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一类混杂动态系统的能控性(III)
——含多时滞的情形*

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摘要: 首次将时滞现象引入到线性切换系统的模型中, 研究含有时滞线性切换系统的能控性及其判定条件• 全部工作由三部分组成• 第III部分, 主要研究含多时滞的线性切换系统的能控性及其判定规则• 首先给出周期型系统的单周能控性和多周期能控性的定义和充要条件, 其次给出非周期系统的能控性的定义和充要条件• 最后, 研究时滞大小不一致的情形, 指出能控性与时滞大小无关•

关 键 词: 混杂动态系统; 线性切换系统; 时滞; 能控性; 能控集; 切换序列; 切换路径

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引 言

输入函数含多个时滞的线性切换系统可描述如下:

$$\dot{\boldsymbol{x}}(t) = \boldsymbol{A}_{r(t)}\boldsymbol{x}(t) + \boldsymbol{B}_{r(t)}\boldsymbol{u}(t) + \sum_{k=1}^{K}\boldsymbol{D}_{r(t),k}\boldsymbol{u}(t-\tau_k), \tag{1}$$

其中, $\boldsymbol{x}(t)$, $\boldsymbol{u}(t)$ 和 $r(t)$ 如前, $\left\{(\boldsymbol{A}_i, \boldsymbol{B}_i, \boldsymbol{D}_{i,1}, \cdots, \boldsymbol{D}_{i,K}) \mid i = 1, \cdots, N\right\}$ 为一族切换模式• 进一步, $r(t) = i$ 表示在时刻t, 系统以模式$(\boldsymbol{A}_i, \boldsymbol{B}_i, \boldsymbol{D}_{i,1}, \cdots, \boldsymbol{D}_{i,K})$ 进行演变• $K < \infty$ 为系统包含时滞的总数, $0 < \tau_1 < \cdots < \tau_K$ 为K 个固定长度的时间滞后•

注 1 类似地, 我们也可以引入切换序列来描述切换路径• 为避免不必要的复杂性, 我们假设所有切换序列中持续时间满足 $h_m > \tau_K$•

下面我们研究系统(1) 的能控性• 首先研究周期型系统的单周期能控性和多周期能控性, 给出几何形式的充要判据; 然后研究非周期型系统的能控性, 同样给出几何形式的充要判据•

在整个工作的第 I 部分给出如下一些引理, 这些引理将作为讨论能控性的基本工具•

引理 1 给定矩阵 $\boldsymbol{A} \in \mathscr{R}^{n\times n}$, $\boldsymbol{B} \in \mathscr{R}^{n\times m}$, 对任意可逆矩阵 $\boldsymbol{P} \in \mathscr{R}^{m\times m}$, 都有

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$$\langle A \mid BP \rangle \langle A \mid B \rangle \cdot \tag{2}$$

引理 2 给定矩阵 $A \in \mathscr{R}^{n\times n}$, $B \in \mathscr{R}^{n\times m}$, 对任意常数 $h \in \mathbf{R}$, 有

$$\exp(Ah)\langle A \mid B \rangle = \langle A \mid B \rangle, \tag{3}$$

$$\langle A \mid \exp(Ah)B \rangle = \langle A \mid B \rangle \cdot \tag{4}$$

引理 3(分离引理) 给定矩阵 $A_1, A_2 \in \mathscr{R}^{n\times n}$; $B_1, B_2 \in \mathscr{R}^{n\times m}$, 有

$$\langle A_1 \mid B_1 + A_2 \mid B_2 \rangle + \langle A_2 \mid B_2 \rangle = \langle A_1 \mid B_1 \rangle + \langle A_2 \mid B_2 \rangle \cdot \tag{5}$$

引理 4 给定矩阵 $A_1, \cdots, A_N \in \mathscr{R}^{n\times n}$ 和 $B_1, \cdots, B_N \in \mathscr{R}^{n\times m}$, 对任意 $0 \leqslant t_0 < t_f < +\infty$, 有

$$\left\{ x \mid x = \sum_{i=1}^{N} \int_{t_0}^{t_f} \exp(A_i(t_f - s)) B_i u(s)\,\mathrm{d}s,\ \forall \text{ 分段连续 } u \right\} = \langle A_1 \mid B_1 + \cdots + A_N \mid B_N \rangle \cdot \tag{6}$$

特别地

$$\left\{ x \mid x = \int_{t_0}^{t_f} \exp(A_1(t_f - s)) B_1 u(s)\,\mathrm{d}s,\ \forall \text{ 分段连续 } u \right\} = \langle A_1 \mid B_1 \rangle \cdot \tag{7}$$

关于不含时滞的线性切换系统的能控性、稳定性和镇定等问题, 可参考[1~21]•

1 多滞后系统的能控性

1.1 周期型系统的能控性

不失一般性, 我们仍选择切换序列 $\pi = \{(1, h_1), \cdots, (N, h_N)\}$ 作为系统(1)的周期•

定理 1(充分必要性) 系统(1)为 1_周期能控的充要条件为

$$\sum_{i=1}^{N-1} \prod_{l=N}^{i+1} \exp(A_l h_l) \langle A_i \mid [B_i, D_{i,1}, \cdots, D_{i,K}] \rangle + \langle A_N \mid [B_N, D_{N,1}, \cdots, D_{N,K}] \rangle = \mathscr{R}^n \cdot \tag{8}$$

