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多变量样条元法分析弹性地基板的弯曲、振动与稳定问题*

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摘　要

本文应用双三次乘积型二元B样条函数来构造弹性地基板的位移、弯矩和扭矩等多种场函数，由混合变分原理导出多变量样条元法方程．文中，对弹性地基板的弯曲、振动与稳定问题作了分析与计算．由于，本文方法设定了各自独立的场函数，因此，所算得的场未知量如位移、弯矩和扭矩值的精度均比较高．

关键词　多变量样条元法　双三次B样条　弹性地基板

一、引　言

弹性地基薄板结构在工程上有着广泛应用，随着科学技术和工程建设的迅速发展，弹性地基模型及计算方法也得到了相应的发展．如地基模型有文克尔地基，符拉索夫地基及半无限弹性体地基等．它们适应于不同的实际地基情况，在计算方法上有有限差分法，有限元法和样条有限元法等．

样条有限元法在分析板壳结构的静力、振动、动力响应和稳定问题方面，已有了不少研究．恩梯斯(H．Antes)[1]提出了板弯曲问题上采用双三次基样条；石钟慈[2]应用三次B样条函数求解薄板弯曲问题；水泽富作(T．Mizusawa)[3]提出了应用样条函数求解斜板的振动与屈曲问题；张佑啟(Y．K．Cheung)[4]提出了样条有限条法；沈鹏程等[5-9]应用三次、五次样条函数分析加劲板壳的静力、振动、动力响应和稳定问题．本文采用双三次B样条函数来构造弹性地基薄板的横向位移及多种内力的独立的场函数，应用二类变量广义变分原理导出多变量样条元法分析基础板的方程组．文中，给出了若干数值算例，计算精度满意．

二、弹性地基板的多变量样条元方程

有限元法是基于各种变分原理建立起来的，如有限元位移法是基于最小总势能原理，它把求解偏微分方程的边值问题转化为求总势能的极小值问题．弹性力学中的二类变量的广义

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变分原理是由哈林格—赖斯纳(Hellinger-Reissner)[18]p.384 提出，它是混合有限元法的理论基础。弹性薄板的二类变量广义变分原理是胡海昌[18]提出来的。本节根据上述理论推导出弹性基地板的多变量样条元法方程。

2.1 弹性地基板的混合能量泛函

$$\Pi_2=\iint_\Omega\{M\}^T\{x\}dxdy-\iint_\Omega\frac{1}{2}\{M\}^T[D]^{-1}\{M\}dxdy-\iint_\Omega qwdxdy$$
$$+\iint_\Omega\frac{1}{2}k_0w^2dxdy-\int_{C_1+C_2}\left(\frac{\partial M_{ns}}{\partial s}+Q_n\right)(w-\bar{w})ds$$
$$-\int_{C_3}\bar{q}wds+\int_{C_1}M_n\left(\frac{\partial w}{\partial n}-\bar{\psi}\right)ds+\int_{C_2+C_3}\bar{M}_n\frac{\partial w}{\partial n}ds \tag{2.1}$$

式中

$$\left.\begin{array}{ll}C_1\text{——固支边界上，} & w=\bar{w},\ \partial w/\partial n=\bar{\psi}\\ C_2\text{——简支边界上，} & w=\bar{w},\ M_n=\bar{M}_n\\ C_3\text{——自由边界上，} & M_n=\bar{M}_n,\ \partial M_{ns}/\partial s+Q_n=\bar{q}\end{array}\right\} \tag{2.2}$$

t—薄板厚度；E—薄板弹性模量；μ—泊松比。

容易验证，若以位移和内力分别对混合能量泛函变分为零，可得弹性地基板的几何与平衡方程及其边界条件。

对于弹性地基板弯曲、振动与稳定问题，在齐次边界条件下，其混合能量泛函为

$$\Pi_2=\iint_\Omega\{M\}^T\{\chi\}dxdy-\iint_\Omega\frac{1}{2}\{M\}^T[D]^{-1}\{M\}dxdy-\iint_\Omega\frac{1}{2}\lambda\bar{m}w^2dxdy$$
$$-\iint_\Omega\frac{1}{2}\{Q\}^T[N]\{Q\}dxdy-\iint_\Omega\frac{1}{2}k_0w^2dxdy-\iint_\Omega qwdxdy \tag{2.3}$$

