{C}_1 \mid \boldsymbol{D}_1 + \cdots + \boldsymbol{C}_M \mid \boldsymbol{D}_M\rangle + \langle \boldsymbol{C}_1 \mid \boldsymbol{D}_1 + \cdots + \boldsymbol{C}_M \mid \boldsymbol{D}_M\rangle = \langle \boldsymbol{A}_1 \mid \boldsymbol{B}_1 + \cdots + \boldsymbol{A}_N \mid \boldsymbol{B}_N\rangle + \langle \boldsymbol{C}_1 \mid \boldsymbol{D}_1 + \cdots + \boldsymbol{C}_M \mid \boldsymbol{D}_M\rangle\cdot \tag{17}$$

下面给出的基本引理是[24] 中的定理 7.8.1 的推广形式• 它将作为描述能控性判定条件的出发点•

引理 5 给定矩阵 $\boldsymbol{A}_1, \cdots, \boldsymbol{A}_N \in \mathscr{R}^{n\times n}$ 和 $\boldsymbol{B}_1, \cdots, \boldsymbol{B}_N \in \mathscr{R}^{n\times m}$, 对任意 $0 \leqslant t_0 < t_f < +\infty$, 有

$$\left\{\boldsymbol{x} \mid \boldsymbol{x} = \sum_{i=1}^{N}\int_{t_0}^{t_f}\exp(\boldsymbol{A}_i(t_f - s))\boldsymbol{B}_i u(s)\mathrm{d}s,\ \forall\ \text{分段连续}\ \boldsymbol{u}\right\} = \langle \boldsymbol{A}_1 \mid \boldsymbol{B}_1 + \cdots + \boldsymbol{A}_N \mid \boldsymbol{B}_N\rangle, \tag{18}$$

特别地,

$$\left\{x \mid x=\int_{t_0}^{t_f}\exp(A_i(t_f-s))B_1u(s)\mathrm{d}s,\ \forall\ 分段连续\ u\right\}=\langle A_1 \mid B_1\rangle. \tag{19}$$

证明　首先,有

$$\begin{aligned}&\left\{x \mid x=\sum_{i=1}^{N}\int_{t_0}^{t_f}\exp\left[A_i(t_f-s)\right]B_iu(s)\mathrm{d}s,\ \forall\ 分段连续\ u\right\}=\\&\quad\left\{x \mid x=\sum_{i=1}^{N}\int_{t_0}^{t_f}\sum_{m=0}^{\infty}\frac{\left[A_i(t_f-s)\right]^m}{m!}B_iu(s)\mathrm{d}s,\ \forall\ 分段连续\ u\right\}=\\&\quad\left\{x \mid x=\sum_{m=0}^{\infty}\int_{t_0}^{t_f}\frac{(t_f-s)^m}{m!}u(s)\mathrm{d}s\sum_{i=1}^{N}(A_i)^mB_i,\ \forall\ 分段连续\ u\right\}\subseteq\\&\quad\sum_{m=0}^{\infty}R\left(\sum_{i=1}^{N}(A_i)^mB_i\right)=\langle A_1 \mid B_1+\cdots+A_N \mid B_N\rangle,\end{aligned} \tag{20}$$

其次,令 $h=t_f-t_0$, 可得

$$\begin{aligned}&\left\{x \mid x=\sum_{i=1}^{N}\int_{t_0}^{t_f}\exp\left[A_i(t_f-s)\right]B_iu(s)\mathrm{d}s,\ \forall\ 分段连续\ u\right\}=\\&\quad\left\{x \mid x=\sum_{i=1}^{N}\int_{0}^{h}\exp\left[A_i(h-t)\right]B_iu(t)\mathrm{d}t,\ \forall\ 分段连续\ u\right\}.\end{aligned}$$

以下证明

$$\begin{aligned}&\left\{x \mid x=\sum_{i=1}^{N}\int_{0}^{h}\exp(A_i(h-t))B_iu(t)\mathrm{d}t,\ \forall\ 分段连续\ u\right\}\supseteq\\&\quad\langle A_1 \mid B_1+\cdots+A_N \mid B_N\rangle.\end{aligned} \tag{21}$$

考虑矩阵

$$W=\int_0^h\left\{\sum_{i=1}^{N}\exp\left[A_i(h-s)\right]B_i\right\}\left\{\sum_{i=1}^{N}\exp\left[A_i(h-s)\right]B_i\right\}^{\mathrm{T}}\mathrm{d}s. \tag{22}$$

