应用数学和力学编委会编
重 庆 出 版 社 出 版

论文编号: 1000_0887(1999)02_0193_199

受弯正交异性复合材料板的裂纹尖端场*

杨维阳, 张少琴

太原重型机械学院 数理系, 太原 030024

(罗海安推荐)

摘要: 本文对受对称弯曲载荷作用的线弹性正交异性复合材料板的裂纹尖端场进行了有关的力学分析· 采用复变函数方法推出了裂纹尖端附近的弯矩、扭矩、应力、应变和位移的计算公式·

关键词: 裂纹尖端场; 弯曲载荷; 复合材料

分类号: O346.1; O174.5 文献标识码: A

1 力学模型

本文和文[1]讨论相同的断裂问题, 即对于厚为 h, 中心有长为 $2a$ 的穿透裂纹, 受对称弯曲载荷 M_0 作用, x 轴和 y 轴平行于弹性主方向, 如图 1 所示的无限大线弹性正交异性复合材料板的裂纹尖端的力学性态进行分析· 这归结为求解下列偏微分方程的边值问题:

$$Q_{11}\frac{\partial^4 w}{\partial x^4}+2(Q_{12}+2Q_{66})\frac{\partial^4 w}{\partial x^2\partial y^2}+Q_{22}\frac{\partial^4 w}{\partial y^4}=0, \tag{1.1}$$

$$|y|\to\infty \text{ 时}, \quad M_y=M_0,\ M_{xy}=0, \tag{1.2}$$

$$y=0,\ |x|<a \text{ 时}, \quad M_y=0,\ M_{xy}=0, \tag{1.3}$$

其中 w 为中面挠度, M_x, M_y 为弯矩, M_{xy} 为扭矩· 且抗弯刚度

$$Q_{11}=\frac{h^3E_1}{12(1-\nu_1\nu_2)},\quad Q_{22}=\frac{h^3E_2}{12(1-\nu_1\nu_2)},\quad Q_{12}=\nu_1Q_{22}=\nu_2Q_{11};$$

抗扭刚度 $Q_{66}=\dfrac{h^3G}{12}$·

根据后面讨论的需要, 我们将板的弯曲公式[2]摘录如下:

$$\left.\begin{aligned} M_x &= -\left(Q_{11}\frac{\partial^2 w}{\partial x^2}+Q_{12}\frac{\partial^2 w}{\partial y^2}\right),\\ M_y &= -\left(Q_{12}\frac{\partial^2 w}{\partial x^2}+Q_{22}\frac{\partial^2 w}{\partial y^2}\right),\\ M_{xy} &= -2Q_{66}\frac{\partial^2 w}{\partial x\partial y}, \end{aligned}\right\} \tag{1.4}$$

$$\varepsilon_x=-z\frac{\partial^2 w}{\partial x^2},\quad \varepsilon_y=-z\frac{\partial^2 w}{\partial y^2},\quad \gamma_{xy}=-2z\frac{\partial^2 w}{\partial x\partial y}, \tag{1.5}$$

* 收稿日期: 1997_03_26; 修订日期: 1997_07_30
基金来源: 山西省自然科学基金资助项目(901050)
作者简介: 杨维阳(1937~), 男, 教授, 机电部有突出贡献的专家

$$\varepsilon_x = a_{11}\sigma_x + a_{12}\sigma_y,\quad \varepsilon_y = a_{12}\sigma_x + a_{22}\sigma_y,\quad \gamma_{xy} = a_{66}\tau_{xy}, \tag{1.6}$$

$$u = -z\frac{\partial w}{\partial x},\quad v = -z\frac{\partial w}{\partial y}, \tag{1.7}$$

其中 ε_x, ε_y, γ_{xy} 是应变, σ_x, σ_y, τ_{xy} 是应力, u, v 是位移, a_{11}, a_{12}, a_{22}, a_{66} 是复合材料板关于弹性主方向的柔度系数•