式中

$$\{M\}=[M_x\quad M_y\quad M_{xy}]^T \tag{2.4}$$

$$\{\chi\}=[-w_{,xx}\quad -w_{,yy}\quad -2w_{,x,y}]^T \tag{2.5}$$

$$[N]=\begin{bmatrix}N_x & N_{xy}\\ N_{yx} & N_y\end{bmatrix},\quad \{Q\}=[w_{,x}\quad w_{,y}]^T \tag{2.6}$$

$$[D]=\frac{Et^3}{12(1-\mu^2)}\begin{bmatrix}1 & \mu & 0\\ \mu & 1 & 0\\ 0 & 0 & (1-\mu)/2\end{bmatrix} \tag{2.7}$$

k_0—文克尔地基常数，$k=k_0L^4/D$；$\bar{m}$—薄板的质量密度；λ—振动特征值。

2.2 内力与位移场函数

现取弹性地基薄板的弯矩 M_x，M_y，扭矩 M_{xy} 和横向位移w作为各自独立的场变量，应用双三次乘积型B样条函数来构造多变量的场函数，即有

$$\{M\}=\begin{Bmatrix}M_x\\ M_y\\ M_{xy}\end{Bmatrix}=\begin{bmatrix}[\Phi(x)]\otimes[\Phi(y)] & & 0\\ & [\Phi(x)]\otimes[\Phi(y)] & \\ 0 & & [\Phi(x)]\otimes[\Phi(y)]\end{bmatrix}\begin{Bmatrix}\{A\}\\ \{B\}\\ \{C\}\end{Bmatrix} \tag{2.8}$$

$$w=[\Phi(x)]\otimes[\Phi(y)]\{D\} \tag{2.9}$$

$$[\Phi(x)]=[\phi_{-1}(x)\phi_0(x)\phi_1(x)\cdots\phi_N(x)\phi_{N+1}(x)] \quad (x=x,\ y)$$

$$\{A\}=[a_{-1}\ a_0\ a_1\ \cdots\ a_S\ \cdots\ a_{N+1}]^T$$

$$a_S=[a_{-1S}\ a_{0S}\ a_{1S}\ \cdots\ a_{SS}\ \cdots\ a_{N+1\,S}], \quad (S=-1,\ 0,\ 1,\ \cdots,\ M+1)$$

式中，记号$\otimes$为克尔尼克(Kronecker)乘积；$\{B\}$，$\{C\}$，$\{D\}$形式与$\{A\}$相同，它们都是待定常数.

当$N\geqslant 4$

$$\left.\begin{aligned}
&\phi_{-1}(x)=\varphi_3(x/h+1)\\
&\phi_0(x)=\varphi_3(x/h)-4\varphi_3(x/h+1)\\
&\phi_1(x)=\varphi_3\left(\frac{x}{h}-1\right)-\frac{1}{2}\varphi_3\left(\frac{x}{h}\right)+\varphi_3\left(\frac{x}{h}+1\right)\\
&\phi_2(x)=\varphi_3(x/h-2)\\
&\qquad\cdots\cdots\\
&\phi_{N-2}(x)=\varphi_3(x/h-N+2)\\
&\phi_{N-1}(x)=\varphi_3\left(\frac{x}{h}-N+1\right)-\frac{1}{2}\varphi_3\left(\frac{x}{h}-N\right)+\varphi_3\left(\frac{x}{h}-N-1\right)\\
&\phi_N(x)=\varphi_3(x/h-N)-4\varphi_3(x/h-N-1)\\
&\phi_{N+1}(x)=\varphi_3(x/h-N-1)
\end{aligned}\right\} \tag{2.10}$$

基样条$\varphi_3(x/h+1)$，$\varphi_3(x/h)$，$\varphi_3(x/h-1)$，…，其图形见图1；式(2.10)图形见图2.