证明 下面, 我们仅对 $K = 2$ 时给出证明, 对 $K > 2$ 的情形可类证明•

对系统(1), 给定初始状态 x_0, 和初始控制函数 $u_0(t)$, $t \in [t_0 - \tau, t_0]$, 令 $t_m = t_0 + \sum_{l=1}^{m} h_l$, $(m = 1, \cdots, N)$• 那么, 终止状态 x_f 可以表示为:

$$\begin{aligned} x_f = {} & \prod_{i=N}^{1} \exp(A_i h_i) x_0 + \sum_{i=1}^{N-1} \prod_{l=N}^{i+1} \exp(A_l h_l) \int_{t_{i-1}}^{t_i} \exp[A_i(t_i - s)] (B_i u(s) + \\ & D_{i,1} u(s - \tau_1) + D_{i,2} u(s - \tau_2))\,\mathrm{d}s + \\ & \int_{t_{N-1}}^{t_N} \exp[A_N(t_N - s)] (B_N u(s) + D_{N,1} u(s - \tau_1) + D_{N,2} u(s - \tau_2))\,\mathrm{d}s \cdot \end{aligned} \tag{9}$$

考虑到

$$\begin{aligned} & \int_{t_{i-1}}^{t_i} \exp[A_i(t_i - s)] (D_{i,1} u(s - \tau_1) + D_{i,2} u(s - \tau_2))\,\mathrm{d}s = \\ & \int_{t_{i-1}-\tau_1}^{t_i - \tau_1} \exp[A_i(t_i - s)] \exp(-A_i \tau_1) D_{i,1} u(s)\,\mathrm{d}s + \\ & \int_{t_{i-1}-\tau_2}^{t_i - \tau_2} \exp[A_i(t_i - s)] \exp(-A_i \tau_2) D_{i,2} u(s)\,\mathrm{d}s = \\ & \int_{t_{i-1}-\tau_2}^{t_{i-1} - \tau_1} \exp[A_i(t_i - s)] \exp(-A_i \tau_2) D_{i,2} u(s)\,\mathrm{d}s + \\ & \int_{t_{i-1}-\tau_1}^{t_{i-1}} \exp[A_i(t_i - s)] [\exp(-A_i \tau_1) D_{i,1} + \exp(-A_i \tau_2) D_{i,2}] u(s)\,\mathrm{d}s + \end{aligned}$$

$$\int_{t_{i-1}}^{t_i-\tau_2}\exp[\boldsymbol{A}_i(t_i-s)][\boldsymbol{B}_i+\exp(-\boldsymbol{A}_i\tau_1)\boldsymbol{D}_{i,1}+\exp(-\boldsymbol{A}_i\tau_2)\boldsymbol{D}_{i,2}]\boldsymbol{u}(s)\mathrm{d}s+$$
$$\int_{t_i-\tau_2}^{t_i-\tau_1}\exp[\boldsymbol{A}_i(t_i-s)][\boldsymbol{B}_i+\exp(-\boldsymbol{A}_i\tau_1)\boldsymbol{D}_{i,1}]\boldsymbol{u}(s)\mathrm{d}s+$$
$$\int_{t_i-\tau_1}^{t_i}\exp[\boldsymbol{A}_i(t_i-s)]\boldsymbol{B}_i\boldsymbol{u}(s)\mathrm{d}s=$$
$$\exp(A_ih_i)\int_{t_{i-1}-\tau_2}^{t_{i-1}-\tau_1}\exp[\boldsymbol{A}_i(t_{i-1}-s)]\exp(-\boldsymbol{A}_i\tau_2)\boldsymbol{D}_{i,2}\boldsymbol{u}(s)\mathrm{d}s+$$
$$\exp(A_ih_i)\int_{t_{i-1}-\tau_2}^{t_{i-1}}\exp[\boldsymbol{A}_i(t_{i-1}-s)][\exp(-\boldsymbol{A}_i\tau_1)+\exp(-\boldsymbol{A}_i\tau_2)\boldsymbol{D}_{i,2}]\boldsymbol{u}(s)\mathrm{d}s+$$
$$\int_{t_{i-1}}^{t_i-\tau_2}\exp[\boldsymbol{A}_i(t_i-s)][\boldsymbol{B}_i+\exp(\boldsymbol{A}_i\tau_i)\boldsymbol{D}_{i,1}+\exp(-\boldsymbol{A}_i\tau_2)\boldsymbol{D}_{i,2}]\boldsymbol{u}(s)\mathrm{d}s+$$
$$\int_{t_i-\tau_2}^{t_i-\tau_1}\exp[\boldsymbol{A}_i(t_i-s)](\boldsymbol{B}_i+\exp(-\boldsymbol{A}_i\tau_1)\boldsymbol{D}_{i,1})\boldsymbol{u}(s)\mathrm{d}s+$$
$$\int_{t_i-\tau_1}^{t_i}\exp[\boldsymbol{A}_i(t_i-s)]\boldsymbol{B}_i\boldsymbol{u}(s)\mathrm{d}s,$$

