明渠中变密度变粘度牛顿流体紊动基本方程式

郑永刚[1] 方 铎[2] 李桂芬[3]

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摘 要

本文利用Navier-Stokes方程及雷诺时均法则，导出了变密度变粘度牛顿流体的紊动微分方程式，并进一步导出了变密度变粘度牛顿流体在明渠中紊流流动时的运动微分方程式。文中首次提出了密度紊动应力与粘度紊动应力的概念。

关键词 明渠 变粘度流体 紊动 基本方程式 牛顿流体

一、前 言

关于不可压缩流体紊动问题，已有许多学者进行了研究[1]、[2]，如文献[1]中，窦国仁给出了恒密度恒粘度流体在明渠中的紊动微分方程式。文献[3]中，肖天铎研究了变密度、恒粘度流体在明渠中的紊流流动，并将密度的紊动与浓度的紊动联系起来，给出了浓度紊动应力的概念。但对粘度的紊动变化几乎没有考虑。

事实上，流体在紊动过程中，粘度是不断变化的。如对掺气水流而言，由于气体浓度是位置和时间的函数，并随流体的紊动而紊动，这使其混合流体的粘度也成为一个不断变化的紊动量[4]。因此，在推导变密度变粘度流体紊动基本方程式时，就必须考虑流体粘度的紊动变化，以及由此而产生的粘度紊动应力。

二、变密度变粘度流体紊动基本方程式

直角坐标系下的N-S方程

$$\rho\frac{Dv_x}{Dt}=\rho g_x-\frac{\partial p}{\partial x}+\left(\frac{\partial\tau_{xx}}{\partial x}+\frac{\partial\tau_{yx}}{\partial y}+\frac{\partial\tau_{zx}}{\partial z}\right)\tag{2.1a}$$

$$\rho\frac{Dv_y}{Dt}=\rho g_y-\frac{\partial p}{\partial y}+\left(\frac{\partial\tau_{xy}}{\partial x}+\frac{\partial\tau_{yy}}{\partial y}+\frac{\partial\tau_{zy}}{\partial z}\right)\tag{2.1b}$$

1 石油大学(华东)，山东，东营 257062。
2 四川联合大学，成都 610065。
3 中国水利水电科学研究院，北京 100044。

$$\rho\frac{Dv_z}{Dt}=\rho g_z-\frac{\partial p}{\partial z}+\left(\frac{\partial\tau_{xz}}{\partial x}+\frac{\partial\tau_{yz}}{\partial y}+\frac{\partial\tau_{zz}}{\partial z}\right) \tag{2.1c}$$

连续性方程 $$\frac{D\rho}{dt}+\rho\,\mathrm{div}\,\boldsymbol{v}=0 \tag{2.1d}$$

式中 $$\frac{D}{Dt}=\frac{\partial}{\partial t}+\boldsymbol{v}\cdot\nabla$$

$$\tau_{xy}=\mu\left(\frac{\partial v_x}{\partial y}+\frac{\partial v_y}{\partial x}\right)\qquad \tau_{xx}=2\mu\frac{\partial v_x}{\partial x}-\frac{2}{3}\mu\,\mathrm{div}\,\boldsymbol{v}$$

$$\tau_{yz}=\mu\left(\frac{\partial v_z}{\partial y}+\frac{\partial v_y}{\partial z}\right)\qquad \tau_{yy}=2\mu\frac{\partial v_y}{\partial y}-\frac{2}{3}\mu\,\mathrm{div}\,\boldsymbol{v}$$

$$\tau_{zx}=\mu\left(\frac{\partial v_z}{\partial x}+\frac{\partial v_x}{\partial z}\right)\qquad \tau_{zz}=2\mu\frac{\partial v_z}{\partial z}-\frac{2}{3}\mu\,\mathrm{div}\,\boldsymbol{v}$$

$$\boldsymbol{v}=v_x\boldsymbol{i}+v_y\boldsymbol{j}+v_z\boldsymbol{k}$$

$$\nabla=\frac{\partial}{\partial x}\boldsymbol{i}+\frac{\partial}{\partial y}\boldsymbol{j}+\frac{\partial}{\partial z}\boldsymbol{k}$$