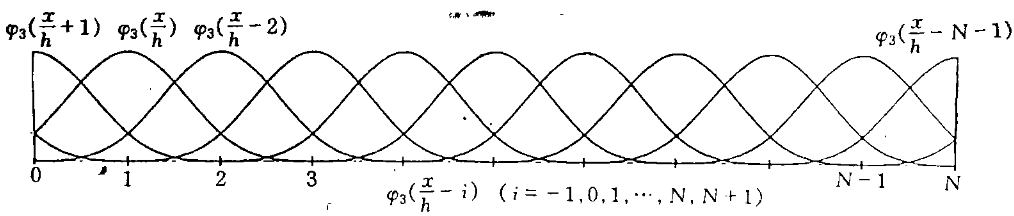

图 1

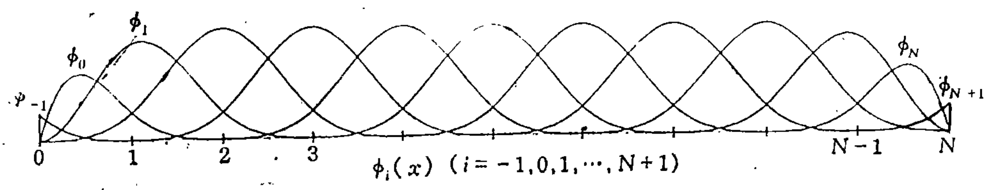

图 2

2.3 弹性地基薄板的多变量样条元方程

现将式(2.8)、(2.9)代入式(2.3)，并有混合能量原理，即

$$\frac{\partial\Pi_2}{\partial\{A\}}=\{0\},\quad \frac{\partial\Pi_2}{\partial\{B\}}=\{0\},\quad \frac{\partial\Pi_2}{\partial\{C\}}=\{0\},\quad \frac{\partial\Pi_2}{\partial\{D\}}=\{0\} \tag{2.11}$$

$$\begin{bmatrix}[F] & [H]\\ [H]^T & [K]\end{bmatrix}\begin{Bmatrix}\{A\}\\\{B\}\\\{C\}\\\{D\}\end{Bmatrix}=\begin{bmatrix}[0] & [0]\\ [0] & \lambda[M]\end{bmatrix}\begin{Bmatrix}\{A\}\\\{B\}\\\{C\}\\\{D\}\end{Bmatrix}$$

$$+\begin{Bmatrix}\{0\}\\\{0\}\\\{0\}\\\hline\{P\}\end{Bmatrix}+\left[\begin{array}{c|c}[0]&[0]\\\hline[0]&[G]\end{array}\right]\begin{Bmatrix}\{A\}\\\{B\}\\\{C\}\\\hline\{D\}\end{Bmatrix}\tag{2.12}$$

$$[F]=\begin{bmatrix}-\frac{12}{Et^3}[K_x^{00}]\otimes[K_y^{00}] & \frac{12\mu}{Et^3}[K_x^{00}]\otimes[K_y^{00}] & 0\\ & -\frac{12}{Et^3}[K_x^{00}]\otimes[K_y^{00}] & 0\\ \text{对称} & & -\frac{24(1+\mu)}{Et^3}[K_x^{00}]\otimes[K_y^{00}]\end{bmatrix}\tag{2.13}$$

$$[H]=\begin{bmatrix}-[K_x^{22}]\otimes[K_y^{00}]\\-[K_x^{00}]\otimes[K_y^{22}]\\-2[K_x^{01}]\otimes[K_y^{10}]\end{bmatrix}\tag{2.14}$$

