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液固耦合系统中液体的有限幅晃动力及晃动力矩*

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(我刊编委马兴瑞来稿)

摘要: 研究弹簧_质量系统与圆柱贮箱类液体有限幅晃动系统间的非线性耦合动力学问题· 在建立了六自由度非线性耦合动力学模型的基础上, 导出了液体有限幅晃动力和力矩解析表达式· 指出在终了构形上积分及压力表达式中的非线性项是有限幅晃动作用力、作用力矩非线性的根源· x、y 方向结果之间良好的对称性在很大程度上证明了结果的正确性· 通过耦合机理分析可知, 这样的理论结果应具有较大的普适性· 数值仿真结果与有关实验结果进行了对比· 分析认为, 在终了构形上求晃动力、晃动力矩较为合理· 舍去的高维模态基底及高阶非线性项以及液体晃动阻尼的复杂性是导致偏差的重要原因·

关 键 词: 液体晃动; 力; 力矩; 液固耦合系统
中图分类号: V415.4 文献标识码: A

引 言

在航天工程中, 经常遇到与贮箱中液体晃动有关的耦合动力学问题[1]· 在复杂航天器系统中, 液体晃动总是与卫星的刚体运动、挠性附件的振动及控制作用等交耦在一起的· 这些运动及晃动初始条件以种种惯性力的方式去激发液体的晃动, 而液体晃动将产生晃动力、晃动力矩, 从而反作用于弹性振动、刚体运动及控制作用· 液体晃动本质上是一个非线性问题, 尤其是液体的有限幅晃动必须采用非线性模型才能给予正确的描述[2, 3]· 然而液体非线性晃动的解析研究并非易事, 一般性地进行液体_航天器的非线性耦合动力学研究将更加困难· Peterson, Crawley 等人采用一个简单的弹簧振子来代表航天器的线性弹性振动, 在质量块上固定一充液圆柱贮箱, 通过独到的实验及部分理论分析考察了该液体_航天器非线性耦合系统的动力学行为[4, 5]· 本文作者曾借助 Mathematica 软件的符号运算功能[6] 最终建立了在所考察自由度意义上完整的耦合动力学微分方程组· 通过数值实验发现, 在无挡板情况下, 该数学模型能够预见[4] 文实验发现的面外模态晃动分叉现象, 并且这是一种特殊的同步分叉现象; 在有档板

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情况下, 存在所谓阻尼渗透现象[6]• 本文进一步导出了耦合系统中液体大幅晃动力、晃动力矩的表达式, 并将数值仿真结果与 Peterson 等的实验结果进行了比较•

1　动力学模型简介

本文研究对象如图 1 所示[4,5]• 圆柱贮箱及箱底集中质量块的总质量为 M, 圆柱贮箱半径为 a, 平均充液深度为 h, 液体密度为 ρ• 贮箱在平面内 x 方向受到弹簧 k_x 及阻尼器 c_x 的约束• 均匀重力或加速度矢量 $-g\boldsymbol{k}$ 作用于贮箱轴向方向, 并足以保证将液体收集于贮箱底部•

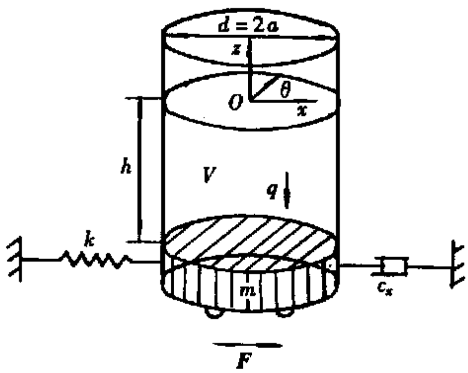

图 1　液固耦合系统动力学模型

将箱体固联坐标系 $Oxyz$ (相应柱坐标系为 $Or\theta z$) 的原点置于液体平均高度中心• 假定贮箱中所充为无粘、无旋、不可压缩理想流体, 则其大幅晃动按位势理论[7,8] 描述为:

$$\nabla^2\phi = 0 \qquad (\text{在 } V \text{ 内}), \tag{1}$$

$$\frac{\partial\phi}{\partial r} = 0 \qquad (\text{当 } r = a), \tag{2}$$

$$\frac{\partial\phi}{\partial z} = 0 \qquad (\text{当 } z = -h), \tag{3}$$

$$\frac{\partial\eta}{\partial t} + \nabla\phi\cdot\nabla\eta = \frac{\partial\phi}{\partial z} \qquad (\text{当 } z = \eta), \tag{4}$$

$$\frac{2\sigma H}{\rho} - \frac{\partial\phi}{\partial t} - \frac{1}{2}|\nabla\phi|^2 - g\eta - (a_x x + a_y y) = \frac{p_g - p_L}{\rho} \qquad (\text{当 } z = \eta), \tag{5}$$

上列诸式中 $\phi(r,\theta,z,t)$ 为速度势函数, $\eta(r,\theta,t)$ 为液高函数• (1) 式为液体质量守恒方程, (2), (3) 式为固壁边界条件, (4), (5) 式为自由液面边界条件• 其中(4) 式为运动学条件, (5) 式为动力学条件• 在小 Bond 数情况下, 应取展开式为:

$$\phi(r,\theta,z,t) = \sum_{i=1}^{N}\frac{\mathrm{ch}k_i(z+h)}{\mathrm{ch}k_i h}\psi_i(r,\theta)\phi_i(t), \tag{6}$$

$$\eta(r,\theta,t) = f(r) + \sum_{n=1}^{N}\xi_n(r,\theta)q_n(t), \tag{7}$$

其中 $f(r)$ 为弯曲自由静液面形状函数, ψ_n 为速度势内模态, ξ_n 为计入接触角线性迟滞 Γ 情况下的液高模态[4]• 在低频段, 参照[4] 文, 取保留模态最高阶数为 $N = 5$• 将伽辽金方法推广运用于非线性问题, 将(6), (7) 直接代入(4) 式进行处理最终得到从液高函数广义坐标 $q_i(i = 1, \cdots, 5)$ 到速度势函数广义坐标 $\phi_j(j = 1, \cdots, 5)$ 的非线性波长变换关系矩阵 $[l_{rn}]$, 其元素一般表达式为[6]:

$$l_{rn} = \delta_{rn}l_{nn}^{(0)} + \sum_{m=1}^{N}l_{rnm}^{(1)}q_m + \sum_{j=1}^{N}\sum_{m=1}^{N}l_{rnmj}^{(2)}q_jq_m + \cdots, \tag{8}$$