对(2.1)式取时均，有

$$\rho\overline{\frac{Dv_x}{Dt}}=\bar\rho g_x-\frac{\partial\bar p}{\partial x}+\left(\frac{\partial\bar\tau_{xx}}{\partial x}+\frac{\partial\bar\tau_{yx}}{\partial y}+\frac{\partial\bar\tau_{zx}}{\partial z}\right) \tag{2.2a}$$

$$\rho\overline{\frac{Dv_y}{Dt}}=\bar\rho g_y-\frac{\partial\bar p}{\partial y}+\left(\frac{\partial\bar\tau_{xy}}{\partial x}+\frac{\partial\bar\tau_{yy}}{\partial y}+\frac{\partial\bar\tau_{zy}}{\partial z}\right) \tag{2.2b}$$

$$\rho\overline{\frac{Dv_z}{Dt}}=\bar\rho g_z-\frac{\partial\bar p}{\partial z}+\left(\frac{\partial\bar\tau_{xz}}{\partial x}+\frac{\partial\bar\tau_{yz}}{\partial y}+\frac{\partial\bar\tau_{zz}}{\partial z}\right) \tag{2.2c}$$

式中

$$\rho\overline{\frac{Dv_x}{Dt}}=\bar\rho\frac{\bar D\bar v_x}{Dt}+\overline{\rho'\frac{D'v'_x}{Dt}}+\bar\rho\,\overline{\boldsymbol{v}'\cdot\nabla v'_x}+\overline{\rho'\bar{\boldsymbol{v}}\cdot\nabla v'_x}+\overline{\rho'\boldsymbol{v}'\cdot\nabla\bar v_x}$$

$$\rho\overline{\frac{Dv_y}{Dt}}=\bar\rho\frac{\bar D\bar v_y}{Dt}+\overline{\rho'\frac{D'v'_y}{Dt}}+\bar\rho\,\overline{\boldsymbol{v}'\cdot\nabla v'_y}+\overline{\rho'\bar{\boldsymbol{v}}\cdot\nabla v'_y}+\overline{\rho'\boldsymbol{v}'\cdot\nabla\bar v_y}$$

$$\rho\overline{\frac{Dv_z}{Dt}}=\bar\rho\frac{\bar D\bar v_z}{Dt}+\overline{\rho'\frac{D'v'_z}{Dt}}+\bar\rho\,\overline{\boldsymbol{v}'\cdot\nabla v'_z}+\overline{\rho'\bar{\boldsymbol{v}}\cdot\nabla v'_z}+\overline{\rho'\boldsymbol{v}'\cdot\nabla\bar v_z}$$

$$\frac{\bar D}{Dt}=\frac{\partial}{\partial t}+\bar v_x\frac{\partial}{\partial x}+\bar v_y\frac{\partial}{\partial y}+\bar v_z\frac{\partial}{\partial z}$$

$$\frac{D'}{Dt}=\frac{\partial}{\partial t}+v'_x\frac{\partial}{\partial x}+v'_y\frac{\partial}{\partial y}+v'_z\frac{\partial}{\partial z}$$

$$\boldsymbol{v}'=v'_x\boldsymbol{i}+v'_y\boldsymbol{j}+v'_z\boldsymbol{k}$$

$$\bar{\boldsymbol{v}}=\bar v_x\boldsymbol{i}+\bar v_y\boldsymbol{j}+\bar v_z\boldsymbol{k}$$

$$\bar\tau_{xx}=2\bar\mu\frac{\partial\bar v_x}{\partial x}+2\overline{\mu'\frac{\partial v'_x}{\partial x}}-\frac{2}{3}\bar\mu\,\overline{\mathrm{div}\,\boldsymbol{v}}-\frac{2}{3}\overline{\mu'\mathrm{div}\,\boldsymbol{v}'}$$

$$\bar\tau_{yy}=2\bar\mu\frac{\partial\bar v_y}{\partial y}+2\overline{\mu'\frac{\partial v'_y}{\partial y}}-\frac{2}{3}\bar\mu\,\overline{\mathrm{div}\,\boldsymbol{v}}-\frac{2}{3}\overline{\mu'\mathrm{div}\,\boldsymbol{v}'}$$

$$\bar\tau_{zz}=2\bar\mu\frac{\partial\bar v_z}{\partial z}+2\overline{\mu'\frac{\partial v'_z}{\partial z}}-\frac{2}{3}\bar\mu\,\overline{\mathrm{div}\,\boldsymbol{v}}-\frac{2}{3}\overline{\mu'\mathrm{div}\,\boldsymbol{v}'}$$

$$\mathrm{div}\boldsymbol{v}=\frac{\partial v_x}{\partial x}+\frac{\partial v_y}{\partial y}+\frac{\partial v_z}{\partial z}$$