$$\{P\}=\int_0^a\int_0^b q(x,\ y)\lfloor\Phi(x)\rfloor^T\otimes\lfloor\Phi(y)\rfloor^T dxdy\tag{2.15}$$

$$[M]=\bar{m}[K_x^{00}]\otimes[K_y^{00}]\tag{2.16}$$

$$[G]=[N_x\ \ 2N_{xy}\ \ N_y]\begin{Bmatrix}-[K_x^{11}]\otimes[K_y^{00}]\\-[K_x^{01}]\otimes[K_y^{10}]\\-[K_x^{00}]\otimes[K_y^{11}]\end{Bmatrix}\tag{2.17}$$

$$[K]=k_0[K_x^{00}]\otimes[K_y^{00}]\tag{2.18}$$

$$[K_x^{ij}]=\int_0^a[\Phi^i(x)]^T[\Phi^j(x)]dx\qquad(x=x,\ y),\ (i,\ j=0,\ 1,\ 2),\ \text{详见[2]}\tag{2.19}$$

荷载项，对均布荷载，三角形荷载与集中荷载分别为

1. 均布荷载

$$\int_0^L\lfloor\Phi(x)\rfloor dx=h_x\left[\frac{1}{24}\ \ \frac{1}{3}\ \ \frac{3}{4}\ \ 1\ \cdots\ 1\ \ \frac{3}{4}\ \ \frac{1}{3}\ \ \frac{1}{24}\right]\tag{2.20}$$

2. 三角形荷载

$$\int_0^L\lfloor x\Phi(x)\rfloor dx=h_x^2\left[\frac{1}{120}\ \ \frac{1}{5}\ \ \frac{9}{10}\ \ 2\ \ 3\ \cdots\ 3(N-2)\right.$$
$$\left.\left(\frac{3N}{4}-\frac{9}{10}\right)\left(\frac{N}{3}-\frac{1}{5}\right)\left(\frac{N}{24}-\frac{1}{120}\right)\right]\tag{2.21}$$

3. 集中荷载

$$\int_0^a\int_0^b\delta(x-\zeta,\ y-\eta)\lfloor\Phi(x)\rfloor\otimes\lfloor\Phi(y)\rfloor dxdy=\lfloor\Phi(\zeta)\rfloor\otimes\lfloor\Phi(\eta)\rfloor\tag{2.22}$$

其中，ζ，η为荷载作用点坐标。

现根据式(2.12)可分别得到弹性地基板的弯曲、振动及稳定方程如下：

$$\begin{bmatrix} [F] & [H] \\ [H]^T & [K] \end{bmatrix} \begin{Bmatrix} \{A\} \\ \{B\} \\ \{C\} \\ \{D\} \end{Bmatrix} = \begin{Bmatrix} \{0\} \\ \{0\} \\ \{0\} \\ \{P\} \end{Bmatrix} \tag{2.23}$$

$$([H]^T[F]^{-1}[H]-[K]-\lambda[M])\{D\}=\{0\} \tag{2.24}$$

$$([H]^T[F]^{-1}[H]-[K]-[G])\{D\}=\{0\} \tag{2.25}$$

由式(2.23)，(2.24)和(2.25)，可对薄板进行弯曲、振动和稳定的分析与计算。

三、数 值 算 例

根据上面各公式，我们编制了FORTRAN-77计算程序.下面一些数值算例均在VAX-11/780超级小型计算机上实现的。其计算成果均列于下述各表中。

表1　四边简支弹性地基板中点处挠度与弯矩值，板承受均布荷载

网格		wD/qL^4	M_x/qL^2	M_y/qL^2
4×4		0.00406299	0.04783820	0.04783819
6×6		0.00406251	0.04787836	0.04787837
8×8		0.00406247	0.04788553	0.04788527
精确解[19]		0.00406	0.0479	0.0479
$k=5$	4×4	0.00401026	0.04716437	0.04716437
	6×6	0.00400984	0.04720538	0.04720547
	8×8	0.00400980	0.04721240	0.04721251
$k=100$	4×4	0.00321323	0.03699015	0.03699016
	6×6	0.00321370	0.03704277	0.03704283
	8×8	0.00321376	0.03705036	0.03705054