由此,(9)可以改写为

$$\boldsymbol{x}_f=\sum_{i=N}^{1}\exp(\boldsymbol{A}_ih_i)\Big\{\boldsymbol{x}_0+\int_{t_0-\tau_2}^{t_0-\tau_1}\exp[\boldsymbol{A}_1(t_0-s)]\exp(-\boldsymbol{A}_1\tau_2)\boldsymbol{D}_{1,1}\boldsymbol{u}_0(s)\mathrm{d}s+$$
$$\int_{t_0-\tau_2}^{t_0-\tau_1}\exp[\boldsymbol{A}_1(t_0-s)]\exp(-\boldsymbol{A}_1\tau_2)\boldsymbol{D}_{1,1}\boldsymbol{u}_0(s)\mathrm{d}s\Big\}+$$
$$\sum_{i=1}^{N-1}\prod_{l=N}^{i+1}\exp(\boldsymbol{A}_lh_l)\Big\{\int_{t_{i-1}}^{t_i-\tau_2}\exp[\boldsymbol{A}_i(t_i-s)][\boldsymbol{B}_i+$$
$$\exp[-\boldsymbol{A}_i\tau_1]\boldsymbol{D}_{i,1}+\exp[-\boldsymbol{A}_i\tau_2]\boldsymbol{D}_{i,2}]\boldsymbol{u}(s)\mathrm{d}s+$$
$$\int_{t_i-\tau_2}^{t_i-\tau_1}\Big\{\exp[\boldsymbol{A}_i(t_i-s)][\boldsymbol{B}_i+\exp[-\boldsymbol{A}_i\tau_1]\boldsymbol{D}_{i,1}]+$$
$$\exp[\boldsymbol{A}_{i+1}(t_i-s)]\exp[-\boldsymbol{A}_{i+1}\tau_2]\boldsymbol{D}_{i+1,2}\Big\}\boldsymbol{u}(s)\mathrm{d}s+$$
$$\int_{t_i-\tau_2}^{t_i}\Big\{\exp[\boldsymbol{A}_i(t_i-s)]\boldsymbol{B}_i+\exp[\boldsymbol{A}_{i+1}(t_i-s)]]\exp[-\boldsymbol{A}_{i+1}\tau_1]\boldsymbol{D}_{i+1,1}+$$
$$\exp[-\boldsymbol{A}_{i+1}\tau_2]\boldsymbol{D}_{i+1,2}\Big\}\boldsymbol{u}(s)\mathrm{d}s\Big\}+$$
$$\int_{t_{N-1}}^{t_N-\tau_2}\exp[\boldsymbol{A}_N(t_N-s)][\boldsymbol{B}_N+\exp(-\boldsymbol{A}_N\tau_1)\boldsymbol{D}_{N,1}+\exp(-\boldsymbol{A}_N\tau_2)\boldsymbol{D}_{N,2}]\boldsymbol{u}(s)\mathrm{d}s+$$
$$\int_{t_N-\tau_2}^{t_N-\tau_1}\exp[\boldsymbol{A}_N(t_N-s)][\boldsymbol{B}_N+\exp(-\boldsymbol{A}_N\tau_1)\boldsymbol{D}_{N,1}]\boldsymbol{u}(s)\mathrm{d}s+$$
$$\int_{t_N-\tau_1}^{t_N}\exp[\boldsymbol{A}_N(t_N-s)]\boldsymbol{B}_N\boldsymbol{u}(s)\mathrm{d}s. \tag{10}$$

那么,系统为1_周期能控当且仅当如下定义的线性空间为全空间

$$\mathscr{K}=\Big\{\boldsymbol{x}\mid\boldsymbol{x}=f(\boldsymbol{u}),\text{分段连续 }\boldsymbol{u}\Big\}, \tag{11}$$

其中

$$
\begin{aligned}
f(\boldsymbol{u}) = &\sum_{i=1}^{N-1}\prod_{l=N}^{i+1}\exp(\boldsymbol{A}_l h_l)\Big\{\int_{t_{i-1}}^{t_i-\tau_2}\exp[\boldsymbol{A}_i(t_i-s)][\boldsymbol{B}_i+\\
&\exp[-\boldsymbol{A}_i\tau_1]\boldsymbol{D}_{i,1}+\exp[-\boldsymbol{A}_i\tau_2]\boldsymbol{D}_{i,2}]\boldsymbol{u}(s)\mathrm{d}s+\\
&\int_{t_i-\tau_2}^{t_i-\tau_1}\Big\{\exp[\boldsymbol{A}_i(t_i-s)][\boldsymbol{B}_i+\exp[(-\boldsymbol{A}_i\tau_1)]\boldsymbol{D}_{i,1}]+\\
&\exp[\boldsymbol{A}_{i+1}(t_i-s)]\exp(-\boldsymbol{A}_{i+1}\tau_2)\boldsymbol{D}_{i+1,2}\Big\}\boldsymbol{u}(s)\mathrm{d}s+\\
&\int_{t_i-\tau_2}^{t_i}\Big\{\exp[\boldsymbol{A}_i(t_i-s)]\boldsymbol{B}_i+\exp[\boldsymbol{A}_{i+1}(t_i-s)][\exp[-\boldsymbol{A}_{i+1}\tau_1]\boldsymbol{D}_{i+1,1}]+\\
&\exp[-\boldsymbol{A}_{i+1}\tau_2]\boldsymbol{D}_{i+1,2}\Big\}\boldsymbol{u}(s)\mathrm{d}s\Big\}+\\
&\int_{t_{N-1}}^{t_N-\tau_2}\exp[\boldsymbol{A}_N(t_N-s)][\boldsymbol{B}_N+\exp(-\boldsymbol{A}_N\tau_1)\boldsymbol{D}_{N,1}+\exp(-\boldsymbol{A}_N\tau_2)\boldsymbol{D}_{N,2}]\boldsymbol{u}(s)\mathrm{d}s+\\
&\int_{t_N-\tau_2}^{t_N-\tau_1}\exp[\boldsymbol{A}_N(t_N-s)][\boldsymbol{B}_N+\exp(-\boldsymbol{A}_N\tau_1)\boldsymbol{D}_{N,1}]\boldsymbol{u}(s)\mathrm{d}s+\\
&\int_{t_N-\tau_1}^{t_N}\exp[\boldsymbol{A}_N(t_N-s)]\boldsymbol{B}_N\boldsymbol{u}(s)\mathrm{d}s. \qquad (12)
\end{aligned}
$$