需要说明的是, 文[1] 给出了仅适用于 $\Delta > 0$ 的复合材料的力学公式, 而本文除此还在第 3 节中给出了适用于 $\Delta < 0$ 的复合材料的力学公式, 在第 4 节中给出了 $\Delta > 0$ 和 $\Delta < 0$ 的复合材料均适用的统一表示式, 所以本文是文[1] 的补充和完善•

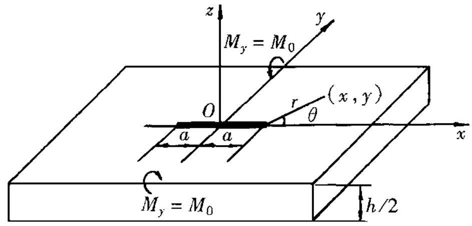

图1　受弯裂纹板

2　挠　　度

下面我们仿照文[3]~[5]采用复变函数方法对边值问题(1.1)、(1.2)、(1.3)进行求解和讨论•　偏微分方程(1.1)的解为

$$w = \sum_{j=1}^{2}\left[a_j \mathrm{Re}(\bar{\bar{w}}_j) + b_j \mathrm{Im}(\bar{\bar{w}}_j)\right], \tag{2.1}$$

其中

$$\bar{\bar{w}}_j = \bar{\bar{w}}_j(x + s_j y) = \bar{\bar{w}}_j(z_j) \tag{2.2}$$

且

$$\frac{d\bar{\bar{w}}_j}{dz_j} = \bar{w}_j,\quad \frac{d\bar{w}_j}{dz_j} = w_j,\quad \frac{dw_j}{dz_j} = w_j', \tag{2.3}$$

这时 s_j 满足下列双二次方程:

$$Q_{22}s^4 + 2(Q_{12} + 2Q_{66})s^2 + Q_{11} = 0• \tag{2.4}$$

记

$$\Delta = \left[\frac{2(Q_{12} + 2Q_{66})}{Q_{22}}\right]^2 - 4\frac{Q_{11}}{Q_{22}}, \tag{2.5}$$

则 $\Delta > 0$ 时, 方程(2.4)有两对共轭纯虚根:

$$s_1 = i\mu_1,\quad \bar{s}_1 = -i\mu_1,\quad s_2 = i\mu_2,\quad \bar{s}_2 = -i\mu_2, \tag{2.6}$$

其中 $\mu_1 > 0$, $\mu_2 > 0$•　当 $\Delta < 0$ 时, 方程(2.4) 有两对共轭复根:

$$s_1 = \lambda + i\mu,\quad \bar{s}_1 = \lambda - i\mu,\quad s_2 = -\lambda + i\mu,\quad \bar{s}_2 = -\lambda - i\mu, \tag{2.7}$$

其中 $\lambda > 0$, $\mu > 0$•

由(2.1)、(2.2)、(2.3), 可以得到

$$\frac{\partial^2 w}{\partial x^2} = \sum_{j=1}^{2}\left[a_j \mathrm{Re}(w_j) + b_j \mathrm{Im}(w_j)\right], \tag{2.8a}$$

$$\frac{\partial^2 w}{\partial y^2} = \sum_{j=1}^{2}\left[a_j \mathrm{Re}(s_j^2 w_j) + b_j \mathrm{Im}(s_j^2 w_j)\right], \tag{2.8b}$$

$$\frac{\partial^2 w}{\partial x \partial y} = \sum_{j=1}^{2}\left[a_j \mathrm{Re}(s_j w_j) + b_j \mathrm{Im}(s_j w_j)\right]• \tag{2.8c}$$