表2　四边简支弹性地基板中点处挠度与弯矩值，板中点承受集中荷载

	网格	wD/PL^2	M_x/P	M_y/P
$k=0$	8×8	0.01158290	0.30245459	0.30245376
$k=5$	8×8	0.01145236	0.30075213	0.30075252
$k=100$	8×8	0.00947683	0.27489784	0.27489838
精确解[19]p.143($k=0$)		0.0116		

表3　四边固支弹性地基板中点处挠度与弯矩值，板承受均布荷载

	网格	wD/qL^4	M_x/qL^2	M_y/qL^2
$k=0$	8×8	0.00126541	0.02282157	0.02282151
*精确解[22]		0.00126532	0.0229051	0.0229051
$k=5$	8×8	0.00126032	0.02271701	0.02271698
$k=100$	8×8	0.00117055	0.02087563	0.02087559

表4　两对边简支，另两对边固定弹性地基板中点处挠度与弯矩值，板承受均布荷载

	网格	wD/qL^4	M_x/qL^2	M_y/qL^2
$k=0$	8×8	0.00191725	0.02437094	0.03319115
精确解[19]p.187		0.00192	0.0244	0.0332
$k=5$	8×8	0.00190544	0.02419899	0.03296581
$k=100$	8×8	0.00170500	0.02128551	0.02914535

表5　一边固支，另三边简支弹性地基板中点处挠度与弯矩值，板承受均布荷载

	网格	wD/qL^4	M_x/qL^2	M_y/qL^2
$k=0$	10×10	0.00278551	0.03388780	0.03918416
精确解[19]p.194		0.0028	0.034	0.039
$k=5$	10×10	0.00276019	0.03354631	0.03879767
$k=100$	10×10	0.00235216	0.02804886	0.03257535

表6　四边简支弹性地基板频率，板剖分为不同网格

	网格	a^*			
$k=100$	4×4	22.12675	50.38742	79.64095	101.24675
	6×6	22.12721	50.35295	79.59053	99.29522
	8×8	22.12417	50.35065	79.58733	99.21275
$k=5$	4×4	19.86536	49.43576	79.04226	100.77653
	6×6	19.86563	49.40061	78.99137	98.81565
	8×8	19.86105	49.39798	78.98844	98.73292
$k=0$	4×4	19.73923	49.38513	79.01062	100.75174
	6×6	19.73886	49.34992	78.95966	98.79032
	8×8	19.73381	49.34739	78.95649	98.70766
[4]		19.74	49.35	78.95	98.64

$$\omega=a^*\frac{1}{L^2}(D/\bar{m}l)^{1/2}$$

表7　四边固支弹性地基板频率，板剖分为不同网格

	网格	a^*			
$k=0$	4×4	36.01731	73.74091	108.82559	134.59386
	6×6	35.99366	73.43199	108.32896	132.00426
	8×8	35.98793	73.40506	108.25762	131.64781
[20]		35.99	73.41	108.3	131.6
$k=5$	4×4	36.08667	73.77477	108.84856	134.61244
	6×6	36.06297	73.46604	108.35204	132.02321
	8×8	36.05709	73.43912	108.28067	131.66678
$k=100$	4×4	37.37978	74.41585	109.28408	134.96483
	6×6	37.35698	74.10978	108.78954	132.38249
	8×8	37.35135	74.08309	108.71854	132.02705