式中 $l_{rn}^{(0)}, l_{rnm}^{(1)}$ 及 $l_{rnmj}^{(2)}$ 为模态组分系数•

接下来建立耦合系统的 Lagrange 函数, 从而导出耦合系统的动力学方程为[6]

$$\{\ddot{P}\} = [R]\{F\}, \tag{9}$$

其中

$$\{\ddot{P}\} = \{\ddot{X}, \ddot{q}_1, \ddot{q}_2, \ddot{q}_3, \ddot{q}_4, \ddot{q}_5\}^{\mathrm{T}}, \tag{10}$$

$$[R] = [M]^{-1}, \tag{11}$$

式中对称阵 $[M]_{6\times 6}$ 为非线性耦合系统的时变质量阵, 它是系统广义坐标的函数•

(9) 式中 X 自由度方程为

$$(1+\mu)\ddot{X}+2\zeta_x\dot{X}+X+\alpha_1\ddot{q}_1=F_{ex}, \tag{12}$$

式中, $\mu=m/M$•

2 晃动力及晃动力矩

作用在贮箱上的晃动力与晃动力矩是贮箱中各点液体压力

$$p=-\rho\left[\Pi+\frac{\partial\phi}{\partial t}+\frac{1}{2}|\nabla\phi|^2+c(t)\right] \tag{13}$$

沿边界积分的结果• 式中 ρ 为液体密度, Π 为作用于液体质点上的单位质量力势函数, $c(t)$ 为 Bernoulli_Lagrange 积分引入的 t 的任意函数• 于是晃动力、力矩表达式分别为[9]

1) x 方向晃动力:

$$F_x=\iint_{S_w+S_f}p\cos(\boldsymbol{n},x)\,\mathrm{d}S, \tag{14}$$

2) y 方向晃动力:

$$F_y=\iint_{S_w+S_f}p\cos(\boldsymbol{n},y)\,\mathrm{d}S, \tag{15}$$

3) 绕 Ox 轴晃动力矩:

$$M_x=\iint_{S_w+S_f}p[z\cos(\boldsymbol{n},y)-y\cos(\boldsymbol{n},z)]\,\mathrm{d}S, \tag{16}$$

4) 绕 Oy 轴晃动力矩:

$$M_y=\iint_{S_w+S_f}p[z\cos(\boldsymbol{n},x)-x\cos(\boldsymbol{n},z)]\,\mathrm{d}S, \tag{17}$$

在上列(14)~(17)四式中, S_w 代表固壁边界, S_f 代表自由液面边界• 晃动液体对贮箱的作用力和力矩, 从严格意义上讲应该包括自由液面的表面张力在与贮箱固壁接触线上的作用力• 由于该作用力等于液体对自由表面的作用力, 因此计算作用力与力矩时, 要在 S_w 和 S_f 上均作积分• 本文推导的目的是将晃动作用力与力矩最终表达为广义坐标 $\phi_i(i=1,\cdots,5)$ 及 $q_i(i=1,\cdots,5)$ 的函数•

在(13)式中, 假定重力为 0, 晃动外激励 $\ddot{X}$ 亦为 0, 则单位质量力势函数

$$\Pi=\ddot{X}x+gz=0• \tag{18}$$

再假定液体恒静止, 即速度势函数

$$\phi(r,\theta,z,t)=0• \tag{19}$$

这时静止自由液面是球面的一部分• 于是运用 Laplace 公式[9]

$$p_g-p_L=\frac{2\sigma}{a^*}, \tag{20}$$

不确定出(13)式中的积分系数

$$c(t)=-\frac{p_L(t)}{\rho}=-\frac{p_g(t)}{\rho}+\frac{2\sigma}{a^*\rho}, \tag{21}$$

式中 $p_L(t)$ 为满足条件(18)及(19)时静止自由液面液体一侧的压力, $p_g(t)$ 为自由液面气枕一侧的压力, σ 为表面张力系数, a^* 为静止自由液面的球半径• 由于整个系统处于大气等压氛围中, 故应将 $p_g(t)$ 略去, 从而

$$c(t)=\frac{2\sigma}{a^{*}\rho}\cdot \tag{22}$$

现在我们可以看到,贮箱中液体晃动时内部各点压力形成可分解为:

$$p(\boldsymbol{R},t)=f_1(g)+f_2(\sigma)+f_3(\ddot{X})+f_4[\phi(g,\sigma)]\cdot \tag{23}$$

换言之,重力、外激励所致的惯性力及表面张力均分别导致了独立的液体压力分量• 它们同时又提供了液体晃动的恢复力• 在恢复力作用下,形成了液体无旋晃动的速度势场分布,这种速度势又导致了另外的液体压力分量,即(23)式中的第四项•

在导出晃动力、力矩最终表达式之前,明确定义积分边界是十分重要的一步• 定义速度势函数的初始构形的边界为:

a) 贮箱底:

$$z=-h\qquad(0\leqslant r\leqslant a,0\leqslant\theta\leqslant 2\pi), \tag{24}$$

b) 贮箱壁:

$$r=a\qquad(-h\leqslant z\leqslant 0,0\leqslant\theta\leqslant 2\pi), \tag{25}$$

c) 平均静液面:

$$z=0\qquad(0\leqslant r\leqslant a,0\leqslant\theta\leqslant 2\pi)\cdot \tag{26}$$

对于任意时间步长,液体晃动的终了构形为:

a) 贮箱底:

$$z=-h\qquad(0\leqslant r\leqslant a,0\leqslant\theta\leqslant 2\pi), \tag{27}$$

b) 贮箱壁:

$$r=a\qquad(-h\leqslant z\leqslant\eta,0\leqslant\theta\leqslant 2\pi), \tag{28}$$

c) 自由液面:

$$z=\eta\qquad(0\leqslant r\leqslant a,0\leqslant\theta\leqslant 2\pi)\cdot \tag{29}$$

不难看到,导致液体有限幅晃动作用力、力矩非线性特征的根源为:其一,压力表达式的非线性项 $|\nabla\phi(r,\theta,z,t)|^2$;其二,在终了构形上积分•

在推导过程中保留所有项将使表达式变得十分庞大• 根据在建模期间的研究工作[4,6],这里仍然指定 X, q_1, q_2, ϕ_1, ϕ_2 及其相应的广义速度为 ε^1 量级的量,q_3, q_4, q_5, ϕ_3, ϕ_4, ϕ_5 及其相应的广义速度为 ε^2 量级的量,而在推导晃动力、力矩的过程中则只保留 ε^1, ε^2, ε^3 量级的量•

经过大量演算,我们最终得到耦合系统中液体有限幅晃动力和力矩的表达式,以 x 方向晃动力为例,作用在固壁上的部分为:

$$\begin{aligned}F_x|_{S_w}={}&C_x^{Fxw}\ddot{X}+C_{q_1}^{Fxw}q_1+C_{\dot\phi_1}^{Fxw}\dot\phi_1+C_{xq_3}^{Fxw}\ddot{X}q_3+\\&C_{xq_4}^{Fxw}\ddot{X}q_4+C_{q_1q_3}^{Fxw}q_1q_3+C_{q_1q_4}^{Fxw}q_1q_4+C_{q_2q_5}^{Fxw}q_2q_5+\\&C_{\phi_1\phi_3}^{Fxw}\phi_1\phi_3+C_{\phi_1\phi_4}^{Fxw}\phi_1\phi_4+C_{\phi_2\phi_5}^{Fxw}\phi_2\phi_5+C_{\dot\phi_1q_3}^{Fxw}\dot\phi_1q_3+\\&C_{\dot\phi_1q_4}^{Fxw}\dot\phi_1q_4+C_{\dot\phi_2q_5}^{Fxw}\dot\phi_2q_5+C_{\dot\phi_3q_1}^{Fxw}\dot\phi_3q_1+C_{\dot\phi_4q_1}^{Fxw}\dot\phi_4q_1+\\&C_{\dot\phi_5q_2}^{Fxw}\dot\phi_5q_2+C_{\phi_1^2q_1}^{Fxw}\phi_1^2q_1+C_{\phi_1\phi_2q_2}^{Fxw}\phi_1\phi_2q_2+\\&C_{\phi_2^2q_1}^{Fxw}\phi_2^2q_1+C_{\dot\phi_1q_1^2}^{Fxw}\dot\phi_1q_1^2+C_{\dot\phi_1q_2^2}^{Fxw}\dot\phi_1q_2^2+C_{\dot\phi_2q_1q_2}^{Fxw}\dot\phi_2q_1q_2,\end{aligned} \tag{30}$$