将该式代入(1.4), 有

$$M_x = -\sum_{j=1}^{2}\left\{a_j \mathrm{Re}\left[(Q_{11} + s_j^2 Q_{12})w_j\right] + b_j \mathrm{Im}\left[(Q_{11} + s_j^2 Q_{12})w_j\right]\right\}, \tag{2.9a}$$

$$M_y = -\sum_{j=1}^{2}\left\{a_j \mathrm{Re}\left[(Q_{12} + s_j^2 Q_{22})w_j\right] + b_j \mathrm{Im}\left[(Q_{12} + s_j^2 Q_{22})w_j\right]\right\}, \tag{2.9b}$$

$$M_{xy} = -2Q_{66}\sum_{j=1}^{2}[a_j\mathrm{Re}(s_jw_j) + b_j\mathrm{Im}(s_jw_j)]. \tag{2.9c}$$

考虑到复合材料板受对称弯曲载荷 M_0 作用，特别选取函数

$$w_j = w_j(z_j) = \frac{g(z_j)}{(z_j^2 - a^2)^{1/2}}, \tag{2.10}$$

其中

$$g(z_j) = M_0z_j, \qquad (z_j \to a). \tag{2.11}$$

对于图1所示情况，有

$$y \to +\infty: \qquad w_j = M_0; \tag{2.12}$$

$$y = 0, |x| < a: \ w_j = -i\frac{M_0x}{(a^2 - x^2)^{1/2}}. \tag{2.13}$$

1. 对于 $\Delta > 0$ 的复合材料

将(2.12)、(2.13)、(2.9)代入边界条件(1.2)、(1.3)，注意到(2.6)，有

$$Q_{12}(a_1 + a_2) - Q_{22}(\mu_1^2a_1 + \mu_2^2a_2) = -1, \tag{2.14a}$$

$$\mu_1b_1 + \mu_2b_2 = 0, \tag{2.14b}$$

$$Q_{12}(b_1 + b_2) - Q_{22}(\mu_1^2b_1 + \mu_2^2b_2) = 0, \tag{2.14c}$$

$$\mu_1a_1 + \mu_2a_2 = 0. \tag{2.14d}$$

求解该方程组，得到

$$a_1 = -\frac{\mu_2}{\mu_1}a_2 = \frac{\mu_2}{(\mu_1 - \mu_2)(Q_{12} + \mu_1\mu_2Q_{22})}, \tag{2.15a}$$

$$b_1 = b_2 = 0, \tag{2.15b}$$

将其代入(2.1)，有

$$w = \frac{1}{(\mu_1 - \mu_2)(Q_{12} + \mu_1\mu_2Q_{22})}[\mu_2\mathrm{Re}(\overline{\overline{w}}_1) - \mu_1\mathrm{Re}(\overline{\overline{w}}_2)]. \tag{2.16}$$

2. 对于 $\Delta < 0$ 的复合材料

将(2.12)、(2.13)、(2.9)代入(1.2)、(1.3)，注意到(2.7)，有

$$[Q_{12} + (\lambda^2 - \mu^2)Q_{22}](a_1 + a_2) + 2\lambda\mu Q_{22}(b_1 - b_2) = -1, \tag{2.17a}$$

$$\lambda(a_1 - a_2) + \mu(b_1 + b_2) = 0, \tag{2.17b}$$

$$2\lambda\mu Q_{22}(a_1 - a_2) - [Q_{12} + (\lambda^2 - \mu^2)Q_{22}](b_1 + b_2) = 0, \tag{2.17c}$$

$$\mu(a_1 + a_2) - \lambda(b_1 - b_2) = 0. \tag{2.17d}$$

求解该方程组，得到

$$a_1 = a_2 = -\frac{1}{2[Q_{12} + (\lambda^2 + \mu^2)Q_{22}]}, \tag{2.18a}$$

$$b_1 = -b_2 = -\frac{\mu}{2\lambda[Q_{12} + (\lambda^2 + \mu^2)Q_{22}]}, \tag{2.18b}$$

将其代入(2.1)，有

$$w = -\frac{1}{2\lambda[Q_{12} + (\lambda^2 + \mu^2)Q_{22}]}[\lambda\mathrm{Re}(\overline{\overline{w}}_1 + \overline{\overline{w}}_2) + \mu\mathrm{Im}(\overline{\overline{w}}_1 - \overline{\overline{w}}_2)]. \tag{2.19}$$

3 裂纹尖端场

1. 对于 $\Delta > 0$ 的复合材料

上节1中的推导过程及其结果与文[1]中的(25)～(41)完全吻合，无需讨论.

