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固液两相流中的K-ε双方程湍流模式*

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摘　　要

建立了固液两相流中更一般的K-ε双方程湍流模式．模化了固相和液相的连续方程、动量方程及K方程和ε方程．该湍流模型考虑了固液两相间速度的滑移，颗粒间的作用及相间作用．使用本文所建立的湍流模型，数值预测了一管湍流中的沙水混合流动，其预测结果与实验结果比较一致．

关键词　固液两相流　K-ε双方程　湍流模式

一、前言和基本假设

湍流两相流的研究很多年来一直是工程中的重要研究课题．由于其流动的复杂性，尤其是分散相浓度较大的流动，因此使其预测相当困难．近年来随着计算技术的快速发展，已出现了一些近似的湍流模型．Yuu等人[1]建立了两相湍射流模型，他们用经验关系式给定Lagrangian颗粒运动方程中的平均速度．Danon等人[2]基于一组抛物型方程给出了两相湍流中的湍流模型，在模型中假设颗粒平均速度等于流体平均速度．Genchev & Karpuzov[3]给出了一两相湍流模型，模型中考虑了颗粒间作用，并假设了均匀颗粒浓度分布和两相平均速度相等．刘大有[4]从Boltzman方程出发建立了两相流基本方程．蔡树棠等人[5]用雷诺平均方法得到了两相流动的雷诺方程组．另外，Rizk & Elghobashi[6]，Ounis & Ahmadi[7]，刘小兵等人[8]用Lagrange方法分析了两相湍流中的颗粒运动．

本文建立了固液两相流中K-ε双方程湍流模型．模化了固相和液相的连续方程、动量方程及K方程和ε方程．该湍流模型考虑了固液两相间速度的滑移，颗粒间的作用及相间作用．应用本文所发展的湍流模型，数值预测了一管湍流中的沙水混合流动，其预测结果与实验结果比较一致．

本湍流模型提供下列基本假设：

1．流体相为不可压缩流体，固相（分散相）为连续流体，每相的物理特性均为常数；

2．固相的颗粒为球形，且尺寸均匀；

3．不考虑相变．

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二、运动方程

从两相流动瞬时变量的基本运动方程出发，液相连续方程为

$$\frac{\partial \Phi_L}{\partial t}+\frac{\partial(\Phi_L U_i)}{\partial x_i}=0 \tag{2.1}$$

固相连续方程

$$\frac{\partial \Phi_S}{\partial t}+\frac{\partial(\Phi_S V_i)}{\partial x_i}=0 \tag{2.2}$$

液相动量方程

$$\frac{\partial}{\partial t}(\Phi_L U_i)+\frac{\partial}{\partial x_k}(\Phi_L U_i U_k)=-\frac{1}{\rho_L}\Phi_L\frac{\partial P}{\partial x_i}+\nu_L\frac{\partial}{\partial x_k}\left[\Phi_L\left(\frac{\partial U_i}{\partial x_k}+\frac{\partial U_k}{\partial x_i}\right)\right]-\frac{B}{\rho_L}\Phi_L\Phi_S(U_i-V_i)+\Phi_L g_i \tag{2.3}$$

$$\frac{\partial}{\partial t}(\Phi_L U_j)+\frac{\partial}{\partial x_k}(\Phi_L U_j U_k)=-\frac{1}{\rho_L}\Phi_L\frac{\partial P}{\partial x_j}+\nu_L\frac{\partial}{\partial x_k}\left[\Phi_L\left(\frac{\partial U_j}{\partial x_k}+\frac{\partial U_k}{\partial x_j}\right)\right]-\frac{B}{\rho_L}\Phi_L\Phi_S(U_j-V_j)+\Phi_L g_j \tag{2.4}$$

方程(2.3)和(2.4)分别为流体动量方程的 i 和 j 方向。

固相动量方程

$$\frac{\partial}{\partial t}(\Phi_S V_i)+\frac{\partial}{\partial x_k}(\Phi_S V_i V_k)=-\frac{1}{\rho_S}\Phi_S\frac{\partial P}{\partial x_i}+\nu_S\frac{\partial}{\partial x_k}\left[\Phi_S\left(\frac{\partial V_i}{\partial x_k}+\frac{\partial V_k}{\partial x_i}\right)\right]-\frac{B}{\rho_S}\Phi_L\Phi_S(V_i-U_i)+\Phi_S g_i \tag{2.5}$$