其中各项系数及其它晃动力表达式在附录中给出• 限于篇幅,晃动力矩表达式未列出•

由附录可见，在各表达式中不含常值项，各项量纲均一致• 从附录中还可以看到，F_x 及 F_y 各表达式系数之间存在一种特殊的对称性，这是系统结构的对称性、所保留模态之间的对称性所导致的必然结果• 以上分析在很大程度上表明本文关于晃动力、晃动力矩的推导是正确的•

由(30)式可见，晃动力仅取决于刚性贮箱的运动加速度及各阶模态晃动的广义坐标、广义速度，故表达式应具有普适性• 即无论贮箱是处在什么样的耦合系统中，只要它仅作 x 方向的平动，则其中有限幅晃动力、晃动力矩均可由(30)式及附录中各式给出•

如果积分是在初始构形上进行，则(30)式简化为：

$$F_x\mid_{S_w} = C_x^{Fxw}\ddot{X} + C_{\phi_1}^{Fxw}\dot{\phi}_1 + C_{\phi_1\phi_3}^{Fxw}\phi_1\phi_3 + C_{\phi_1\phi_4}^{Fxw}\phi_1\phi_4 + C_{\phi_2\phi_5}^{Fxw}\phi_2\phi_5 • \tag{31}$$

其它晃动力、力矩表达式亦相应得到简化• 若忽略掉高阶非线性项，则上式继续简化为：

$$F_x\mid_{S_w} = C_x^{Fxw}\ddot{X} + C_{\phi_1}^{Fxw}\dot{\phi}_1 • \tag{32}$$

正如文献[4]所指出的那样，从概念上讲，由(12)式可直接获得 x 方向晃动力的一种表达式：

$$F_{xs} = -\mu\ddot{X} - \alpha_1\ddot{q}_1 • \tag{33}$$

3 数值仿真及讨论

取[4]文中图 9 对应系统作为算例，其有关参数为，贮箱半径 $d = 3.1\text{cm}$，充液深度 $h = 3.1\text{cm}$，常重力加速度 $g = 9.8\text{m/s}^2$，Bond 数 $B_0 = 66$，表面模态线性接触角迟滞常数 $\Gamma = 0$，质量比 $\mu = 0.1600$，液体一阶模态晃动阻尼比为 $\zeta_1 = \zeta_2 = 0.0348$，弹性振动阻尼比参照[4]文取为 $\zeta_x = 0.05$，无量纲外激励幅值为 $F_{ex}/kd = 0.0100$• 接触角[4]文末给出，据工程经验一般取为 $\theta_c = 5°$，实际上这对液体晃动特性的影响是微小的• 通过实验分别确定模态晃动的阻尼是困难的• 在以下计算中，我们取次生对称模态晃动的阻尼比为 $\zeta_3 = \zeta_4 = \zeta_5 = 0.035$• 实际上，这些阻尼比对晃动的影响是很大的，这一点将另文详细谈到•

本文数值仿真采用四阶定步长 Runde_Kutta 方法完成• 积分步长曾在 0.02～0.5 的范围取值，计算表明结果基本不依赖于步长• 无量纲数值仿真结果由图 2～图 7 给出• 在图中，“○”为[4]文实验结果，虚线为线性结果，点划线为在初始构形上的积分结果，实线为在终了构形上积分的结果，点线为(33)式仿真结果• f 为外激励频率，f_0 为弹簧_质量系统的特征频率•