2. 对于 $\Delta < 0$ 的复合材料

上节 2 中的推导过程及其结果, 文[1] 中的相应部分未有给出• 在这种情况下, 将(2.18)、(2.7) 依次代入(2.8)、(2.9), 有

$$\frac{\partial^2 w}{\partial x^2} = -\frac{1}{2\lambda[Q_{12}+(\lambda^2+\mu^2)Q_{22}]}[\lambda \mathrm{Re}(w_1+w_2)+\mu \mathrm{Im}(w_1-w_2)], \tag{3.1a}$$

$$\frac{\partial^2 w}{\partial y^2} = -\frac{\lambda^2+\mu^2}{2\lambda[Q_{12}+(\lambda^2+\mu^2)Q_{22}]}[\lambda \mathrm{Re}(w_1+w_2)-\mu \mathrm{Im}(w_1-w_2)], \tag{3.1b}$$

$$\frac{\partial^2 w}{\partial x \partial y} = -\frac{\lambda^2+\mu^2}{2\lambda[Q_{12}+(\lambda^2+\mu^2)Q_{22}]}\mathrm{Re}(w_1-w_2); \tag{3.1c}$$

$$M_x = \frac{1}{2\lambda[Q_{12}+(\lambda^2+\mu^2)Q_{22}]}\Big\{\lambda[Q_{11}+(\lambda^2+\mu^2)Q_{12}]\mathrm{Re}(w_1+w_2)+ \mu[Q_{11}-(\lambda^2+\mu^2)Q_{12}]\mathrm{Im}(w_1-w_2)\Big\}, \tag{3.2a}$$

$$M_y = \frac{1}{2\lambda[Q_{12}+(\lambda^2+\mu^2)Q_{22}]}\Big\{\lambda[Q_{12}+(\lambda^2+\mu^2)Q_{22}]\mathrm{Re}(w_1+w_2)+ \mu[Q_{12}-(\lambda^2+\mu^2)Q_{22}]\mathrm{Im}(w_1-w_2)\Big\}, \tag{3.2b}$$

$$M_{xy} = \frac{(\lambda^2+\mu^2)Q_{66}}{\lambda[Q_{12}+(\lambda^2+\mu^2)Q_{22}]}\mathrm{Re}(w_1-w_2). \tag{3.2c}$$

引进应力强度因子

$$K_{\mathrm{I}} = \lim_{z_j \to a}[2\pi(z_j-a)]^{1/2}w_j(z_j), \tag{3.3}$$

由(2.10)、(2.11) 可知, 在裂纹尖端附近 $(z_j \to a)$, 有

$$w_j = w_j(z_j) = \frac{K_{\mathrm{I}}}{\sqrt{2\pi}}\frac{1}{(z_j-a)^{1/2}}, \tag{3.4}$$

其中 $z_j - a = x - a + s_j y, \qquad (j = 1, 2).$ (3.5)