由引理4, 可得

$$
\begin{aligned}
\mathscr{K}' = &\sum_{i=1}^{N-1}\prod_{l=N}^{i+1}\exp(\boldsymbol{A}_l h_l)\Big\{\langle\boldsymbol{A}_i \mid (\boldsymbol{B}_i+\exp(-\boldsymbol{A}_i\tau_1)\boldsymbol{D}_{i,1}+\exp(-\boldsymbol{A}_i\tau_2)\boldsymbol{D}_{i,2})\rangle+\\
&\langle\boldsymbol{A}_i \mid \boldsymbol{B}_i+\boldsymbol{A}_i \mid \exp(-\boldsymbol{A}_i\tau_1)\boldsymbol{D}_{i,1}+\boldsymbol{A}_{i+1}\mid\exp(-\boldsymbol{A}_{i+1}\tau_2)\boldsymbol{D}_{i+1,2}\rangle+\\
&\langle\boldsymbol{A}_i \mid \boldsymbol{B}_i+\boldsymbol{A}_{i+1}\mid\exp(-\boldsymbol{A}_{i+1}\tau_1)\boldsymbol{D}_{i+1,1}+\boldsymbol{A}_{i+1}\mid\exp(-\boldsymbol{A}_{i+1}\tau_2)\boldsymbol{D}_{i+1,2}\rangle\Big\}+\\
&\langle\boldsymbol{A}_N \mid \boldsymbol{B}_N+\exp(-\boldsymbol{A}_N\tau_1)\boldsymbol{D}_{N,1}+\exp(-\boldsymbol{A}_N\tau_2)\boldsymbol{D}_{N,2}\rangle+\\
&\langle\boldsymbol{A}_N \mid \boldsymbol{B}_N+\exp(-\boldsymbol{A}_N\tau_1)\boldsymbol{D}_{N,1}\rangle+\langle\boldsymbol{A}_N \mid \boldsymbol{B}_N\rangle. \qquad (13)
\end{aligned}
$$

再根据引理3和引理2, 容易推得

$$
\begin{aligned}
\mathscr{K} = &\sum_{i=1}^{N-1}\prod_{l=N}^{i+1}\exp(\boldsymbol{A}_l h_l)\langle\boldsymbol{A}_i \mid [\boldsymbol{B}_i, \exp(-\boldsymbol{A}_i\tau_1)\boldsymbol{D}_{i,1}, \exp(-\boldsymbol{A}_i\tau_2)\boldsymbol{D}_{i,2}]\rangle+\\
&\langle\boldsymbol{A}_N \mid [\boldsymbol{B}_N, \exp(-\boldsymbol{A}_N\tau_1)\boldsymbol{D}_{N,1}, \exp(-\boldsymbol{A}_N\tau_2)\boldsymbol{D}_{N,2}]\rangle. \qquad (14)
\end{aligned}
$$

由引理1, 可知定理结论成立. (Q. E. D)

类似可得

定理2(充分必要性) 系统(1)为 m_周期能控的充要条件为

$$
\mathscr{K}+\left(\prod_{i=N}^{1}\exp(\boldsymbol{A}_i h_i)\right)\mathscr{K}+\cdots+\left(\prod_{i=N}^{1}\exp(\boldsymbol{A}_i h_i)\right)^{m-1}\mathscr{K} = \mathscr{R}^n, \qquad (15)
$$

其中, $\mathscr{K}$ 如(14)所定义.

注2 对任意 $m \geqslant n$, 系统(1)为 m_周期能控的充要条件为

$$
\langle\prod_{i=N}^{1}\exp(\boldsymbol{A}_i h_i) \mid \mathscr{K}\rangle = \mathscr{R}^n \qquad (16)
$$

1.2 非周期型系统的能控性

类似周期型系统能控性的讨论, 我们很容易将含单时滞情形的结论推广到含多时滞的情形, 下面不加证明将相应结论列出.

定理3 对系统(1), 给定状态 $\boldsymbol{x}_0$ 和初始控制函数 $\boldsymbol{u}_0(t)$, $t \in [t_0-\tau, t_0]$, 给定切换序列

列 $\pi=\{(i_m,h_m)\}_{m=1}^{M}$, 则切换序列 π 的 $(\boldsymbol{x}_0,\boldsymbol{u}_0)$_能控集 $\mathscr{C}(\boldsymbol{x}_0,\boldsymbol{u}_0,\pi)$ 为如下的集合:

$$\mathscr{C}(\boldsymbol{x}_0,\boldsymbol{u}_0,\pi)=\mathscr{T}n(\boldsymbol{x}_0,\boldsymbol{u}_0,\pi)+\sum_{m=1}^{M-1}\prod_{l=M}^{m+1}\exp[\boldsymbol{A}_{i_l}h_l]\langle\boldsymbol{A}_{i_m}\mid[\boldsymbol{B}_{i_m},\boldsymbol{D}_{i_m,1},\cdots,\boldsymbol{D}_{i_m,K}]\rangle+\langle\boldsymbol{A}_{i_M}\mid[\boldsymbol{B}_{i_M},\boldsymbol{D}_{i_M},\cdots,\boldsymbol{D}_{i_M,K}]\rangle,\tag{17}$$