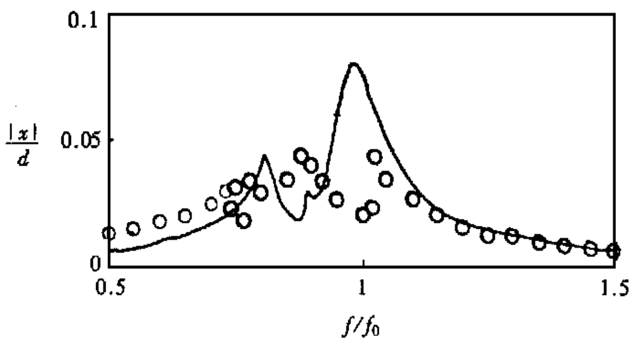

图 2 耦合系统中 X 自由度弹性振动频率响应曲线

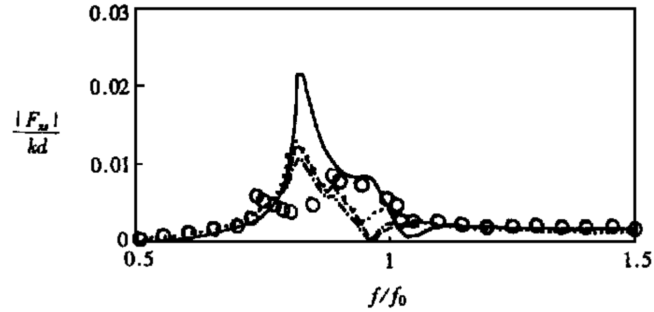

图 3 耦合系统中液体在 x 方向晃动力的频率响应曲线

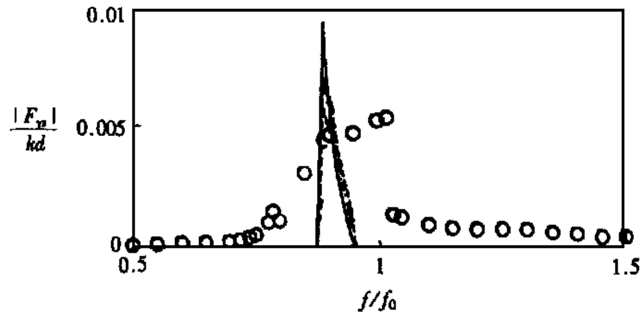

图 4 耦合系统中液体在 y 方向晃动力的频率响应曲线

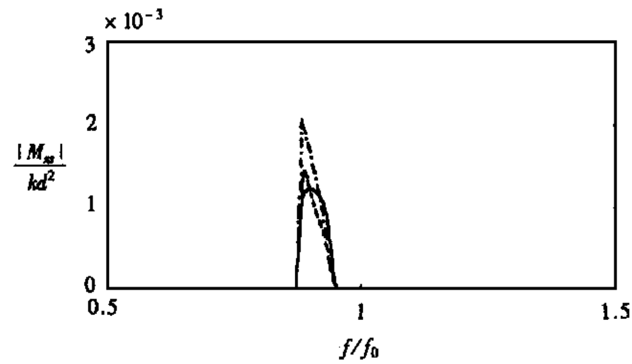

图 5 耦合系统中液体绕 x 轴晃动力矩的频率响应曲线

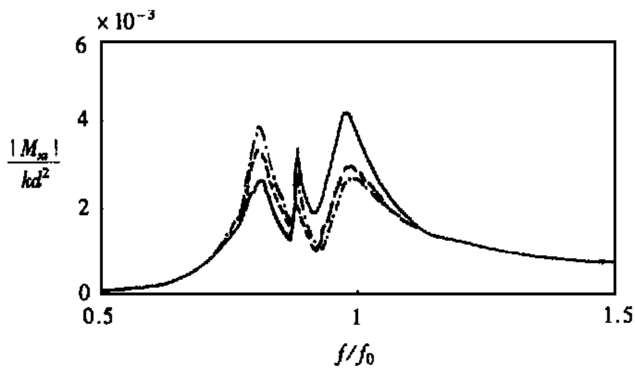

图 6 耦合系统中液体绕 y 轴晃动力矩的频率响应曲线

仿真结果中图 2、图 4 与实验结果相比误差较大, 这是由于理论模型所预示的面外晃动失稳频带与实验结果相差较大所致• 理论模型所预示的面外晃动失稳频带为 0.87~ 0.96 之间, 而实验所给出的面外晃动失稳频带为 0.75~ 1.05 之间• 前面已经提到, 液体面外晃动失稳频带受到晃动阻尼比的强烈影响, 晃动阻尼的微小变分将导致失稳频带的明显变化, 而准确确定液体晃动阻尼比是十分困难的•

图 3 给出了液体在平面内晃动力的变化规律, 仿真结果与实验结果的可比性较大• 从图 3 不难看出, 在液体面外晃动失稳区之外的近线性平庸晃动区, 晃动力解析表达式的仿真结果吻合很好• 其中在高频段上, 本文推导的结果同直接由(33) 式给出的结果相比, 前者与实验更为接近• 在实验所给出的失稳频带上, 晃动力理论与实验结果在液体一阶晃动特征频率点 0.90 的附近相当吻合• 在面外晃动失稳的其他频带上, 仿真结果与实验相差较大, 这反映出非线性液固耦合动力学解析建模研究的局限性• 其一, 以线性晃动模态系列作为基底对液体晃动速度势函数及液高函数进行展开时, 无限维基底的截取必然是有限的, 本文取为 5; 其二, 高阶非线性项的截断是必要的, 但当液体晃动幅度相当大时, 截断的高阶非线性项也会带来较大的误差; 其三, 液体在作大幅度的非线性晃动时, 其各自由度的晃动阻尼仍然沿用相应的线性晃动阻尼还是也具有非线性特征, 各阶晃动阻尼因子是否一致, 这仍然是值得探讨的问题•

从图 3 可见, 当无量纲外激励频率为 0.95 左右时线性结果及在初始构型上积分的结果接近于零, 这是二自由度线性系统受迫振动从以准同相模态响应为主向以准反相模态响应为主过渡的典型特征, 而在终了构形上积分所得(30) 式的仿真结果在趋势上与实验更为接近• 因此分析认为, 对晃动力的计算在终了构形上积分更为合理, 因为这更能反映有限幅晃动的非线性本质•

图 5 及图 6 分别给出了耦合系统的液体绕 x 轴及 y 轴晃动力矩的频率响应曲线, 图 5 的尖峰及图 6 的中间峰均是液体出现面外晃动的直接反映•

附　　录

1. $F_x|_{S_w}$ 表达式各项系数:

$$C^{Fxw}_{\phi_1^2 q_1} = -\frac{\rho\pi A_1^2 B_1 J_1^2(k_1 a) J_1(\lambda_1 a)}{16 a \cosh^2 k_1 h}\left\{1 - 3k_1^2 a^2 + (1 + 3k_1^2 a^2)\cosh 2k_1[f(a) + h]\right\},$$

$$C^{Fxw}_{\phi_1\phi_2 q_2} = -\frac{\rho\pi A_1^2 B_1 J_1^2(k_1 a) J_1(\lambda_1 a)}{8 a \cosh^2 k_1 h}\left\{-1 - k_1^2 a^2 + (-1 + k_1^2 a^2)\cosh 2k_1[f(a) + h]\right\},$$

$$C^{Fxw}_{\phi_2^2 q_1} = -\frac{\rho\pi A_1^2 B_1 J_1^2(k_1 a) J_1(\lambda_1 a)}{16 a \cosh^2 k_1 h}\left\{3 - k_1^2 a^2 + (3 + k_1^2 a^2)\cosh 2k_1[f(a) + h]\right\},$$

$$C^{Fxw}_{\phi_1 q_1^2} = -\frac{3\rho\pi a k_1 A_1 B_1^2 J_1(k_1 a) J_1^2(\lambda_1 a)}{8\cosh k_1 h}\sinh k_1[f(a) + h],$$

$$C^{Fxw}_{\phi_1 q_2^2} = -\frac{\rho\pi a k_1 A_1 B_1^2 J_1(k_1 a) J_1^2(\lambda_1 a)}{8\cosh k_1 h}\sinh k_1[f(a) + h],$$

$$C^{Fxw}_{\phi_2 q_1 q_2} = -\frac{\rho\pi a k_1 A_1 B_1^2 J_1(k_1 a) J_1^2(\lambda_1 a)}{4\cosh k_1 h}\sinh k_1[f(a) + h],$$

$$C^{Fxw}_{q_1 q_3} = -\rho g\pi a B_1 B_3 J_0(\lambda_3 a) J_1(\lambda_1 a), \quad C^{Fxw}_{q_1 q_4} = -\rho g\pi a B_1 B_4 J_1(\lambda_1 a) J_2(\lambda_4 a)/2,$$

$$C^{Fxw}_{q_2 q_5} = C^{Fxw}_{q_1 q_4},$$

$$C^{Fxw}_{\phi_1\phi_3} = \frac{\rho\pi a k_1 k_3 A_1 A_3 J_0(k_3 a) J_1(k_1 a)}{2(k_1 - k_3)(k_1 + k_3)\cosh k_1 h \cosh k_3 h}\times \left\{(k_1 + k_3)\sinh(k_1 - k_3)[f(a) + h] + (-k_1 + k_3)\sinh(k_1 + k_3)[f(a) + h]\right\},$$

$$C^{Fxw}_{\phi_1\phi_4} = \frac{\rho\pi A_1 A_4 J_1(k_1 a) J_2(k_4 a)}{4a(k_1 - k_4)(k_1 + k_4)\cosh k_1 h \cosh k_4 h} \times$$
$$\Big\{ [-2k_4 + k_1^2 k_4 a^2 + k_1(-2 + k_4^2 a^2)] \sinh(k_1 - k_4)[f(a) + h] +$$
$$[2k_4 - k_1^2 k_4 a^2 + k_1(-2 + k_4^2 a^2)] \sinh(k_1 + k_4)[f(a) + h] \Big\},$$