式中，U 和 V 分别为液相和固相的速度；ρ 为相材质密度；ν 为相材质粘性系数；P 为压强；g 为重力加速度；x_i 为坐标分量；$B=18(1+B_0)\rho_L\nu_L/d^2$，表示相间作用系数；$d$ 为颗粒直径；B_0 项的引入是为了考虑除 Stokes 线性阻力作用外的其它作用因素，如虚拟质量力、Saffman升力以及 Magus升力等，一般情况下，B_0 不为常数，它与颗粒雷诺数等流动参数有关，本文暂假设为常数；Φ_L 和 Φ_S 分别为液相和固相体积分数，并有关系方程

$$\Phi_L+\Phi_S=1 \tag{2.6}$$

脚标：L 和 S 分别表示液相和固相；i，j，k，l 为张量坐标。

描述湍流运动的一般方法是将瞬时变量分解成平均和脉动两部分来进行处理，即定义

$$U_i=\bar{U}_i+u_i,\quad V_i=\bar{V}_i+v_i,\quad P=\bar{P}+p,\quad \Phi_L=\bar{\Phi}_L+\phi_L,\quad \Phi_S=\bar{\Phi}_S+\phi_S \tag{2.7}$$

式中，U，V，P，Φ 为瞬时变量；u，v，p，ϕ 表示脉动量；符号"—"表示平均值。将式(2.7)代入到方程(2.1)～(2.6)中，并考虑到脉动量的平均值为零，可写出固液两相流的平均方程

液相平均连续方程

$$\frac{\partial \bar{\Phi}_L}{\partial t}+\frac{\partial}{\partial x_i}(\bar{\Phi}_L\bar{U}_i+\overline{\phi_L u_i})=0 \tag{2.8}$$

固相平均连续方程

$$\frac{\partial \bar{\Phi}_S}{\partial t}+\frac{\partial}{\partial x_i}(\bar{\Phi}_S\bar{V}_i+\overline{\phi_S v_i})=0 \tag{2.9}$$

液相平均动量方程

$$\frac{\partial}{\partial t}(\bar{\Phi}_L\bar{U}_i+\overline{\phi_L u_i})+\frac{\partial}{\partial x_k}(\bar{\Phi}_L\bar{U}_i\bar{U}_k+\bar{\Phi}_L\overline{u_iu_k}+\bar{U}_i\overline{\phi_Lu_k}+\bar{U}_k\overline{\phi_Lu_i}+\overline{\phi_Lu_iu_k})$$

$$=-\frac{1}{\rho_L}\left(\bar{\Phi}_L\frac{\partial\bar{P}}{\partial x_i}+\overline{\phi_L\frac{\partial p}{\partial x_i}}\right)+\nu_L\frac{\partial}{\partial x_k}\left[\bar{\Phi}_L\left(\frac{\partial\bar{U}_i}{\partial x_k}+\frac{\partial\bar{U}_k}{\partial x_i}\right)+\overline{\phi_L\left(\frac{\partial u_i}{\partial x_k}+\frac{\partial u_k}{\partial x_i}\right)}\right]$$

$$-\frac{B}{\rho_L}[\bar{\Phi}_L\bar{\Phi}_S(\bar{U}_i-\bar{V}_i)+(\bar{\Phi}_S-\bar{\Phi}_L)\overline{\phi_L(u_i-v_i)}$$

$$+(\bar{U}_i-\bar{V}_i)\overline{\phi_L\phi_S}+\overline{\phi_L\phi_S(u_i-v_i)}]+\bar{\Phi}_Lg_i \tag{2.10}$$

固相平均动量方程

$$\frac{\partial}{\partial t}(\bar{\Phi}_S\bar{V}_i+\overline{\phi_S v_i})+\frac{\partial}{\partial x_k}(\bar{\Phi}_S\bar{V}_i\bar{V}_k+\bar{\Phi}_S\overline{v_iv_k}+\bar{V}_i\overline{\phi_Sv_k}+\bar{V}_k\overline{\phi_Sv_i}+\overline{\phi_Su_iv_k})$$

$$=-\frac{1}{\rho_S}\left(\bar{\Phi}_S\frac{\partial\bar{P}}{\partial x_i}+\overline{\phi_S\frac{\partial p}{\partial x_i}}\right)+\nu_S\frac{\partial}{\partial x_k}\left[\bar{\Phi}_S\left(\frac{\partial\bar{V}_i}{\partial x_k}+\frac{\partial\bar{V}_k}{\partial x_i}\right)+\overline{\phi_S\left(\frac{\partial v_i}{\partial x_k}+\frac{\partial v_k}{\partial x_i}\right)}\right]$$

$$-\frac{B}{\rho_S}[\bar{\Phi}_L\bar{\Phi}_S(\bar{V}_i-\bar{U}_i)+(\bar{\Phi}_S-\bar{\Phi}_L)\overline{\phi_L(v_i-u_i)}$$

$$+(\bar{V}_i-\bar{U}_i)\overline{\phi_L\phi_S}+\overline{\phi_L\phi_S(v_i-u_i)}]+\bar{\Phi}_Sg_i \tag{2.11}$$