由于 $W^T=W$ 且为半正定,因而

$$R(W)=\mathcal{N}(W)^{\perp},$$

其中 $\mathcal{N}(W)$ 为矩阵的零空间,定义为

$$\mathcal{N}(W)=\left\{x \mid Wx=0\right\}, \tag{23}$$

而"$\perp$"表示正交补空间. 进一步

$$\begin{aligned}y\in\mathcal{N}(W)&\Leftrightarrow y^{\mathrm{T}}Wy=0\\&\Leftrightarrow\int_0^h y^{\mathrm{T}}\left\{\sum_{i=1}^{N}\exp\left[A_i(h-s)\right]B_i\right\}\left\{\sum_{i=1}^{N}\exp\left[A_i(h-s)\right]B_i\right\}^{\mathrm{T}}y\,\mathrm{d}s=0\\&\Leftrightarrow\left\{\sum_{i=1}^{N}\exp\left[A_i(h-s)\right]B_i\right\}^{\mathrm{T}}y=0\qquad(0\leqslant s\leqslant h).\end{aligned} \tag{24}$$

由(24),可知$\left\{\sum_{i=1}^{N}\exp\left[A_i(h-s)\right]B_i\right\}^{\mathrm{T}}y$ 对 s 的任何次微商在 $s=h$ 处应为零,即有

$$\left\{\sum_{i=1}^{N}B_i^{\mathrm{T}}\right\}y=0,\ \left\{\sum_{i=1}^{N}B_i^{\mathrm{T}}A_i^{\mathrm{T}}\right\}y=0,\ \left\{\sum_{i=1}^{N}B_i^{\mathrm{T}}(A_i^{\mathrm{T}})^{Nn-1}\right\}y=0,\ \cdots, \tag{25}$$

由此有

$$y\in\mathcal{N}\left(\sum_{i=1}^{N}B_i^{\mathrm{T}}\right)\cap\mathcal{N}\left(\sum_{i=1}^{N}B_i^{\mathrm{T}}A_i^{\mathrm{T}}\right)\cap\cdots\cap\mathcal{N}\left(\sum_{i=1}^{N}B_i^{\mathrm{T}}(A_i^{\mathrm{T}})^{Nn-1}\right)=$$

$$\left[R\left(\sum_{i=1}^{N}\boldsymbol{B}_i\right)+R\left(\sum_{i=1}^{N}\boldsymbol{A}_i\boldsymbol{B}_i\right)+\cdots+R\left(\sum_{i=1}^{N}(\boldsymbol{A}_i)^{Nn-1}\boldsymbol{B}_i\right)\right]^{\perp}=$$
$$\langle\boldsymbol{A}_1\mid\boldsymbol{B}_1+\cdots+\boldsymbol{A}_N\mid\boldsymbol{B}_N\rangle^{\perp}\cdot$$

反之也能证明 $\boldsymbol{y}\in\langle\boldsymbol{A}_1\mid\boldsymbol{B}_1+\cdots+\boldsymbol{A}_N\mid\boldsymbol{B}_N\rangle^{\perp}$ 意味着(24) 成立, 即有 $\boldsymbol{y}\in\mathscr{N}(\boldsymbol{W})$· 由此可知

$$\mathscr{N}(\boldsymbol{W})=\langle\boldsymbol{A}_1\mid\boldsymbol{B}_1+\cdots+\boldsymbol{A}_N\mid\boldsymbol{B}_N\rangle^{\perp},$$

或等价地改写成

$$R(\boldsymbol{W})=\langle\boldsymbol{A}_1\mid\boldsymbol{B}_1+\cdots+\boldsymbol{A}_N\mid\boldsymbol{B}_N\rangle\cdot\tag{26}$$

对任意 $\boldsymbol{x}\in\langle\boldsymbol{A}_1\mid\boldsymbol{B}_1+\cdots+\boldsymbol{A}_N\mid\boldsymbol{B}_N\rangle$, 由(26), 存在 $\boldsymbol{z}$ 使 $\boldsymbol{x}=\boldsymbol{W}\boldsymbol{z}$· 现令

$$\boldsymbol{u}(s)=\left\{\sum_{i=1}^{N}\boldsymbol{B}_i^{\mathrm{T}}(\exp[\boldsymbol{A}_i(h-s)])^{\mathrm{T}}\right\}\boldsymbol{z}\qquad(s\in[0,h])$$

则

$$\boldsymbol{x}=\boldsymbol{W}\boldsymbol{z}=\int_0^h\left\{\sum_{i=1}^{N}\exp[\boldsymbol{A}_i(h-s)]\boldsymbol{B}_i\right\}\left\{\sum_{i=1}^{N}\boldsymbol{B}_i^{\mathrm{T}}(\exp[\boldsymbol{A}_i(h-s)])^{\mathrm{T}}\right\}\boldsymbol{z}\,\mathrm{d}s=$$
$$\int_0^h\left\{\sum_{i=1}^{N}\exp[\boldsymbol{A}_i(h-s)]\boldsymbol{B}_i\right\}\boldsymbol{u}(s)\,\mathrm{d}s=$$
$$\sum_{i=1}^{N}\int_0^h(\exp[\boldsymbol{A}_i(h-s)])\boldsymbol{B}_i\boldsymbol{u}(s)\,\mathrm{d}s\cdot$$