$$\mathrm{div}\boldsymbol{v}'=\frac{\partial v'_x}{\partial x}+\frac{\partial v'_y}{\partial y}+\frac{\partial v'_z}{\partial z}$$

$$\overline{\mathrm{div}\boldsymbol{v}}=\frac{\partial \bar{v}_x}{\partial x}+\frac{\partial \bar{v}_y}{\partial y}+\frac{\partial \bar{v}_z}{\partial z}$$

$$\bar{\tau}_{xy}=\bar{\mu}\left(\frac{\partial \bar{v}_x}{\partial y}+\frac{\partial \bar{v}_y}{\partial x}\right)+\overline{\mu'\left(\frac{\partial v'_x}{\partial y}+\frac{\partial v'_y}{\partial x}\right)}$$

$$\bar{\tau}_{yz}=\bar{\mu}\left(\frac{\partial \bar{v}_y}{\partial z}+\frac{\partial \bar{v}_z}{\partial y}\right)+\overline{\mu'\left(\frac{\partial v'_y}{\partial z}+\frac{\partial v'_z}{\partial y}\right)}$$

$$\bar{\tau}_{xz}=\bar{\mu}\left(\frac{\partial \bar{v}_x}{\partial z}+\frac{\partial \bar{v}_z}{\partial x}\right)+\overline{\mu'\left(\frac{\partial v'_x}{\partial z}+\frac{\partial v'_z}{\partial x}\right)}$$

将上述各式，代入(2.2)式，就可以得到直角坐标系下变粘度、变密度的紊动微分方程式。

三、明渠中变密度变粘度流体紊动基本方程式

对明渠流，不计气泡的滑脱并考虑恒定紊流有

$$\frac{\partial}{\partial x}=0,\quad \frac{\partial}{\partial z}=0,\quad \bar{v}_y=0,\quad \bar{v}_z=0,\quad g_z=0,\quad \frac{\partial}{\partial t}=0$$

于是，x方向的运动方程(2.2a)式简化成

$$\overline{\rho' v'_y\frac{\partial v'_x}{\partial y}}+\bar{\rho}\overline{v'_y\frac{\partial v'_x}{\partial y}}+\overline{\rho' v'_y}\frac{\partial \bar{v}_x}{\partial y}=\bar{\rho}g_x-\frac{\partial \bar{p}}{\partial x}+\frac{\partial \bar{\tau}_{yx}}{\partial y}\tag{3.1}$$

式中
$$\bar{\tau}_{yx}=\bar{\mu}\frac{\partial \bar{v}_x}{\partial y}+\overline{\mu'\frac{\partial v'_x}{\partial y}}$$

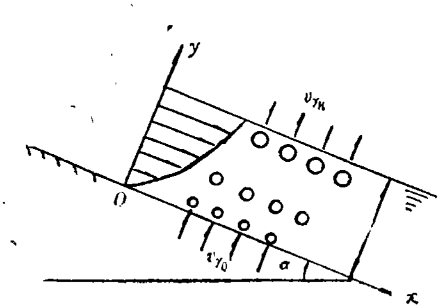

图1 明渠紊流示意图

恒定明渠流连续性方程:

$$\frac{\partial(\rho v_y)}{\partial y}=0$$

或
$$\frac{\partial(\bar{\rho}\bar{v}_y)}{\partial y}+\frac{\partial\overline{\rho' v'_y}}{\partial y}=0\tag{3.2}$$

即
$$\frac{\partial\overline{\rho' v'_y}}{\partial y}=0\tag{3.2$'$}$$

又由(3.2)式，有

$$\frac{\partial(\rho v'_y)}{\partial y}=0\tag{3.2$''$}$$

若ρ=常数，μ=常数，则

$$\rho'=0,\quad \mu'=0$$

由(3.2)式，有

$$\frac{\partial v_y}{\partial y}=0$$

或 $\frac{\partial v'_y}{\partial y}=0$

将上述条件代入(3.1)式，有

$$\bar{\rho}\overline{v'_y\frac{\partial v'_x}{\partial y}}=\bar{\rho}g_x-\frac{\partial\bar{p}}{\partial x}+\bar{\mu}\frac{\partial^2\bar{v}_x}{\partial y^2} \tag{3.1$'$}$$