将(3.4) 代入(3.2), 得到裂纹尖端附近弯矩和扭矩的计算公式:

$$M_x = \frac{K_{\mathrm{I}}}{\sqrt{2\pi}}\frac{1}{2\lambda[Q_{12}+(\lambda^2+\mu^2)Q_{22}]}\Big\{\lambda[Q_{11}+(\lambda^2+\mu^2)Q_{12}]\mathrm{Re}\Big[\frac{1}{(z_1-a)^{1/2}}+\frac{1}{(z_2-a)^{1/2}}\Big]+\mu[Q_{11}-(\lambda^2+\mu^2)Q_{12}]\mathrm{Im}\Big[\frac{1}{(z_1-a)^{1/2}}-\frac{1}{(z_2-a)^{1/2}}\Big]\Big\}, \tag{3.6a}$$

$$M_y = \frac{K_{\mathrm{I}}}{\sqrt{2\pi}}\frac{1}{2\lambda[Q_{12}+(\lambda^2+\mu^2)Q_{22}]}\Big\{\lambda[Q_{12}+(\lambda^2+\mu^2)Q_{22}]\mathrm{Re}\Big[\frac{1}{(z_1-a)^{1/2}}+\frac{1}{(z_2-a)^{1/2}}\Big]+\mu[Q_{12}-(\lambda^2+\mu^2)Q_{22}]\mathrm{Im}\Big[\frac{1}{(z_1-a)^{1/2}}-\frac{1}{(z_2-a)^{1/2}}$$

$$\gamma_{xy} = z\frac{K_{\mathrm{I}}}{\sqrt{2\pi}}\frac{\lambda^2+\mu^2}{\lambda[Q_{12}+(\lambda^2+\mu^2)Q_{22}]}\mathrm{Re}\left[\frac{1}{(z_1-a)^{1/2}}-\frac{1}{(z_2-a)^{1/2}}\right]\cdot \tag{3.7c}$$

由(1.5)、(1.6)解得:

$$\sigma_x = -z\left(a_{22}\frac{\partial^2 w}{\partial x^2}-a_{12}\frac{\partial^2 w}{\partial y^2}\right)\Big/(a_{11}a_{22}-a_{12}^2), \tag{3.8a}$$

$$\sigma_y = -z\left(a_{11}\frac{\partial^2 w}{\partial y^2}-a_{12}\frac{\partial^2 w}{\partial x^2}\right)\Big/(a_{11}a_{22}-a_{12}^2), \tag{3.8b}$$

$$\tau_{xy} = -2z\frac{\partial^2 w}{\partial x\partial y}\Big/a_{66}\cdot \tag{3.8c}$$

将(3.1)、(3.4)代入上式, 得到裂纹尖端附近应力的计算公式:

$$\sigma_x = z\frac{K_{\mathrm{I}}}{\sqrt{2\pi}}\frac{1}{2\lambda[Q_{12}+(\lambda^2+\mu^2)Q_{22}](a_{11}a_{22}-a_{12}^2)}\cdot$$
$$\left\{\lambda[a_{22}-(\lambda^2+\mu^2)a_{22}]\mathrm{Re}\left[\frac{1}{(z_1-a)^{1/2}}+\frac{1}{(z_2-a)^{1/2}}\right]+\right.$$
$$\left.\mu[a_{22}+(\lambda^2+\mu^2)a_{12}]\mathrm{Im}\left[\frac{1}{(z_1-a)^{1/2}}-\frac{1}{(z_2-a)^{1/2}}\right]\right\}, \tag{3.9a}$$

$$\sigma_y = z\frac{K_{\mathrm{I}}}{\sqrt{2\pi}}\frac{1}{2\lambda[Q_{12}+(\lambda^2+\mu^2)Q_{22}](a_{11}a_{22}-a_{12}^2)}\cdot$$
$$\left\{\lambda[a_{11}(\lambda^2+\mu^2)-a_{12}]\mathrm{Re}\left[\frac{1}{(z_1-a)^{1/2}}+\frac{1}{(z_2-a)^{1/2}}\right]-\right.$$
$$\left.\mu[a_{11}(\lambda^2+\mu^2)+a_{12}]\mathrm{Im}\left[\frac{1}{(z_1-a)^{1/2}}-\frac{1}{(z_2-a)^{1/2}}\right]\right\}, \tag{3.9b}$$

$$\tau_{xy} = z\frac{K_{\mathrm{I}}}{\sqrt{2\pi}}\frac{\lambda^2+\mu^2}{\lambda[Q_{12}+(\lambda^2+\mu^2)Q_{22}]a_{66}}\mathrm{Re}\left[\frac{1}{(z_1-a)^{1/2}}-\frac{1}{(z_2-a)^{1/2}}\right]\cdot \tag{3.9c}$$