由此就有(21) 成立, 结合(20), 可知引理结论成立· (Q. E. D)

下面这个引理在证明非周期型切换系统能控性判据的过程中起关键作用, 是构造某一基本切换序列的理论基础·

引理 6 给定矩阵 $\boldsymbol{A}_{n\times n}$ 对几乎所有 $T>0$, 对任意线性空间 $\mathscr{W}\subseteq\mathscr{R}^n$, 成立

$$\langle\boldsymbol{A}\mid\mathscr{W}\rangle=\langle\exp(\boldsymbol{A}T)\mid\mathscr{W}\rangle\cdot\tag{27}$$

证明 设 $\boldsymbol{A}^n=\sum_{i=1}^{n}\alpha_i\boldsymbol{A}^{i-1}$, $\exp(\boldsymbol{A}t)=\sum_{i=1}^{n}\lambda_i(t)\boldsymbol{A}^{i-1}$· 首先, 考虑 $\exp(\boldsymbol{A}t)$ 关于 t 的导数, 可得

$$\frac{\mathrm{d}\exp(\boldsymbol{A}t)}{\mathrm{d}t}=\boldsymbol{A}\exp(\boldsymbol{A}t)=$$
$$\boldsymbol{A}\left(\sum_{i=1}^{n}\lambda_i(t)\boldsymbol{A}^{i-1}\right)=\sum_{i=1}^{n}\lambda_i(t)\boldsymbol{A}^{i},$$

于是有

$$\sum_{i=1}^{n}\lambda_i(t)\boldsymbol{A}^{i}=\sum_{i=1}^{n}\dot{\lambda}_i(t)\boldsymbol{A}^{i-1}\cdot$$

既然 $\boldsymbol{A}^n=\sum_{i=1}^{n}\alpha_i(t)\boldsymbol{A}^{i-1}$, 可令

$$\begin{cases}\dot{\lambda}_1(t)=\alpha_1\lambda_n(t),\\ \dot{\lambda}_2(t)=\lambda_1(t)+\alpha_2\lambda_n(t),\\ \cdots\cdots\\ \dot{\lambda}_n(t)=\lambda_{n-1}(t)+\alpha_n\lambda_n(t),\end{cases}\tag{28}$$

和

$$\begin{bmatrix} \lambda_1(0) \\ \lambda_2(0) \\ \vdots \\ \lambda_n(0) \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ \vdots \\ 0 \end{bmatrix} \cdot \tag{29}$$

方程(28)可以改写为矩阵形式:

$$\begin{bmatrix} \dot{\lambda}_1(t) \\ \dot{\lambda}_2(t) \\ \vdots \\ \dot{\lambda}_n(t) \end{bmatrix} = A \begin{bmatrix} \lambda_1(t) \\ \lambda_2(t) \\ \vdots \\ \lambda_n(t) \end{bmatrix}, \tag{30}$$

其中,

$$A = \begin{bmatrix} 0 & \cdots & 0 & \alpha_1 \\ 1 & \cdots & 0 & \alpha_2 \\ & \cdots & \vdots & \vdots \\ & & 1 & \alpha_n \end{bmatrix} \cdot \tag{31}$$

初值问题(30)和(29)的解为

$$\begin{bmatrix} \lambda_1(t) \\ \lambda_2(t) \\ \vdots \\ \lambda_n(t) \end{bmatrix} = \exp(At) \begin{bmatrix} 1 \\ 0 \\ \vdots \\ 0 \end{bmatrix} \cdot \tag{32}$$

其次,考虑线性定常系统

$$\dot{y}(t) = Ay(t) + bv(t), \tag{33}$$

其中 A 如(31)所定义, $b = [1, 0, \cdots, 0]^{T}$. 显然,系统(33)为完全能控. 根据连续系统的离散化理论,对几乎所有 $T > 0$ 都有如下的离散化系统

$$y(k+1) = Hy(k) + Dv(k) \cdot \tag{34}$$

为完全能控,其中

$$H = \exp(AT), \quad D = \left(\int_0^T \exp(At)\,\mathrm{d}t\right) b \cdot \tag{35}$$

因此,矩阵 $[D, HD, \cdots, H^{n-1}D]$ 为非奇异. 注意到

$$[D, HD, \cdots, H^{n-1}D] = \left(\int_0^T \exp(At)\,\mathrm{d}t\right)[b, Hb, \cdots, H^{n-1}b] \cdot \tag{36}$$

和矩阵 $\int_0^T \exp(At)\,\mathrm{d}t$ 也为非奇异. 所以,矩阵 $[b, Hb, \cdots, H^{n-1}b]$ 也为非奇异.