又因为

$$\frac{\partial(v'_xv'_y)}{\partial y}=v'_x\frac{\partial v'_y}{\partial y}+v'_y\frac{\partial v'_x}{\partial y}$$

$$=v'_y\frac{\partial v'_x}{\partial y}$$

所以，(3.1)′式成为

$$\bar{\rho}\frac{\partial\overline{(v'_xv'_y)}}{\partial y}=\bar{\rho}g_x-\frac{\partial\bar{p}}{\partial x}+\bar{\mu}\frac{\partial^2\bar{v}_x}{\partial y^2} \tag{3.1$''$}$$

式(3.1)″即为窦国仁的恒密度、恒粘度，x方向运动微分方程式．变化(3.1)式，有

$$\overline{\rho'v'_y\frac{\partial(\bar{v}_x+v'_x)}{\partial y}}+\bar{\rho}\overline{v'_y\frac{\partial v'_x}{\partial y}}=\bar{\rho}g_x-\frac{\partial\bar{p}}{\partial x}+\frac{\partial\bar{\tau}_{yx}}{\partial y} \tag{3.3a}$$

同理，有

$$\overline{\rho'v'_y\frac{\partial(\bar{v}_y+v'_y)}{\partial y}}+\bar{\rho}\overline{v'_y\frac{\partial v'_y}{\partial y}}=\bar{\rho}g_y-\frac{\partial\bar{p}}{\partial y}+\frac{\partial\bar{\tau}_{yy}}{\partial y} \tag{3.3b}$$

$$\overline{\rho'v'_y\frac{\partial(\bar{v}_z+v'_z)}{\partial y}}+\bar{\rho}\overline{v'_y\frac{\partial v'_z}{\partial y}}=\bar{\rho}g_z-\frac{\partial\bar{p}}{\partial z}+\frac{\partial\bar{\tau}_{yz}}{\partial y} \tag{3.3c}$$

或

$$\overline{\rho'v'_y\frac{\partial(\bar{v}_x+v'_x)}{\partial y}}+\bar{\rho}\overline{v'_y\frac{\partial v'_x}{\partial y}}=\bar{\rho}g_x+\frac{\partial\bar{\tau}_{yx}}{\partial y} \tag{3.4a}$$

$$\overline{\rho'v'_y\frac{\partial v'_y}{\partial y}}+\bar{\rho}\overline{v'_y\frac{\partial v'_y}{\partial y}}=\bar{\rho}g_y-\frac{\partial\bar{p}}{\partial y}+\frac{\partial\bar{\tau}_{yy}}{\partial y} \tag{3.4b}$$

$$\overline{\rho'v'_y\frac{\partial v'_z}{\partial y}}+\bar{\rho}\overline{v'_y\frac{\partial v'_z}{\partial y}}=\frac{\partial\bar{\tau}_{yz}}{\partial y} \tag{3.4c}$$

式中：

$$\bar{\tau}_{yx}=\bar{\mu}\frac{\partial\bar{v}_x}{\partial y}+\overline{\mu'\frac{\partial v'_x}{\partial y}}$$

$$\bar{\tau}_{yy}=\frac{4}{3}\overline{\mu'\frac{\partial v'_y}{\partial y}}$$

$$\bar{\tau}_{yz}=\overline{\mu'\frac{\partial v'_z}{\partial y}}$$

变化(3.4a)式，有

$$\overline{(\bar{\rho}+\rho')v'_y\frac{\partial v'_x}{\partial y}}+\overline{\rho'v'_y}\frac{\partial\bar{v}_x}{\partial y}=\bar{\rho}g_x+\frac{\partial\bar{\tau}_{yx}}{\partial y}$$

由(3.2)′、(3.2)″式，有

$$\frac{\partial\overline{\rho'v'_y}}{\partial y}=0,\quad \frac{\partial\overline{(\bar{\rho}+\rho')v'_y}}{\partial y}=0$$

因此，积分上式，有

$$\overline{(\bar{\rho}+\rho')v'_yv'_x}-\overline{(\bar{\rho}+\rho')v'_yv'_x}\,|_{y=0}+\overline{\rho'v'_y}\bar{v}_x-\overline{\rho'v'_y}\bar{v}_x\,|_{y=0}$$

$$=g_x\int_0^y\bar{\rho}dy+\bar{\tau}_{yx}-\bar{\tau}_{yx}|_{y=0}$$

因为，在渠底上（$y=0$处），所有脉动量为零，并令$\tau_w=\bar{\tau}_{yx}|_{y=0}$，所以，上式成为

$$\overline{(\bar{\rho}+\rho')v_x'v_y'}+\overline{\rho'v_y'}\bar{v}_x=g_x\int_0^y\bar{\rho}dy+\bar{\mu}\frac{\partial\bar{v}_x}{\partial y}+\overline{\mu'\frac{\partial v_x'}{\partial y}}-\tau_w$$