其中状态

$$\mathscr{T}n(\boldsymbol{x}_0,\boldsymbol{u}_0,\pi)=\prod_{l=N}^{1}\exp(A_{i_l}h_l)\Big\{\boldsymbol{x}_0+\int_{t_0-\tau_K}^{t_0-\tau_{K-1}}\exp[\boldsymbol{A}_{i_1}(t_0-s)]\times\exp(-\boldsymbol{A}_{i_1}\tau_K)\boldsymbol{D}_{i_1,K}\boldsymbol{u}_0(s)\mathrm{d}s+\int_{t_0-\tau_{K-1}}^{t_0-\tau_{K-2}}\exp[\boldsymbol{A}_{i_1}(t_0-s)]\times(\exp(-\boldsymbol{A}_{i_1}\tau_K)\boldsymbol{D}_{i_1,K}+\exp(-\boldsymbol{A}_{i_1}\tau_{K-1})\boldsymbol{D}_{i_1,K-1})\boldsymbol{u}_0(s)\mathrm{d}s+\cdots+\int_{t_0-\tau_1}^{t_0}\exp[\boldsymbol{A}_{i_1}(t_0-s)](\exp(-\boldsymbol{A}_{i_1}\tau_K)\boldsymbol{D}_{i_1,K}+\cdots+\exp(-\boldsymbol{A}_{i_1}\tau_1)\boldsymbol{D}_{i_1,1})\boldsymbol{u}_0(s)\mathrm{d}s\Big\}.\tag{18}$$

特别地, 当 $\boldsymbol{x}_0=\boldsymbol{0}$, $\boldsymbol{u}_0=\boldsymbol{0}$ 时, 有

$$\mathscr{C}(\boldsymbol{0},\boldsymbol{0},\pi)=\sum_{m=1}^{M-1}\prod_{l=M}^{m+1}\exp[\boldsymbol{A}_{i_l}h_l]\langle\boldsymbol{A}_{i_m}\mid[\boldsymbol{B}_{i_m},\boldsymbol{D}_{i_m,1},\cdots,\boldsymbol{D}_{i_m,K}]\rangle+\langle\boldsymbol{A}_{i_M}\mid[\boldsymbol{B}_{i_M},\boldsymbol{D}_{i_M},\cdots,\boldsymbol{D}_{i_M,K}]\rangle\tag{19}$$

为 $\mathscr{R}^n$ 中的线性子空间, 我们将其简写为 $\mathscr{C}(\pi)$.

对系统(1), 我们递归定义如下一个线性子空间序列

$$\mathscr{W}_1=\sum_{i=1}^{N}\langle\boldsymbol{A}_i\mid[\boldsymbol{B}_i,\boldsymbol{D}_{i,1},\cdots,\boldsymbol{D}_{i,K}]\rangle,\quad\mathscr{W}_2=\sum_{i=1}^{N}\langle\boldsymbol{A}_i\mid\mathscr{W}_1\rangle,\ \cdots,$$
$$\mathscr{W}_n=\sum_{i=1}^{N}\langle(\boldsymbol{A}_i)\mid\mathscr{W}_{n-1}\rangle.\tag{20}$$

定理 4　对系统(1), 必存在某一切换序列 π_b, 使得

$$\mathscr{C}(\pi_b)=\mathscr{W}_n.\tag{21}$$

推论 1(充分必要性)　系统(1)能控的充分必要性条件为

$$\mathscr{W}_n=\mathscr{R}^n.\tag{22}$$

2　时滞不一致系统的能控性

回顾关于能控性的所有判据, 不论单时滞, 还是多时滞, 不论周期型系统, 还是非周期型系统, 都是在假设各个切换模式的时滞一致时得到的. 而得到的结果有一个共同的特点, 即所有判据都与时滞本身的大小无关. 下面, 我们讨论当各切换模式所含时滞不一致时, 系统能控性的判据问题.

输入函数含单个不一致时滞的线性切换系统可描述如下:

$$\dot{\boldsymbol{x}}(t)=\boldsymbol{A}_{r(t)}\boldsymbol{x}(t)+\boldsymbol{B}_{r(t)}\boldsymbol{u}(t)+\boldsymbol{D}_{r(t)}\boldsymbol{u}(t-\tau_{r(t)}),\tag{23}$$

其中, $\boldsymbol{x}(t)$, $\boldsymbol{u}(t)$, $r(t)$ 如前, $\{(\boldsymbol{A}_i,\boldsymbol{B}_i,\boldsymbol{D}_i,\tau_i)\mid i=1,\cdots,N\}$ 为一族切换模式. 进一步, $r(t)=i$ 表示在时刻 t, 系统以模式 $(\boldsymbol{A}_i,\boldsymbol{B}_i,\boldsymbol{D}_i,\tau_i)$ 进行演变. $\tau_i>0$, 为固定长度的时间滞后, $i=1,\cdots,N$.

定理 5　对系统(23)，对任意的切换序列 $\pi=\{(i_m, h_m)\}_{m=1}^{M}$，若其满足对 $m=1, \dots, M$，都有 $h_m > \tau_{i_m}$，则其能控集为

$$\mathscr{C}(\pi)=\sum_{m=1}^{M-1}\prod_{l=M}^{m+1}\exp[A_{i_l}h_l]\langle A_{i_m} \mid [B_{i_m}, D_{i_m}]\rangle+ \langle A_{i_M} \mid [B_{i_M}, D_{i_M}]\rangle \tag{24}$$

证明　我们仅对切换序列 $\pi=\{(1, h_1), (2, h_2)\}$ 进行验证．这个证明过程可以很容易推广到一般的切换序列．对切换序列 $\pi=\{(1, h_1), (2, h_2)\}$，有

$$\mathscr{C}(\pi)=\Big\{x \mid x=\exp(A_2h_2)\int_{t_0}^{t_1}[(A_1(t_1-s))](B_1u(s)+D_1u(s-\tau)\mathrm{d}s+ \int_{t_1}^{t_2}\exp[(A_2(t_2-s))](B_2u(s)+D_2u(s-\tau)\mathrm{d}s, \forall \text{ 分段连续 } u\Big\} \tag{25}$$