表8　　两对边简支，另两边固支板频率，板剖分为不同网格

	网格	α^*					
$k=0$	4×4	28.95979	54.86541	69.59309	94.90040	104.38136	131.90736
	6×6	28.95244	54.76415	69.33943	94.62939	102.36386	129.46341
	8×8	28.95030	54.74785	69.32851	94.59936	102.24642	129.13942
[20]		28.95	54.74	69.32	94.59	102.2	
$k=100$	4×4	30.63766	55.76926	70.30791	95.42583	104.85929	132.28589
	6×6	30.63063	55.66972	70.05677	95.15636	102.85112	129.84901
	8×8	30.62778	55.65341	70.04597	95.12680	102.85112	129.51577

表9　　一边固支，另三边简支板频率，板剖分为不同网格

	网格	α^*				
$k=0$	4×1	23.65015	51.74423	58.74947	86.28146	102.36620
	6×6	23.64658	51.68347	58.65156	85.15998	100.38531
	8×8	23.64513	51.67646	58.64064	86.14058	100.28543
[20]		23.64	51.67	58.65	86.13	100.3
$k=100$	4×4	25.68563	52.85313	59.92971	87.22285	103.43564
	6×6	25.67655	52.67282	59.56432	86.80229	101.29420
	8×8	25.67234	52.64376	59.51242	86.73985	100.91338

表10　　两对边简支，另两对边固支弹性地基板的临界荷载，板剖分为不同网格

	网格	β^*					
$k=100$	4×4	78.70324	95.10642	122.44077	224.88707	230.42537	233.14458
	6×6	78.49639	95.06429	118.96029	186.40219	222.87892	229.34409
	8×8	78.45819	95.05717	118.80257	184.64641	222.60085	229.21323
	10×10	78.44795	95.05546	118.76863	184.45959	222.49974	229.17358
$k=0$	4×4	76.17279	84.97434	121.32851	223.77817	227.89500	232.63531
	6×6	75.96350	84.93226	117.83567	185.77420	221.75438	226.81117
	8×8	75.92520	84.92478	117.67709	184.01385	221.47537	226.68044
[21]		75.8973					

表11　　一边固支，另三边简支弹性地基板的临界荷载，板剖分为不同网格

	网格	β^*					
$k=100$	4×4	66.80338	70.28313	117.80657	190.91060	204.38481	230.11893
	6×6	66.78832	70.19011	114.44904	183.31866	190.54396	202.83008
	8×8	66.78594	70.17582	114.34003	181.61801	190.48865	202.71489
	10×10	66.78571	70.17223	114.32134	181.45645	190.47241	202.67201

$$N_{cr}=\beta^*\frac{D}{L^2}$$

表12 四边简支板临界荷载

m,n	网格	M.S.E.M.	精确解[21]
1,1	8×8	39.47793	39.4784
2,1	8×8	61.68469	61.6850
3,1	8×8	109.67264	109.6513
2,2	8×8	157.90319	157.9137
3,2	8×8	185.3254	185.3292
		β^*	

M.S.E.M.——多变量样条元法

四、结 语

本文方法基于二类变量广义变分原理及多种场变量的概念．场函数均应用双三次乘积型二元B样条函数来构造，其优点是未知量少，连续性强，逼近精度高，系统方程直接得到，易于在计算机上实施工程问题的计算，便于推广应用．由于同时采用内力与位移作为独立未知量，因此，内力与位移的计算精度均比较高．

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Analysis of Bending, Vibration and Stability for Thin Plate on Elastic Foundation by the Multivariable Spline Element Method

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Abstract

In this paper, the bicubic splines in product form are used to construct the multi-field functions for bending moments, twisting moment and transverse displacement of the plate on elastic foundation. The multivariable spline element equations are derived,based on the mixed variational principle. The analysis and calculations of bending, vibration and stability of the plates on elastic foundation are presented in the paper.Because the field funcuions of plate on elastic foundation are assumed independently, the precision of the field variables of bending moments and displacement is high.

Key words multivariable spline element method, bicubic B spline,plate on elastic foundation