或

$$\tau_w=g_x\int_0^y\bar{\rho}dy+\bar{\mu}\frac{\partial\bar{v}_x}{\partial y}+\overline{\mu'\frac{\partial v_x'}{\partial y}}-\overline{\rho'(\bar{v}_x+v_x')v_y'}-\bar{\rho}\,\overline{v_x'v_y'}$$

由力平衡方程，有

$$\tau_w=g_x\int_0^h\bar{\rho}dy$$

此式也可由前式中，令$y=h$，并注意$y=h$处，右边各项除第一项外，均为零，即可得到上式。

所以，运动方程式成为

$$g_x\int_0^h\bar{\rho}dy\left(1-\frac{\int_0^y\bar{\rho}dy}{\int_0^h\bar{\rho}dy}\right)=\bar{\mu}\frac{\partial\bar{v}_x}{\partial y}+\overline{\mu'\frac{\partial v_x'}{\partial y}}-\bar{\rho}\,\overline{v_x'v_y'}-\overline{\rho'(\bar{v}_x+v_x')v_y'} \qquad (3.5)$$

这就是变密度、变粘度流体在明渠中紊动微分方程式。

若令$\rho'=0$，$\mu'=0$，上式成为

$$g_x\rho h\left(1-\frac{y}{h}\right)=\bar{\mu}\frac{\partial\bar{v}_x}{\partial y}-\bar{\rho}\,\overline{v_x'v_y'}$$

或

$$\tau_w\left(1-\frac{y}{h}\right)=\bar{\mu}\frac{\partial\bar{v}_x}{\partial y}-\bar{\rho}\,\overline{v_x'v_y'}$$

此式即为常见的恒密度、恒粘度紊动微分方程式。因此，(3.5)式左端也称为总应力项。$\bar{\mu}\frac{\partial\bar{v}_x}{\partial y}$同样称为粘滞应力项。$\overline{\mu'\frac{\partial v_x'}{\partial y}}$称为粘度紊动应力项，该项是由于粘度的紊动变化而增加的应力。$-\bar{\rho}\overline{v_x'v_y'}$同样称为速度紊动应力项。$-\overline{\rho'(\bar{v}_x+v_x')v_y'}$称为密度紊动应力项，该项是由于密度的紊动变化而增加的应力。

在上述推导过程中，没有考虑气泡的滑脱。下面考虑气泡的滑脱。如图1所示，设气泡的滑脱速度为v_s，v_s在三坐标方向的分量为$(-v_s\sin\alpha,\ v_s\cos\alpha,\ 0)$，重力加速度$g$的三分量为$(g\sin\alpha,\ -g\cos\alpha,\ 0)$。

仍然考虑恒定紊流，因此

$$\frac{\partial}{\partial t}=0,\quad \frac{\partial}{\partial x}=0,\quad \frac{\partial}{\partial z}=0,\quad \bar{v}_z=0,\quad g_z=0$$

连续性方程为

$$\frac{\partial(\rho v_y)}{\partial y}=0 \qquad (3.6)$$

由上式有

$$\frac{\partial\overline{\rho v_y}}{\partial y}=0 \qquad (3.6)'$$

或

$$\frac{\partial(\bar{\rho}\bar{v}_y+\overline{\rho'v_y'})}{\partial y}=0 \qquad (3.6)''$$

若假设$\frac{\partial \overline{\rho' v_y'}}{\partial y}=0$，则

$$\frac{\partial(\bar{\rho}\bar{v}_y)}{\partial y}=0 \qquad (3.6)'''$$

x方向的运动方程(2.2a)式成为

$$\bar{\rho}\bar{v}_y\frac{\partial \bar{v}_x}{\partial y}+\overline{\rho' v_y'\frac{\partial v_x'}{\partial y}}+\overline{\bar{\rho}v_y'\frac{\partial v_x'}{\partial y}}+\overline{\rho'\bar{v}_y\frac{\partial v_x'}{\partial y}}+\overline{\rho' v_y'}\frac{\partial \bar{v}_x}{\partial y}$$

$$=\bar{\rho}g_x+\frac{\partial \bar{\tau}_{yx}}{\partial y}$$

或

$$(\bar{\rho}\bar{v}_y+\overline{\rho' v_y'})\frac{\partial \bar{v}_x}{\partial y}+\overline{(\bar{\rho}v_y'+\rho' v_y)\frac{\partial v_x'}{\partial y}}=\bar{\rho}g_x+\frac{\partial \bar{\tau}_{yx}}{\partial y} \qquad (3.7)$$