$$C^{Fxw}_{\phi_2\phi_5} = C^{Fxw}_{\phi_1\phi_4},$$

$$C^{Fxw}_{\dot\phi_1 q_3} = -\frac{\rho\pi a A_1 B_3 J_0(\lambda_3 a) J_1(k_1 a)}{\cosh k_1 h} \cosh k_1 [f(a) + h],$$

$$C^{Fxw}_{\dot\phi_1 q_4} = -\frac{\rho\pi a A_1 B_4 J_1(k_1 a) J_2(\lambda_4 a)}{2\cosh k_1 h} \cosh k_1 [f(a) + h], \quad C^{Fxw}_{\dot\phi_2 q_5} = C^{Fxw}_{\dot\phi_1 q_4},$$

$$C^{Fxw}_{\dot\phi_3 q_1} = -\frac{\rho\pi a A_3 B_1 J_0(k_3 a) J_1(\lambda_1 a)}{\cosh k_3 h} \cosh k_3 [f(a) + h],$$

$$C^{Fxw}_{\dot\phi_4 q_1} = -\frac{\rho\pi a A_4 B_1 J_1(k_1 a) J_2(\lambda_4 a)}{2\cosh k_4 h} \cosh k_4 [f(a) + h], \quad C^{Fxw}_{\dot\phi_5 q_2} = C^{Fxw}_{\dot\phi_4 q_1},$$

$$C^{Fzw}_{xq_3} = -\rho\pi a^2 B_3 J_0(\lambda_3 a), \quad C^{Fxw}_{xq_4} = -\frac{1}{2}\rho\pi a^2 B_4 J_2(\lambda_4 a),$$

$$C^{Fxw}_{q_1} = -a\pi B_1 J_1(\lambda_1 a)\left[\frac{2\sigma}{a^*} + \rho g f(a)\right],$$

$$C^{Fxw}_{\dot\phi_1} = -\frac{\rho a\pi A_1 J_1(k_1 a)}{k_1 \cosh k_1 h} \sinh k_1 [f(a) + h],$$

$$C^{Fxw}_{x} = -\rho a^2 \pi [f(a) + h]\cdot$$

2 $F_x|_{S_f}$ 表达式各项系数

$$F_x|_{S_f} = C^{Fxf}_x \ddot X + C^{Fxf}_{q_1} q_1 + C^{Fxf}_{\phi_1} \dot\phi_1 + C^{Fxf}_{xq_3} \ddot X q_3 + C^{Fxf}_{xq_4} \ddot X q_4 +$$
$$C^{Fxf}_{q_1 q_3} q_1 q_3 + C^{Fxf}_{q_1 q_4} q_1 q_4 + C^{Fxf}_{q_2 q_5} q_2 q_5 + C^{Fxf}_{\phi_1\phi_3} \phi_1\phi_3 + C^{Fxf}_{\phi_1\phi_4} \phi_1\phi_4 + C^{Fxf}_{\phi_2\phi_5} \phi_2\phi_5 +$$
$$C^{Fxf}_{\dot\phi_1 q_3} \dot\phi_1 q_3 + C^{Fxf}_{\dot\phi_1 q_4} \dot\phi_1 q_4 + C^{Fxf}_{\dot\phi_2 q_5} \dot\phi_2 q_5 + C^{Fxf}_{\dot\phi_3 q_1} \dot\phi_3 q_1 + C^{Fxf}_{\dot\phi_4 q_1} \dot\phi_4 q_1 + C^{Fxf}_{\dot\phi_5 q_2} \dot\phi_5 q_2 +$$
$$C^{Fxf}_{\phi_1^2 q_1} \phi_1^2 q_1 + C^{Fxf}_{\phi_1\phi_2 q_2} \phi_1\phi_2 q_2 + C^{Fxf}_{\phi_2^2 q_1} \phi_2^2 q_1 + C^{Fxf}_{\dot\phi_1 q_1^2} \dot\phi_1 q_1^2 + C^{Fxf}_{\dot\phi_1 q_2^2} \dot\phi_1 q_2^2 + C^{Fxf}_{\dot\phi_2 q_1 q_2} \dot\phi_2 q_1 q_2,$$

$$C^{Fxf}_{\phi_1^2 q_1} = -\frac{\rho\pi A_1^2 B_1}{64\cosh^2 k_1 h}\Big\{ 2k_1^2 \int_0^a J_1(\lambda_1 r)(J_0(k_1 r) - J_2(k_1 r))^2 \cosh^2 k_1 [f(r) + h]\, dr +$$
$$\lambda_1 \int_0^a \frac{1}{r}[J_0(\lambda_1 r) - J_2(\lambda_1 r)][3k_1^2 r^2 (J_0(k_1 r) - J_2(k_1 r))^2 +$$
$$4J_1^2(k_1 r)] \cosh^2 k_1 [f(r) + h]\, dr +$$
$$8k_1^2 \int_0^a J_1^2(k_1 r) J_1(\lambda_1 r) \sinh^2 k_1 [f(r) + h]\, dr +$$
$$12 k_1^2 \lambda_1 \int_0^a r J_1^2(k_1 r)[J_0(\lambda_1 r) - J_2(\lambda_1 r)] \sinh^2 k_1 [f(r) + h]\, dr +$$
$$24 k_1^3 \int_0^a r J_1^2(k_1 r) J_1(\lambda_1 r) f'(r) \sinh 2k_1 [f(r) + h]\, dr +$$
$$2k_1 \int_0^a \frac{1}{r} J_1(\lambda_1 r)[3k_1^2 r^2 (J_0(k_1 r) - J_2(k_1 r))^2 +$$
$$4J_1^2(k_1 r)] f'(r) \sinh 2k_1 [f(r) + h]\, dr \Big\},$$

$$C^{Fxf}_{\phi_1\phi_2 q_2} = -\frac{\rho\pi A_1^2 B_1}{32\cosh^2 k_1 h}\Big\{ -2k_1^2 \int_0^a J_1(\lambda_1 r)(J_0(k_1 r) - J_2(k_1 r))^2 \cosh^2 k_1 [f(r) + h]\, dr +$$
$$\lambda_1 \int_0^a \frac{1}{r}[J_0(\lambda_1 r) - J_2(\lambda_1 r)][k_1^2 r^2 (J_0(k_1 r) - J_2(k_1 r))^2 -$$
$$4J_1^2(k_1 r)] \cosh^2 k_1 [f(r) + h]\, dr -$$
$$8k_1^2 \int_0^a J_1^2(k_1 r) J_1(\lambda_1 r) \sinh^2 k_1 [f(r) + h]\, dr +$$

$$4k_1^2\lambda_1\int_0^a rJ_1^2(k_1r)[J_0(\lambda_1r)-J_2(\lambda_1r)]\sinh^2k_1[f(r)+h]\,dr+$$

$$8k_1^3\int_0^a rJ_1^2(k_1r)J_1(\lambda_1r)f'(r)\sinh 2k_1[f(r)+h]\,dr+$$

$$2k_1\int_0^a \frac{1}{r}J_1(\lambda_1r)[k_1^2r^2(J_0(k_1r)-J_2(k_1r))^2-$$

$$4J_1^2(k_1r)]f'(r)\sinh 2k_1[f(r)+h])\,dr\Big\},$$

$$C^{Fxf}_{\phi_2^2q_1}=-\frac{\rho\pi A_1^2B_1}{64\cosh^2k_1h}\Big\{6k_1^2\int_0^a J_1(\lambda_1r)(J_0(k_1r)-J_2(k_1r))^2\cosh^2k_1[f(r)+h]\,dr+$$