我们只考虑两种情况: 1) 当 $\tau_1>\tau_2$ 时, 和 2) 当 $\tau_1<\tau_2$ 时, 而二者相等时已不必考虑．

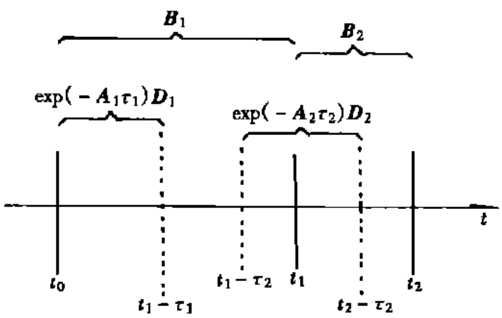

图 1　当 $\tau_1>\tau_2$ 时

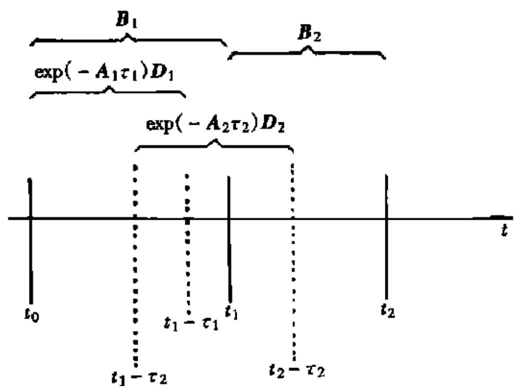

图 2　当 $\tau_1<\tau_2$, $t_0<t_1-\tau_2$ 时

当 $\tau_1>\tau_2$ 时, 如图 1 所示, 我们可以将积分区间分为 5 段, 即

$$\begin{aligned}\mathscr{C}(\pi)=&\exp[A_2h_2]\Big\{x \mid x=\int_{t_0}^{t_1-\tau_1}\exp[A_1(t_1-s)](B_1+\\&\exp[-A_1\tau_1]D_1)u(s)\mathrm{d}s, \forall \text{ 分段连续 } u\Big\}+\\&\exp[A_2h_2]\Big\{x \mid x=\int_{t_1-\tau_1}^{t_1-\tau_2}\exp[A_1(t_1-s)](B_1u(s)\mathrm{d}s, \forall \text{ 分段连续 } u\Big\}+\\&\exp[A_2h_2]\Big\{x \mid x=\int_{t_1-\tau_2}^{t_1}\Big\{\exp[A_1(t_1-s)]\times\\&B_1\exp[-A_2(t_1-s)]\exp[-A_2\tau_2]D_2\Big\}u(s)\mathrm{d}s, \forall \text{ 分段连续 } u\Big\}+\\&\Big\{x \mid x=\int_{t_1}^{t_2-\tau_2}\exp[A_2(t_2-s)](B_2+\\&\exp[-A_2(\tau_2)]D_2u(s)\mathrm{d}s, \forall \text{ 分段连续 } u\Big\}+\\&\Big\{x \mid x=\int_{t_2-\tau_2}^{t_2}\exp[A_2(t_2-s)]B_2u(s)\mathrm{d}s, \forall \text{ 分段连续 } u\Big\}=\\&\exp(A_2h_2)\Big\{\langle A_1 \mid B_1+\exp[-A_1\tau_1]D_1\rangle+\langle A_1 \mid B_1\rangle+\end{aligned}$$

$$\langle A_1 \mid B_1 + A_2 \mid \exp[-A_2\tau_2]D_2\rangle\} +$$
$$\langle A_2 \mid B_2 + \exp[-A_2\tau_2]D_2\rangle + \langle A_2 \mid B_2\rangle =$$
$$\exp(A_2h_2)\langle A_1 \mid [B_1, D_1] + A_2 \mid [B_2, D_2]\rangle. \tag{26}$$

当 $\tau_1 < \tau_2$ 时, 又分两种情况, $t_0 < t_1 - \tau_2$ 和 $t_0 \geqslant t_1 - \tau_2$. 当 $t_0 < t_1 - \tau_2$ 时, 如图 2 所示, 我们可以将积分区间分为 5 段, 即

$$\mathscr{C}(\pi) = \exp[A_2h_2]\left\{x \mid x = \int_{t_0}^{t_1-\tau_2}\exp[A_1(t_1 - s)](B_1 + \exp[-A_1\tau_1]D_1)u(s)\mathrm{d}s,\ \forall \text{ 分段连续 } u\right\} +$$
$$\exp[A_2h_2]\left\{x \mid x = \int_{t_1-\tau_2}^{t_1-\tau_1}\exp[A_1(t_1 - s)](B_1 + \exp[-A_1\tau_1]D_1) + \exp[A_2(t_1 - s)]\exp[-A_2\tau_2]D_2\right\}u(s)\mathrm{d}s,\ \forall \text{ 分段连续 } u\} +$$
$$\exp[A_2h_2]\left\{x \mid x = \int_{t_1-\tau_1}^{t_1}\left\{\exp[A_1(t_1 - s)]B_1 + \exp[A_2(t_1 - s)]\exp[-A_2\tau_2]D_2\right\}u(s)\mathrm{d}s,\ \forall \text{ 分段连续 } u\right\} +$$
$$\left\{x \mid x = \int_{t_1}^{t_2-\tau_2}\exp[A_2(t_2 - s)](B_2 + \exp[-A_2(\tau_2)]D_2u(s))\mathrm{d}s,\ \forall \text{ 分段连续 } u\right\} +$$
$$\left\{x \mid x = \int_{t_2-\tau_2}^{t_2}\exp[A_2(t_2 - s)]B_2u(s)\mathrm{d}s,\ \forall \text{ 分段连续 } u\right\} =$$
$$\exp(A_2h_2)\{\langle A_1 \mid B_1 + \exp[-A_1\tau_1]D_1\rangle +$$
$$\langle A_1 \mid (B_1 + \exp[-A_1\tau_1]D_1) + A_2 \mid \exp[-A_2\tau_2]D_2)\rangle +$$
$$\langle A_1 \mid B_1 + A_2 \mid \exp[-A_2\tau_2]D_2\rangle\} +$$
$$\langle A_2 \mid B_2 + \exp[-A_2\tau_2]D_2\rangle + \langle A_2 \mid B_2\rangle =$$
$$\exp(A_2h_2)\langle A_1 \mid [B_1, D_1] + A_2 \mid [B_2, D_2]\rangle. \tag{27}$$