体积分数平均关系方程

$$\bar{\Phi}_L+\bar{\Phi}_S=1,\quad \phi_S=-\phi_L \tag{2.12}$$

将$(2.3)\times u_j+(2.4)\times u_i$进行平均，再令$j=i$，并考虑到这时$i$为迭加标，整理所得方程可得到精确的固液两相流湍动能K方程（见附录Ⅰ）。

将方程(2.3)对x_l偏微分，且在方程两侧乘以$\nu_L\partial u_i/\partial x_l$，再对所得方程进行平均，最后整理可得出精确的能耗率ε方程（见附录Ⅱ）。

三、平均运动方程的模化

在进行平均运动方程模化前，首先对基本相关项$\overline{\phi_Lu_i}$，$\overline{\phi_Sv_i}$，$\overline{u_iu_j}$，$\overline{v_iv_j}$，$\overline{u_iv_i}$，$\overline{\phi_Lu_iu_j}$，$\overline{\phi_Sv_iv_j}$，$\overline{\phi_Su_iv_i}$，$\overline{\phi_L\partial p/\partial x_i}$，$\overline{p\partial u_i/\partial x_i}$，$\overline{u_i\partial u_j/\partial x_j}$和$\overline{\phi_Lu_j\partial u_i/\partial x_j}$进行模化，它们将在各平均运动方程中被使用。

相关项$\overline{\phi_Lu_i}$和$\overline{\phi_Sv_i}$一般模化成梯度扩散形式，即

$$\overline{\phi_Lu_i}=-\frac{\nu_t}{\sigma_{\Phi L}}\frac{\partial\bar{\Phi}_L}{\partial x_i},\quad \overline{\phi_Sv_i}=-\frac{\nu_t}{\sigma_{\Phi S}}\frac{\partial\bar{\Phi}_S}{\partial x_i} \tag{3.1}$$

式中，ν_t为湍流运动粘性系数($\nu_t=C_\mu K^2/\varepsilon$，$C_\mu\approx0.09$)。$\sigma_{\Phi L}\approx1.0$。$\sigma_{\Phi S}$由Peskin[6]的研究可确定

$$\sigma_{\Phi S}=1/\{1-[T_L^2\varepsilon/(15\nu_L)\cdot6A^2/(A+1)]\} \tag{3.2}$$

这里A为颗粒响应时间τ_m与流体 Lagrangian 积分时间标尺T_L的比值[$A=\tau_m/T_L$，$\tau_m=\rho_Sd^2/(18\rho_L\nu_L)$，$T_L=C_TK/\varepsilon$，$C_T\approx0.20-0.41$]。

流体相雷诺应力$\overline{u_iu_j}$和颗粒相雷诺应力$\overline{v_iv_j}$分别可模化成

$$\overline{u_iu_j}=-\nu_t\left(\frac{\partial\bar{U}_i}{\partial x_j}+\frac{\partial\bar{U}_j}{\partial x_i}\right)+\frac{2}{3}\delta_{ij}K \tag{3.3}$$

$$\overline{v_i v_j} = -\frac{\nu_t}{\sigma_{\Phi S}}\left(\frac{\partial V_i}{\partial x_j}+\frac{\partial V_j}{\partial x_i}\right)+\frac{2}{3}\delta_{ij}K_S \tag{3.4}$$

式中，δ_{ij}为Kronecker δ函数；K_S表示固相湍动能($K_S\equiv\overline{v_i v_i}/2$)，对于大流动雷诺数和短颗粒响应时间，$K_S$可近似模化成

$$K_S \approx K/(A+1) \tag{3.5}$$

同样，$\overline{u_i v_i}$可近似模化成

$$\overline{u_i v_i} \approx 2K/(A+1) \tag{3.6}$$

$\overline{\phi_L u_i u_j}$，$\overline{\phi_S v_i v_j}$和$\overline{\phi_S u_i v_i}$可分别模化成

$$\overline{\phi_L u_i u_j} = -C_k\frac{K}{\varepsilon}\left[\overline{u_i u_l}\frac{\partial}{\partial x_l}(\overline{\phi_L u_j})+\overline{u_j u_l}\frac{\partial}{\partial x_l}(\overline{\phi_l u_i})\right] \tag{3.7}$$