第三,考虑如下方程

$$(\exp(AT))^m = \sum_{i=1}^{n} \lambda_i(mT) A^{i-1} \qquad (m = 0, 1, \cdots, n-1) \cdot \tag{37}$$

其中 T 为如上所取. 方程(37)可以改写为

$$\begin{bmatrix} I \\ \exp(AT) \\ \vdots \\ (\exp(AT))^{n-1} \end{bmatrix} = (\Xi \otimes I) \cdot \begin{bmatrix} I \\ A \\ \vdots \\ A^{n-1} \end{bmatrix}, \tag{38}$$

其中

$$\Xi = \begin{bmatrix} 1 & 0 & \cdots & 0 \\ \lambda_1(T) & \lambda_2(T) & \cdots & \lambda_n(T) \\ \cdots & \cdots & \cdots & \cdots \\ \lambda_1((n-1)T) & \lambda_2((n-1)T) & \cdots & \lambda_n((n-1)T) \end{bmatrix}, \tag{39}$$

这里 $\otimes$ 表示矩阵的 Kornecker 积• 由(32),容易验证

$$\Xi = [\boldsymbol{b}, \boldsymbol{Hb}, \cdots, \boldsymbol{H}^{n-1}\boldsymbol{b}]^{\mathrm{T}}• \tag{40}$$

这表明 Ξ 为非奇异• 由 Kronecker 积的定义, $(\Xi \times \boldsymbol{I})^{-1} = \Xi^{-1} \otimes \boldsymbol{I}$• 因此, 有

$$\begin{bmatrix} \boldsymbol{I} \\ \boldsymbol{A} \\ \vdots \\ \boldsymbol{A}^{n-1} \end{bmatrix} = (\Xi^{-1} \otimes \boldsymbol{I}) • \begin{bmatrix} \boldsymbol{I} \\ \exp(\boldsymbol{A}T) \\ \vdots \\ (\exp(\boldsymbol{A}T))^{n-1} \end{bmatrix}• \tag{41}$$

记 $\Xi^{-1} = [\xi_{ij}]_{n\times n}$, 可得

$$\boldsymbol{A}^{i-1} = \sum_{j=1}^{n} \xi_{ij} (\exp(\boldsymbol{A}T))^{j-1} \qquad (i = 1, \cdots, n)• \tag{42}$$

最后, 对任意的线性空间 $\mathscr{W} \subseteq \mathscr{R}^n$, 有

$$\boldsymbol{A}^{i-1}\mathscr{W} \doteq \sum_{j=1}^{n} \xi_{ij} (\exp(\boldsymbol{A}T))^{j-1}\mathscr{W} \subseteq \langle \exp(\boldsymbol{A}T) \mid \mathscr{W} \rangle \qquad (i = 1, \cdots, n)• \tag{43}$$

因此, 可得

$$\langle \boldsymbol{A} \mid \mathscr{W} \rangle \subseteq \langle \exp(\boldsymbol{A}T) \mid \mathscr{W} \rangle• \tag{44}$$

既然 $\langle \boldsymbol{A} \mid \mathscr{W} \rangle \supseteq \langle \exp(\boldsymbol{A}T) \mid \mathscr{W} \rangle$, 则有

$$\langle \boldsymbol{A} \mid \mathscr{W} \rangle = \langle \exp(\boldsymbol{A}T) \mid \mathscr{W} \rangle• \tag{45}$$

(Q. E. D)

3 结 论

本文首次将时滞引入到线性切换系统中, 提出含时滞的线性切换系统模型• 这里先给出问题描述和相关的准备知识, 在第 II 部分和第 III部分中将分别讨论含单时滞情形和多时滞情形线性切换系统的能控性•

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Controllability of a Class of Hybrid Dynamic Systems(Ⅰ) —Basic Properties and Preliminary Results

XIE Guang_ming, WANG Long, YE Qing_kai
(Department of Mechanics and Engineering Science,
Peking University, Beijing 100871, P.R. China)

Abstract: The controllability for switched linear system with time_delay in controls was first investigated. The whole work contains three parts. This is the first part, including problem formulation and some preliminaries. First, the mathematical model of switched linear systems with time_delay in control functions was presented. Secondly, the concept of column space, cyclic invariant subspace and generalized cyclic invariant subspace were introduced. And some basic properties, such as separation lemma, were presented. Finally, a basic lemma was given to reveal the relation between the solution set of a centain integral equations and the generalized cyclic invariant subspace. This lemma will play an important role in the determination of controllability. All these definitoins and lemmas are necessary research tools for controllability analysis.

Key words: hybrid dynamic system; switched linear system; time_delay; controllability; gener_alized cyclic invariant subspace; switching sequence; switching path