当 $t_0 \geqslant t_1 - \tau_2$ 时, 如图 3 所示, 我们可以将积分区间分为 4 段, 即

$$\mathscr{C}(\pi) = \exp[A_2h_2]\left\{x \mid x = \int_{t_0}^{t_1-\tau_1}\left\{\exp[A_1(t_1 - s)](B_1 + \exp[-A_1\tau_1]D_1) \exp[A_2(t_1 - s)]\exp[-A_2\tau_2]D_2\right\}u(s)\mathrm{d}s,\ \forall \text{ 分段连续 } u\right\} +$$
$$\exp[A_2h_2]\left\{x \mid x = \int_{t_1-\tau_1}^{t_1}\left\{\exp[A_1(t_1 - s)]B_1 + \exp[A_1(t_1 - s)]\exp[-A_2\tau_2]D_2 + \right\}u(s)\mathrm{d}s,\ \forall \text{ 分段连续 } u\right\} +$$
$$\left\{x \mid x = \int_{t_1}^{t_2-\tau_2}\exp[A_2(t_2 - s)](B_2 + \exp[-A_2(\tau_2)]D_2u(s)\mathrm{d}s,\ \forall \text{ 分段连续 } u\right\} +$$
$$\left\{x \mid x = \int_{t_2-\tau_2}^{t_2}\exp[A_2(t_2 - s)]B_2u(s)\mathrm{d}s,\ \forall \text{ 分段连续 } u\right\} =$$

$$\begin{aligned}&\exp(\boldsymbol{A}_2h_2)\Big\{\langle \boldsymbol{A}_1 \mid (\boldsymbol{B}_1+\exp[-\boldsymbol{A}_1\tau_1]\boldsymbol{D}_1)+\boldsymbol{A}_2 \mid \exp[-\boldsymbol{A}_2\tau_2]\boldsymbol{D}_2\rangle+\\&\langle \boldsymbol{A}_1 \mid \boldsymbol{B}_1+\boldsymbol{A}_2 \mid \exp[-\boldsymbol{A}_2\tau_2]\boldsymbol{D}_2)\rangle\Big\}+\\&\langle \boldsymbol{A}_2 \mid \boldsymbol{B}_2+\exp[-\boldsymbol{A}_2\tau_2]\boldsymbol{D}_2\rangle+\langle \boldsymbol{A}_2 \mid \boldsymbol{B}_2\rangle=\\&\exp(\boldsymbol{A}_2h_2)\langle \boldsymbol{A}_1 \mid [\boldsymbol{B}_1,\boldsymbol{D}_1]\rangle+\langle \boldsymbol{A}_2 \mid [\boldsymbol{B}_2,\boldsymbol{D}_2]\rangle\boldsymbol{\cdot}\end{aligned}\tag{28}$$

于是对每种情形,其结果相同,均为(24)的形式•　(Q. E. D.)

推论2　系统(23)与系统(1)的能控性相同,即

1) 对周期型的情形,系统(23) 1_周期能控(m_周期能控)当且仅当系统(1) 1_周期能控(m_周期能控);

2) 对非周期型的情形,系统(23)能控当且仅当系统(1)能控;

注3　对多滞后的系统,仍然有类似的结论,即各时滞大小与系统能性无关•

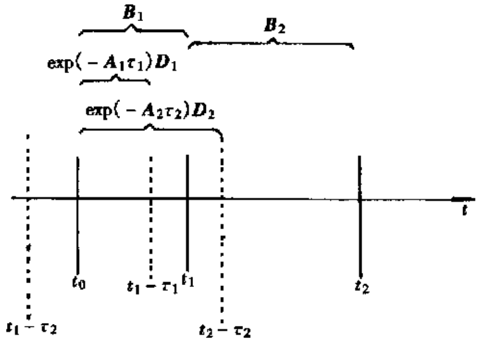

图3　当 $\tau_1<\tau_2$, $t_0\geqslant t_1-\tau_2$ 时

3　例　　子

下面我们给出两个含时滞系统的例子,并判定系统的能控性•

例1　考虑一个5维的含有单时滞的线性切换系统,有两个切换模式,分别如下:

$$\begin{cases}\boldsymbol{A}_1=\begin{bmatrix}1&0&0&0&0\\0&2&0&0&0\\0&0&3&0&0\\0&0&0&0&0\\0&0&0&0&0\end{bmatrix},\ \boldsymbol{B}_1=\begin{bmatrix}0\\0\\0\\1\\1\end{bmatrix},\ \boldsymbol{D}_1=\begin{bmatrix}0\\0\\0\\0\\1\end{bmatrix},\\[2ex]\boldsymbol{A}_2=\begin{bmatrix}0&0&0&0&0\\0&0&0&0&0\\0&0&0&0&0\\0&0&0&0&0\\0&0&0&0&0\end{bmatrix},\ \boldsymbol{B}_2=\begin{bmatrix}1\\1\\1\\0\\0\end{bmatrix},\ \boldsymbol{D}_2=\begin{bmatrix}0\\0\\0\\1\\0\end{bmatrix}\boldsymbol{\cdot}\end{cases}\tag{29}$$