$$\lambda_1\int_0^a\frac{1}{r}[J_0(\lambda_1r)-J_2(\lambda_1r)][k_1^2r^2(J_0(k_1r)-J_2(k_1r))^2+$$

$$12J_1^2(k_1r)]\cosh^2k_1[f(r)+h]\,dr+$$

$$24k_1^2\int_0^a J_1^2(k_1r)J_1(\lambda_1r)\sinh^2k_1[f(r)+h]\,dr+$$

$$4k_1^2\lambda_1\int_0^a rJ_1^2(k_1r)[J_0(\lambda_1r)-J_2(\lambda_1r)]\sinh^2k_1[f(r)+h]\,dr+$$

$$8k_1^3\int_0^a rJ_1^2(k_1r)J_1(\lambda_1r)f'(r)\sinh 2k_1[f(r)+h]\,dr+$$

$$2k_1\int_0^a\frac{1}{r}J_1(\lambda_1r)[k_1^2r^2(J_0(k_1r)-J_2(k_1r))^2+$$

$$12J_1^2(k_1r)]f'(r)\sinh 2k_1[f(r)+h]\,dr\Big\},$$

$$C^{Fxf}_{\phi_1q_1^2}=-\frac{\rho\pi k_1A_1B_1^2}{8\cosh k_1h}\Big\{2\int_0^a J_1(k_1r)J_1^2(\lambda_1r)\sinh k_1[f(r)+h]\,dr+$$

$$3\int_0^a rJ_1(k_1r)J_1(\lambda_1r)(k_1J_1(\lambda_1r)f'(r)\cosh k_1[f(r)+h]+$$

$$\lambda_1[J_0(\lambda_1r)-J_2(\lambda_1r)]\sinh k_1[f(r)+h])\,dr\Big\},$$

$$C^{Fxf}_{\phi_1q_2^2}=\frac{\rho\pi k_1A_1B_1^2}{8\cosh k_1h}\Big\{2\int_0^a J_1(k_1r)J_1^2(\lambda_1r)\sinh k_1[f(r)+h]\,dr-$$

$$\int_0^a rJ_1(k_1r)J_1(\lambda_1r)(k_1J_1(\lambda_1r)f'(r)\cosh k_1[f(r)+h]+$$

$$\lambda_1[J_0(\lambda_1r)-J_2(\lambda_1r)]\sinh k_1[f(r)+h])\,dr\Big\},$$

$$C^{Fxf}_{\phi_2q_1q_2}=-\frac{\rho\pi k_1A_1B_1^2}{4\cosh k_1h}\Big\{2\int_0^a J_1(k_1r)J_1^2(\lambda_1r)\sinh k_1[f(r)+h]\,dr+$$

$$\int_0^a rJ_1(k_1r)J_1(\lambda_1r)(k_1J_1(\lambda_1r)f'(r)\cosh k_1[f(r)+h]+$$

$$\lambda_1[J_0(\lambda_1r)-J_2(\lambda_1r)]\sinh k_1[f(r)+h])\,dr\Big\},$$

$$C^{Fxf}_{q_1q_3}=-\frac{1}{2}\rho g\pi B_1B_3\Big\{2\int_0^a J_0(\lambda_3r)J_1(\lambda_1r)\,dr+$$

$$\lambda_3\int_0^a rJ_1(\lambda_1r)[J_{-1}(\lambda_3r)-J_1(\lambda_3r)]\,dr+\lambda_1\int_0^a rJ_0(\lambda_3r)[J_0(\lambda_1r)-J_2(\lambda_1r)]\,dr,$$

$$C^{Fxf}_{q_1q_4}=-\frac{1}{4}\rho g\pi B_1B_4\Big\{2\int_0^a J_1(\lambda_1r)J_2(\lambda_4r)\,dr+$$

$$\lambda_1\int_0^a rJ_2(\lambda_4r)[J_0(\lambda_1r)-J_2(\lambda_1r)]\,dr+\lambda_4\int_0^a rJ_1(\lambda_1r)[J_1(\lambda_4r)-J_3(\lambda_4r)]\,dr,$$

$$C^{Fxf}_{q_2q_5}=C^{Fxf}_{q_1q_4},$$

$$C^{Fxf}_{\phi_1\phi_3}=-\frac{\rho\pi k_1k_3A_1A_3}{4\cosh k_1h\cosh k_3h}\Big\{\int_0^a rf'(r)[J_{-1}(k_3r)-J_1(k_3r)][J_0(k_1r)-J_2(k_1r)]\times$$

$$\cosh k_1[f(r)+h]\cosh k_3[f(r)+h]\,dr+$$

$$4\int_0^a rf'(r)\mathrm{J}_0(k_3r)\mathrm{J}_1(k_1r)\sinh k_1[f(r)+h]\sinh k_3[f(r)+h]\mathrm{d}r\Big\},$$

$$C_{\phi_1\phi_4}^{Fxf}=-\frac{\rho\pi A_1A_4}{8\cosh k_1h\cosh k_4h}\Big\{\int_0^a\frac{1}{r}f'(r)(k_1k_4r^2[\mathrm{J}_0(k_1r)-\mathrm{J}_2(k_1r)][\mathrm{J}_1(k_4r)-\mathrm{J}_3(k_4r)]+$$
$$8\mathrm{J}_1(k_1r)\mathrm{J}_2(k_4r))\cosh k_1[f(r)+h]\cosh k_4[f(r)+h]\mathrm{d}r+$$
$$4k_1k_4\int_0^a rf'(r)\mathrm{J}_1(k_1r)\mathrm{J}_2(k_4r)\sinh k_1[f(r)+h]\sinh k_4[f(r)+h]\mathrm{d}r\Big\},$$

$$C_{\phi_2\phi_5}^{Fxf}=C_{\phi_1\phi_4}^{Fxf},$$

$$C_{\phi_1q_3}^{Fxf}=-\frac{\rho\pi A_1B_3}{2\cosh k_1h}\int_0^a r\mathrm{J}_1(k_1r)\Big\{\lambda_3[\mathrm{J}_{-1}(\lambda_3r)-\mathrm{J}_1(\lambda_3r)]\cosh k_1[f(r)+h]+$$
$$2k_1f'(r)\mathrm{J}_0(\lambda_3r)\sinh k_1[f(r)+h]\Big\}\mathrm{d}r$$

$$C_{\phi_1q_4}^{Fxf}=-\frac{\rho\pi A_1B_4}{4\cosh k_1h}\Big\{4\int_0^a\mathrm{J}_1(k_1r)\mathrm{J}_2(\lambda_4r)\cosh k_1[f(r)+h]\mathrm{d}r+$$
$$\int_0^a r\mathrm{J}_1(k_1r)(\lambda_4[\mathrm{J}_1(\lambda_4r)-\mathrm{J}_3(\lambda_4r)]\cosh k_1[f(r)+h]+$$
$$2k_1f'(r)\mathrm{J}_2(\lambda_4r)\sinh k_1[f(r)+h])\mathrm{d}r\Big\},$$