注意到(2.19)、(2.2)、(2.3)、(3.4), 由(1.7)得到裂纹尖端附近位移的计算公式:

$$u = z\frac{K_{\mathrm{I}}}{\sqrt{2\pi}}\frac{1}{\lambda[Q_{12}+(\lambda^2+\mu^2)Q_{22}]}\left\{\lambda\mathrm{Re}[(z_1-a)^{1/2}+(z_2-a)^{1/2}]+\right.$$
$$\left.\mu\mathrm{Im}[(z_1-a)^{1/2}-(z_2-a)^{1/2}]\right\}, \tag{3.10a}$$

$$v = z\frac{K_{\mathrm{I}}}{\sqrt{2\pi}}\frac{\lambda^2+\mu^2}{\lambda[Q_{12}+(\lambda^2+\mu^2)Q_{22}]}\mathrm{Re}[(z_1-a)^{1/2}-(z_2-a)^{1/2}]\cdot \tag{3.10b}$$

如图 1 所示, 可记

$$z_j-a = r(\cos\theta+s_j\sin\theta), \qquad (r \ll a, j=1,2), \tag{3.11}$$

则弯矩、扭矩、应变、应力、位移的计算公式(3.6)、(3.7)、(3.9)、(3.10)化为极坐标形式· 如弯矩、扭矩的计算公式(3.6), 其极坐标形式:

$$M_x = \frac{K_{\mathrm{I}}}{\sqrt{2\pi r}}\frac{1}{2\lambda[Q_{12}+(\lambda^2+\mu^2)Q_{22}]}\cdot$$
$$\left\{\lambda[Q_{11}+(\lambda^2+\mu^2)Q_{12}]\mathrm{Re}\left[\frac{1}{(\cos\theta+s_1\sin\theta)^{1/2}}+\frac{1}{(\cos\theta+s_2\sin\theta)^{1/2}}\right]+\right.$$
$$\left.\mu[Q_{11}-(\lambda^2+\mu^2)Q_{12}]\mathrm{Im}\left[\frac{1}{(\cos\theta+s_1\sin\theta)^{1/2}}-\frac{1}{(\cos\theta+s_2\sin\theta)^{1/2}}\right]\right\}, \tag{3.12a}$$

$$M_y = \frac{K_{\mathrm{I}}}{\sqrt{2\pi r}}\frac{1}{2\lambda[Q_{12}+(\lambda^2+\mu^2)Q_{22}]}\cdot$$

$$\left\{\lambda\left[Q_{12}+(\lambda^2+\mu^2)Q_{22}\right]\mathrm{Re}\left[\frac{1}{(\cos\theta+s_1\sin\theta)^{1/2}}+\frac{1}{(\cos\theta+s_2\sin\theta)^{1/2}}\right]+\right.$$

$$\left.\mu\left[Q_{12}-(\lambda^2+\mu^2)Q_{22}\right]\mathrm{Im}\left[\frac{1}{(\cos\theta+s_1\sin\theta)^{1/2}}-\frac{1}{(\cos\theta+s_2\sin\theta)^{1/2}}\right]\right\}, \qquad (3.12b)$$

$$M_{xy}=\frac{K_{\mathrm{I}}}{\sqrt{2\pi r}}\frac{(\lambda^2+\mu^2)Q_{66}}{\lambda\left[Q_{12}+(\lambda^2+\mu^2)Q_{22}\right]}\cdot$$

$$\mathrm{Re}\left[\frac{1}{(\cos\theta+s_1\sin\theta)^{1/2}}-\frac{1}{(\cos\theta+s_2\sin\theta)^{1/2}}\right]\cdot \qquad (3.12c)$$