设时滞 $\tau=1$•

考虑以切换序列 $\pi=\{(1,2),(2,2)\}$ 为周期得到的周期型切换系统,可得

$$\mathscr{C}(\pi)=\exp(\boldsymbol{A}_2h_2)\langle \boldsymbol{A}_1 \mid [\boldsymbol{B}_1,\boldsymbol{D}_1]+\boldsymbol{A}_2 \mid [\boldsymbol{B}_2,\boldsymbol{D}_2]\rangle=$$

$$\operatorname{span}\left\{\begin{bmatrix}1\\1\\1\\0\\0\end{bmatrix},\begin{bmatrix}0\\0\\0\\1\\2\end{bmatrix},\begin{bmatrix}0\\0\\0\\2\\1\end{bmatrix}\right\}\boldsymbol{\cdot}\tag{30}$$

显然,系统不为1_周期能控•　而

$$\langle\exp(2\boldsymbol{A}_2)\exp(2\boldsymbol{A}_1) \mid \mathscr{C}(\pi)\rangle=$$

$$\mathrm{span}\left\{\begin{bmatrix} e^2 \\ e^4 \\ e^6 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} e^4 \\ e^8 \\ e^{12} \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} e^6 \\ e^{12} \\ e^{18} \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \\ 2 \end{bmatrix}\right\}. \tag{31}$$

因此,系统必为多周期能控.

例 2 考虑一个 6 维的含单时滞的线性切换系统, 有 3 个切换模式, 分别如下:

$$\begin{cases} \boldsymbol{A}_1 = \begin{bmatrix} 0&0&0&0&0&0 \\ 0&0&0&0&0&0 \\ 0&0&0&0&0&0 \\ 0&0&0&0&0&0 \\ 0&0&0&0&0&0 \\ 0&0&0&0&0&0 \end{bmatrix}, \ \boldsymbol{B}_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \ \boldsymbol{D}_1 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}, \\ \boldsymbol{A}_2 = \begin{bmatrix} 0&0&0&0&0&0 \\ 1&0&0&0&0&0 \\ 0&0&0&0&0&0 \\ 0&0&0&0&0&0 \\ 0&0&0&0&0&0 \\ 0&0&0&0&0&0 \end{bmatrix}, \ \boldsymbol{B}_2 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \ \boldsymbol{D}_2 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}, \\ \boldsymbol{A}_3 = \begin{bmatrix} 0&0&0&0&0&0 \\ 0&0&0&0&0&0 \\ 0&0&1&0&0&0 \\ 0&0&0&0&0&0 \\ 0&0&0&0&0&0 \\ 0&0&0&0&0&0 \end{bmatrix}, \ \boldsymbol{B}_3 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \ \boldsymbol{D}_3 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}. \end{cases} \tag{32}$$

设时滞 $\tau_1 = 3, \tau_2 = \tau_3 = 1$.

经过简单计算可得

$$\mathscr{W}_1 = \mathrm{span}\left\{\begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}\right\}. \tag{33}$$

$$\mathscr{W}_2 = \mathrm{span}\left\{\begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}\right\}. \tag{34}$$

$$\mathscr{W}_3 = \operatorname{span}\left\{\begin{bmatrix}1\\0\\0\\0\\0\\0\end{bmatrix}, \begin{bmatrix}0\\1\\0\\0\\0\\0\end{bmatrix}, \begin{bmatrix}0\\0\\1\\0\\0\\0\end{bmatrix}, \begin{bmatrix}0\\0\\0\\1\\0\\0\end{bmatrix}, \begin{bmatrix}0\\0\\0\\0\\1\\0\end{bmatrix}, \begin{bmatrix}0\\0\\0\\0\\0\\1\end{bmatrix}\right\}. \tag{35}$$

根据推论 2, 系统应为能控• 事实上, 考虑切换序列 $\pi = \{(1,4),(2,2),(3,2),(1,4),(2,2),(1,4)\}$ 的能控集, 容易验证其为 $\mathscr{W}_3$• 由此可知系统确实能控•

4 结　论

本文首次将时滞引入到线性切换系统中,研究含多时滞的线性切换系统的能判性判定问题• 运用第 I 部分所给出的基本工具, 首先给出周期型系统单周期能控性的充要条件,然后给出多周期能控性的充要条件, 接着给出非周期型系统能控性的充要条件• 最后指出能控性与时滞大小无关•

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Controllability of a Class of Hybrid Dynamic Systems(III) —Multiple Time_Delay Case

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Abstract: The controllability for switched linear systems with time_delay in controls is first investigated. The whole work contains three parts. This is the third part. The definition and determination of controllability of switched linear systems with multiple time_delay in control functions is mainly investigated. The sufficient and necessary conditions for the 1_periodic, multiple_periodic controllability of periodic_type systems and controllability of aperiodic systems are presented, respectively. Finally, the case of distinct delays is discussed, it is shown that the controllability is independent of the size of delays.

Key words: hybrid dynamic system; switched linear system; time_delay; controllability; controllable set; switching sequence; switching path