$$C_{\phi_2q_5}^{Fxf}=C_{\phi_1q_4}^{Fxf},$$

$$C_{\phi_3q_1}^{Fxf}=-\frac{\rho\pi A_3B_1}{2\cosh k_3h}\Big\{2\int_0^a\mathrm{J}_0(k_3r)\mathrm{J}_1(\lambda_1r)\cosh k_3[f(r)+h]\mathrm{d}r+$$
$$\int_0^a r\mathrm{J}_0(k_3r)(\lambda_1[\mathrm{J}_0(\lambda_1r)-\mathrm{J}_2(\lambda_1r)]\cosh k_3[f(r)+h]+$$
$$2k_3f'(r)\mathrm{J}_1(\lambda_1r)\sinh k_3[f(r)+h])\mathrm{d}r\Big\},$$

$$C_{\phi_4q_1}^{Fxf}=\frac{\rho\pi A_4B_1}{4\cosh k_4h}\Big\{2\int_0^a\mathrm{J}_1(\lambda_1r)\mathrm{J}_2(k_4r)\cosh k_4[f(r)+h]-$$
$$\int_0^a r\mathrm{J}_2(k_4r)(\lambda_1[\mathrm{J}_0(\lambda_1r)-\mathrm{J}_2(\lambda_1r)]\cosh k_4[f(r)+h]+$$
$$2k_4f'(r)\mathrm{J}_1(\lambda_1r)\sinh k_4[f(r)+h])\mathrm{d}r\Big\},$$

$$C_{\phi_5q_2}^{Fxf}=C_{\phi_4q_1}^{Fxf},\quad C_{xq_3}^{Fxf}=\rho a^2\pi B_3\mathrm{J}_2(\lambda_3a),\quad C_{xq_4}^{Fxf}=-\frac{1}{2}\rho a^2\pi B_4\mathrm{J}_2(\lambda_4a),$$

$$C_{q_1}^{Fxf}=-\frac{\pi B_1}{2}\Big\{\frac{4a\sigma}{a^*}\mathrm{J}_1(\lambda_1a)+2\rho g\int_0^a f(r)\mathrm{J}_1(\lambda_1r)\mathrm{d}r+$$
$$\rho g\int_0^a r(\lambda_1f(r)[\mathrm{J}_0(\lambda_1r)-\mathrm{J}_2(\lambda_1r)]+2f'(r)\mathrm{J}_1(\lambda_1r))\mathrm{d}r,$$

$$C_{\phi_1}^{Fxf}=-\frac{\rho\pi A_1}{\cosh k_1h}\int_0^a rf'(r)\mathrm{J}_1(k_1r)\cosh k_1[f(r)+h]\mathrm{d}r,\quad C_x^{Fxf}=-\rho\pi\int_0^a r^2f'(r)\mathrm{d}r\cdot$$

3 $F_y|_{S_w}$ 表达式各项系数

$$F_y|_{S_w}=C_{q_2}^{Fyw}q_2+C_{\phi_2}^{Fyw}\dot{\phi}_2+C_{xq_5}^{Fyw}\ddot{X}q_5+C_{q_1q_5}^{Fyw}q_1q_5+C_{q_2q_3}^{Fyw}q_2q_3+C_{q_2q_4}^{Fyw}q_2q_4+$$
$$C_{\phi_1\phi_5}^{Fyw}\phi_1\phi_5+C_{\phi_2\phi_3}^{Fyw}\phi_2\phi_3+C_{\phi_2\phi_4}^{Fyw}\phi_2\phi_4+C_{\phi_1q_5}^{Fyw}\dot{\phi}_1q_5+C_{\phi_2q_3}^{Fyw}\dot{\phi}_2q_3+C_{\phi_2q_4}^{Fyw}\dot{\phi}_2q_4+$$
$$C_{\phi_3q_2}^{Fyw}\dot{\phi}_3q_2+C_{\phi_4q_2}^{Fyw}\dot{\phi}_4q_2+C_{\phi_5q_1}^{Fyw}\dot{\phi}_5q_1+C_{\phi_1^2q_2}^{Fyw}\phi_1^2q_2+C_{\phi_1\phi_2q_1}^{Fyw}\phi_1\phi_2q_1+$$
$$C_{\phi_2^2q_2}^{Fyw}\phi_2^2q_2+C_{\phi_1q_1q_2}^{Fyw}\dot{\phi}_1q_1q_2+C_{\dot{\phi}_2q_1^2}^{Fyw}\dot{\phi}_2q_1^2+C_{\dot{\phi}_2q_2^2}^{Fyw}\dot{\phi}_2q_2^2,$$

$$C_{\phi_1^2q_2}^{Fyw}=C_{\phi_2^2q_1}^{Fxw},\ C_{\phi_1\phi_2q_1}^{Fyw}=C_{\phi_1\phi_2q_2}^{Fxw},\ C_{\phi_1^2q_2}^{Fyw}=C_{\phi_1^2q_1}^{Fxw},\ C_{\dot{\phi}_1q_1q_2}^{Fyw}=C_{\dot{\phi}_2q_1q_2}^{Fxw},\ C_{\dot{\phi}_2q_1^2}^{Fyw}=C_{\dot{\phi}_1q_2^2}^{Fxw},$$

$$C_{\dot{\phi}_2q_2^2}^{Fyw}=C_{\dot{\phi}_1q_1^2}^{Fxw},\ C_{q_1q_5}^{Fyw}=C_{q_1q_4}^{Fxw},\ C_{q_2q_3}^{Fyw}=C_{q_1q_3}^{Fxw},\ C_{q_2q_4}^{Fyw}=-C_{q_1q_4}^{Fxw},\ C_{\phi_1\phi_5}^{Fyw}=C_{\phi_1\phi_4}^{Fxw},$$

$$C^{Fyw}_{\phi_2\phi_3} = C^{Fxw}_{\phi_1\phi_3},\ C^{Fyw}_{\phi_2\phi_4} = -\ C^{Fxw}_{\phi_1\phi_4},\ C^{Fyw}_{\dot\phi_1 q_5} = C^{Fxw}_{\dot\phi_1 q_4},\ C^{Fyw}_{\dot\phi_2 q_3} = C^{Fxw}_{\dot\phi_1 q_3},\ C^{Fyw}_{\dot\phi_2 q_4} = -\ C^{Fxw}_{\dot\phi_1 q_4},$$

$$C^{Fyw}_{\dot\phi_3 q_2} = C^{Fxw}_{\dot\phi_3 q_1},\ C^{Fyw}_{\dot\phi_4 q_2} = -\ C^{Fxw}_{\dot\phi_4 q_1},\ C^{Fyw}_{\dot\phi_5 q_1} = C^{Fxw}_{\dot\phi_4 q_1},\ C^{Fyw}_{xq_5} = C^{Fxw}_{xq_4},\ C^{Fyw}_{q_2} = C^{Fxw}_{q_1},$$