因为 $\frac{\partial(\rho v_y)}{\partial y}=0$

所以 $\frac{\partial(\bar{\rho}\bar{v}_y+\rho'\bar{v}_y+\bar{\rho}v_y'+\rho' v_y')}{\partial y}=0$

又因为 $\frac{\partial(\bar{\rho}\bar{v}_y)}{\partial y}=0$

所以 $\frac{\partial(\rho'\bar{v}_y+\bar{\rho}v_y'+\rho' v_y')}{\partial y}=0$

即 $\frac{\partial(\bar{\rho}v_y'+\rho' v_y)}{\partial y}=0 \qquad (3.8)$

积分(3.7)式，并注意$\frac{\partial \overline{\rho' v_y'}}{\partial y}=0$及(3.6)'''式和(3.8)式，则

$$(\bar{\rho}\bar{v}_y+\overline{\rho' v_y'})\bar{v}_x\Big|_0^y+\overline{(\bar{\rho}v_y'+\rho' v_y)v_x'}\Big|_0^y=g_x\int_0^y\bar{\rho}\,dy+\bar{\tau}_{yx}\Big|_0^y$$

渠底$y=0$处的脉动量及时均流速均为零，则上式成为

$$(\ \bar{v}_y+\overline{\rho' v_y'})\bar{v}_x+\overline{(\bar{\rho}v_y'+\rho' v_y)v_x'}=g_x\int_0^y\bar{\rho}\,dy+\bar{\tau}_{yx}-\tau_w$$

同理$\tau_w=g_x\int_0^h\bar{\rho}\,dy$，则由上式可得

$$g_x\int_0^h\bar{\rho}\,dy\left(1-\frac{\int_0^y\bar{\rho}\,dy}{\int_0^h\bar{\rho}\,dy}\right)=\bar{\mu}\frac{\partial \bar{v}_x}{\partial y}+\mu'\overline{\frac{\partial v_x'}{\partial y}}-\bar{\rho}\bar{v}_x\bar{v}_y-\bar{\rho}\,\overline{v_x'v_y'}$$

$$-\overline{\rho' v_x'}\bar{v}_y-\overline{\rho' v_y'}\bar{v}_x-\overline{\rho' v_x' v_y'} \qquad (3.9)$$

比较(3.5)式与(3.9)式，(3.9)式中增加了$\bar{\rho}\bar{v}_x\bar{v}_y$与$\overline{\rho' v_x'}\bar{v}_y$两项。$\overline{\rho' v_x'}\bar{v}_y$也是由于密度紊动引起的附加应力，因此，也称之为密度紊动应力。$\bar{\rho}\bar{v}_x\bar{v}_y$是由于对流加速度或迁移加速度引起的附加应力，故称之为对流加速度应力。其余各项的意义同(3.5)式。

四、结　束　语

本文导出了变密度、变粘度流体紊流流动时的运动方程式，并进一步导出了变密度、变粘度流体在明渠中紊流流动时的运动微分方程式。文中首次提出了密度紊动应力与粘度紊动应力的概念。

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Basic Equations of Turbulent Flow for Variable Density and Variable Viscosity Newtonian Fluid in Open Channel

Zheng Yonggang

(*University of Petroleum (East China), Dongying, Shandong* 257062, *P. R. China*)

Fang Duo

(*Sichuan Union University, Chengdu* 610065, *P. R. China*)

Li Guifen

(*China Institute of Water Resources and Hydropower Research, Beijing* 100044, *P. R. China*)

Abstract

In this paper, using Navier-Stokes equations and Reynolds time-averaged rules, the turbulent motional differential equations of variable density and variable viscosity Newtonian fluid have been presented, and the turbulent motional differential equations of variable density and variable viscosity Newtonian fluid in open channel have been further proposed. The concepts of the density turbulence stress and the viscosity turbulence stress have been presented in the paper for the first time.

Key words open channel, variable viscosity fluid, turbulence, basic equation, Newtonian fluid