4 统一表示式

由(2.6)、(2.7)、(2.2)，经推导易知情况 Δ> 0 时和情况 Δ< 0 时的挠度公式(2.16)和(2.19)可统一表示为

$$w=\frac{1}{Q_{12}-s_1s_2Q_{12}}\mathrm{Re}\left[\frac{1}{s_1-s_2}(s_2\bar{\bar{w}}_1-s_1\bar{\bar{w}}_2)\right]\cdot \qquad (4.1)$$

同理，文[1]给出的情况 Δ> 0 时和本文给出的情况 Δ< 0 时的弯矩、扭矩、应变、应力、位移的计算公式可统一表示为

$$M_x=-\frac{K_{\mathrm{I}}}{\sqrt{2\pi}}\frac{1}{Q_{12}-s_1s_2Q_{22}}\mathrm{Re}\left\{\frac{1}{s_1-s_2}\left[s_2(Q_{11}+s_1^2Q_{12})\frac{1}{(z_1-a)^{1/2}}-\right.\right.$$

$$\left.\left.s_1(Q_{11}+s_2^2Q_{12})\frac{1}{(z_2-a)^{1/2}}\right]\right\}, \qquad (4.2a)$$

$$M_y=-\frac{K_{\mathrm{I}}}{\sqrt{2\pi}}\frac{1}{Q_{12}-s_1s_2Q_{22}}\mathrm{Re}\left\{\frac{1}{s_1-s_2}\left[s_2(Q_{12}+s_1^2Q_{22})\frac{-1}{(z_1-a)^{1/2}}-\right.\right.$$

$$\left.\left.s_1(Q_{12}+s_2^2Q_{22})\frac{1}{(z_2-a)^{1/2}}\right]\right\}, \qquad (4.2b)$$

$$M_{xy}=-\frac{K_{\mathrm{I}}}{\sqrt{2\pi}}\frac{2Q_{66}}{Q_{12}-s_1s_2Q_{22}}\mathrm{Re}\left\{\frac{s_1s_2}{s_1-s_2}\left[\frac{1}{(z_1-a)^{1/2}}-\frac{1}{(z_2-a)^{1/2}}\right]\right\}; \qquad (4.2c)$$

$$\varepsilon_x=-z\frac{K_{\mathrm{I}}}{\sqrt{2\pi}}\frac{1}{Q_{12}-s_1s_2Q_{22}}\mathrm{Re}\left\{\frac{1}{s_1-s_2}\left[\frac{s_2}{(z_1-a)^{1/2}}-\frac{s_1}{(z_2-a)^{1/2}}\right]\right\}, \qquad (4.3a)$$

$$\varepsilon_y=-z\frac{K_{\mathrm{I}}}{\sqrt{2\pi}}\frac{1}{Q_{12}-s_1s_2Q_{22}}\mathrm{Re}\left\{\frac{s_1s_2}{s_1-s_2}\left[\frac{s_1}{(z_1-a)^{1/2}}-\frac{s_2}{(z_2-a)^{1/2}}\right]\right\}, \qquad (4.3b)$$

$$\gamma_{xy}=-2z\frac{K_{\mathrm{I}}}{\sqrt{2\pi}}\frac{1}{Q_{12}-s_1s_2Q_{22}}\mathrm{Re}\left\{\frac{s_1s_2}{s_1-s_2}\left[\frac{1}{(z_1-a)^{1/2}}-\frac{1}{(z_2-a)^{1/2}}\right]\right\}; \qquad (4.3c)$$

$$\sigma_x=-z\frac{K_{\mathrm{I}}}{\sqrt{2\pi}}\frac{1}{(Q_{12}-s_1s_2Q_{22})(a_{11}a_{22}-a_{12}^2)}\cdot$$