$$C^{Fyw}_{\dot\phi_2} = C^{Fxw}_{\dot\phi_1}\cdot$$

4. $F_y|_{S_f}$ 表达式各项系数

$$\begin{aligned}F_y|_{S_f} = {} & C^{Fyf}_{q_2}q_2 + C^{Fyf}_{\dot\phi_2}\dot\phi_2 + C^{Fyf}_{xq_5}\ddot{X}q_5 + C^{Fyf}_{q_1q_5}q_1q_5 + C^{Fyf}_{q_2q_3}q_2q_3 + C^{Fyf}_{q_2q_4q_2}q_4 + \\ & C^{Fyf}_{\phi_1\phi_5}\phi_1\phi_5 + C^{Fyf}_{\phi_2\phi_3}\phi_2\phi_3 + C^{Fyf}_{\phi_2\phi_4}\phi_2\phi_4 + C^{Fyf}_{\dot\phi_1 q_5}\dot\phi_1 q_5 + C^{Fyf}_{\dot\phi_2 q_3}\dot\phi_2 q_3 + C^{Fyf}_{\dot\phi_2 q_4}\dot\phi_2 q_4 + \\ & C^{Fyf}_{\dot\phi_3 q_2}\dot\phi_3 q_2 + C^{Fyf}_{\dot\phi_4 q_2}\dot\phi_4 q_2 + C^{Fyf}_{\dot\phi_5 q_1}\dot\phi_5 q_1 + C^{Fyf}_{\phi_1^2 q_2}\phi_1^2 q_2 + C^{Fyf}_{\phi_1\phi_2 q_1}\phi_1\phi_2 q_1 + \\ & C^{Fyf}_{\phi_2^2 q_2}\phi_2^2 q_2 + C^{Fyf}_{\dot\phi_1 q_1 q_2}\dot\phi_1 q_1 q_2 + C^{Fyf}_{\dot\phi_2 q_1^2}\dot\phi_2 q_1^2 + C^{Fyf}_{\dot\phi_2 q_2^2}\dot\phi_2 q_2^2,\end{aligned}$$

$$C^{Fyf}_{\phi_1^2 q_2} = C^{Fxf}_{\phi_2^2 q_1},\ C^{Fyf}_{\phi_1\phi_2 q_1} = C^{Fxf}_{\phi_1\phi_2 q_2},\ C^{Fyf}_{\phi_1^2 q_2} = C^{Fxf}_{\phi_1^2 q_1},\ C^{Fyf}_{\dot\phi_1 q_1 q_2} = C^{Fxf}_{\dot\phi_2 q_1 q_2},\ C^{Fyf}_{\dot\phi_2 q_1^2} = C^{Fxf}_{\dot\phi_1 q_2^2},$$

$$C^{Fyf}_{\dot\phi_2 q_2^2} = C^{Fxf}_{\dot\phi_1 q_1^2},\ C^{Fyf}_{q_1q_5} = C^{Fxf}_{q_1q_4},\ C^{Fyf}_{q_2q_3} = C^{Fxf}_{q_1q_3},\ C^{Fyf}_{q_2q_4} = -\ C^{Fxf}_{q_1q_4},\ C^{Fyf}_{\phi_1\phi_5} = C^{Fxf}_{\phi_1\phi_4},$$

$$C^{Fyf}_{\phi_2\phi_3} = C^{Fxf}_{\phi_1\phi_3},\ C^{Fyf}_{\phi_2\phi_4} = -\ C^{Fxf}_{\phi_1\phi_4},\ C^{Fyf}_{\dot\phi_1 q_5} = C^{Fxf}_{\dot\phi_1 q_4},\ C^{Fyf}_{\dot\phi_2 q_3} = C^{Fxf}_{\dot\phi_1 q_3},\ C^{Fyf}_{\dot\phi_2 q_4} = -\ C^{Fxf}_{\dot\phi_1 q_4},$$

$$C^{Fyf}_{\dot\phi_3 q_2} = C^{Fxf}_{\dot\phi_3 q_1},\ C^{Fyf}_{\dot\phi_4 q_2} = -\ C^{Fxf}_{\dot\phi_4 q_1},\ C^{Fyf}_{\dot\phi_5 q_1} = C^{Fxf}_{\dot\phi_4 q_1},\ C^{Fyf}_{xq_5} = C^{Fxf}_{xq_4},\ C^{Fyf}_{q_2} = C^{Fxf}_{q_1},$$

$$C^{Fyf}_{\dot\phi_2} = C^{Fxf}_{\dot\phi_1}\cdot$$

[参 考 文 献]

[1] Cendral J L, Berry W. ESA' s activities in the fields of dynamics and technology related to the presence of liquids on_board spacecraft[A]. Proceedings of the CNES_ESA Conference on Attitude Control of Space Vehicles_Technological and Dynamical Problems Associated with the Presence of Liquids[C]. Toulouse, 10_12, Oct, 1977.

[2] Miles J W. Nonlinear surface waves in closed basins[J]. Journal of Fluid Mechanics, 1976, **75**: 419—448.

[3] Hutton. An investigation of resonant nonlinear nonplanar free surface oscillations of a fluid[R]. NASA TN D_1870, 1963.

[4] Peterson L D, Crawley E F, Hansman R J. The nonlinear dynamics of a spacecraft coupled to the vibration of a contained fluid[R]. AIAA paper 88_2470.

[5] Van Schoor M C, Crawley E F. Nonlinear forced_response characteristics of contained fluids in microgravity[J]. Journal of Spacecraft and Rockets, 1995, **32**(3): 521—532.

[6] 苟兴宇. 航天工程中的贮箱类液固耦合动力学研究[D]. 哈尔滨: 博士学位论文. 哈尔滨工业大学, 1998.

[7] 苟兴宇, 王本利, 马兴瑞, 等. 圆柱贮箱中液体晃动的模部分析[J]. 宇航学报, 1998, **19**(1): 72—76.

[8] 苟兴宇, 王本利, 马兴瑞, 等. 小 Bond 数条件下圆柱贮箱中液体晃动的模部分析[J]. 应用数学和力学, 1999, **20**(9): 913—918.

[9] 黄圳圭, 赵志建. 大型航天器动力学与控制[M]. 长沙: 国防科技大学出版社, 1990, 168.

Forces and Moments of the Liquid Finite Amplitude Sloshing in a Liquid_Solid Coupled System

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Abstract: Nonlinear coupling dynamics between a spring_mass system and a finite amplitude sloshing system with liquid in a cylindrical tank is investigated. Based on a group of nonlinear coupling equations of six degrees of freedoms, analytical formulae of forces and moments of the liquid large amplitude sloshing were obtained. Nonlinearity of the forces and moments of the sloshing was induced by integrating on final configuration of liquid sloshing and the nonlinear terms in the liquid pressure formula. The symmetry between the formula of ox and oy direction proves that the derivation is correct. According to the coupled mechanism, the formulae are available in other liquid_solid coupled systems. Simulations and corresponding experimental results are compared. It is shown that the forces and moments formulae by integrating on the final sloshing configuration are more reasonable. The omitted high_dimensional modal bases and high_order nonlinear terms and the complexity of sloshing damping are main sources of errors.

Key words: liquid sloshing; force; moment; liquid_solid coupled system