$$\mathrm{Re}\left\{\frac{1}{s_1-s_2}\left[s_1(a_{12}s_2^2-a_{22})\frac{1}{(z_2-a)^{1/2}}-s_2(a_{12}s_1^2-a_{22})\frac{1}{(z_1-a)^{1/2}}\right]\right\}, \qquad (4.4a)$$

$$\sigma_y=-z\frac{K_{\mathrm{I}}}{\sqrt{2\pi}}\frac{1}{(Q_{12}-s_1s_2Q_{22})(a_{11}a_{22}-a_{12}^2)}\cdot$$

$$\mathrm{Re}\left\{\frac{1}{s_1-s_2}\left[s_2(a_{11}s_1^2-a_{12})\frac{1}{(z_1-a)^{1/2}}-s_1(a_{11}s_1^2-a_{12})\frac{1}{(z_2-a)^{1/2}}\right]\right\}, \qquad (4.4b)$$

$$\tau_{xy}=-2z\frac{K_{\mathrm{I}}}{\sqrt{2\pi}}\frac{1}{(Q_{12}-s_1s_2Q_{22})a_{66}}\mathrm{Re}\left\{\frac{s_1s_2}{s_1-s_2}\left[\frac{1}{(z_1-a)^{1/2}}-\frac{1}{(z_2-a)^{1/2}}\right]\right\}; \qquad (4.4c)$$

$$u=-zK_{\mathrm{I}}\sqrt{\frac{2}{\pi}}\frac{1}{Q_{12}-s_1s_2Q_{22}}\mathrm{Re}\left\{\frac{1}{s_1-s_2}\left[s_2(z_1-a)^{1/2}-s_1(z_2-a)^{1/2}\right]\right\}, \tag{4.5a}$$

$$v=-zK_{\mathrm{I}}\sqrt{\frac{2}{\pi}}\frac{1}{Q_{12}-\mu s_1s_2Q_{22}}\mathrm{Re}\left\{\frac{s_1s_2}{s_1-s_2}\left[(z_1-a)^{1/2}-(z_2-a)^{1/2}\right]\right\}· \tag{4.5b}$$

将(3.11)代入(4.2)、(4.3)、(4.4)、(4.5),可得到它们的极坐标形式,在此略去•

对于受弯正交异性复合材料板的断裂问题,很少见到过详细的讨论,所以本文的推导过程及其结果在相应断裂分析中具有一定的参考价值•

参 考 文 献

[1] Zhang S Q, Zhu Y Z, Yang W Y. Energy based Z_criterion in fracture analysis of composite plate under bending[J]. Engineering Fracture Mechanics, 1992, **43**(5): 797~ 805

[2] 赫尼茨基. 各向异性板[M]. 胡海昌译, 北京: 科学出版社, 1963, 241~ 250

[3] Corten H T. Fracture mechanics of composites[A]. In: H Liebowitz ed. Fracture[C], Vol, **7**, New York: Academic Press, 1972, 695~ 703

[4] Zhang S Q, Jang B Z, Valaire B T, Suhling J C. An energy based fracture criterion for mode Ⅱ crack in fiber composites[J]. Engineering Fracture Mechanics, 1990, **36**(1): 49~ 59

[5] 杨维阳, 张少琴. 关于复合材料单层板Ⅰ型裂纹尖端应力场的探讨[J]. 太原重型机械学院学报, 1986, **7**(3): 46~ 53

The Crack Tip Fields of Orthotropic Composite Plate under Bending

Yang Weiyang   Zhang Shaoqin

Taiyuan Heavy Machinery Institute, Taiyuan 030024, P R China

Abstract: In this paper, mechanics analysis for crack tip fields of linear elastic orthotropic composite plate under symmetric bending load was done. By using a complex variable method, the equations for bending moment, twisting moment, stress, strain and displacement fields near crack tip are derived.

Key words: crack tip field; bending load; composite material