$$\overline{\phi_S v_i v_j} = -C_k\frac{K}{\varepsilon}\left[\overline{v_i v_l}\frac{\partial}{\partial x_l}(\overline{\phi_S v_j})+\overline{v_j v_l}\frac{\partial}{\partial x_l}(\overline{\phi_S v_i})\right] \tag{3.8}$$

$$\overline{\phi_S u_i v_i} = -C_k\frac{K}{\varepsilon}\left[\overline{u_i v_l}\frac{\partial}{\partial x_l}(\overline{\phi_S v_i})+\overline{v_i u_l}\frac{\partial}{\partial x_l}(\overline{\phi_S u_i})\right] \tag{3.9}$$

式中$C_k \approx 0.1$。

相关项$\overline{\phi_L \partial p/\partial x_i}$和$\overline{p\partial u_i/\partial x_i}$根据Launder等人[10]的研究可模化成

$$\begin{aligned}\overline{\phi_L\frac{\partial p}{\partial x_i}} &= \frac{\partial}{\partial x_i}\overline{\phi_L p} - \overline{p\frac{\partial \phi_L}{\partial x_i}} = \rho_L\frac{\partial}{\partial x_i}\left[\left(C_1 K^{1/2}+C_2\frac{K^{3/2}}{\varepsilon}\frac{\partial \bar U_i}{\partial x_l}\right)\overline{\phi_l u_l}\right] \\ &+\rho_L\frac{\varepsilon}{K}\left[C_3\overline{\phi_L u_i}+C_4\left(\frac{1}{K}\overline{u_i u_l}-\frac{2}{3}\delta_{ij}\right)\overline{\phi_l u_l}\right] \\ &+\rho_L\left(C_5\frac{\partial \bar U_i}{\partial x_l}+C_6\frac{\partial \bar U_l}{\partial x_i}\right)\overline{\phi_L u_l}\end{aligned} \tag{3.10}$$

$$\begin{aligned}\overline{p\frac{\partial u_i}{\partial x_i}} &= \rho_L\left[-C_{p1}\frac{\varepsilon}{K}\left(\frac{1}{2}\overline{u_i u_i}-\frac{1}{3}K\right)+\frac{2C_{p2}+16}{33}\frac{\partial \bar U_i}{\partial x_l}\overline{u_i u_k}\right. \\ &\left.-\frac{30C_{p2}-2}{55}K\frac{\partial \bar U_i}{\partial x_i}+\frac{8C_{p2}-2}{11}\left(\frac{\partial \bar U_k}{\partial x_i}+\frac{1}{3}\frac{\partial \bar U_i}{\partial x_k}\right)\overline{u_i u_k}\right]\end{aligned} \tag{3.11}$$

式中，$C_1=-1.0$，$C_2=1.0$，$C_3=4.3$，$C_4=-3.2$，$C_5=-0.8$，$C_6=0.2$，$C_{p1}=1.5$，$C_{p2}=0.4$。

$\overline{u_i\partial u_j/\partial x_j}$和$\overline{\phi_L u_j \partial u_i/\partial x_j}$可分别模化成

$$\overline{u_i\frac{\partial u_j}{\partial x_j}} = -\nu_t\frac{\partial^2 \bar U_j}{\partial x_i \partial x_j} \tag{3.12}$$

$$\overline{\phi_L u_j\frac{\partial u_i}{\partial x_j}} = -C_k\frac{K}{\varepsilon}\frac{\nu_t}{\sigma_{\Phi L}}\frac{\partial \bar\Phi_L}{\partial x_j} \tag{3.13}$$

3.1 平均连续方程的模化

将方程(3.1)代入到方程(2.8)和(2.9)可分别得到液相和固相平均连续方程的模化式

$$\frac{\partial \bar\Phi_L}{\partial t}+\frac{\partial}{\partial x_i}\left(\bar\Phi_L \bar U_i-\frac{\nu_t}{\sigma_{\Phi L}}\frac{\partial \bar\Phi_L}{\partial x_i}\right)=0 \tag{3.14}$$

$$\frac{\partial \bar\Phi_S}{\partial t}+\frac{\partial}{\partial x_i}\left(\bar\Phi_S \bar V_i-\frac{\nu_t}{\sigma_{\Phi S}}\frac{\partial \bar\Phi_S}{\partial x_i}\right)=0 \tag{3.15}$$

3.2 动量方程的模化

方程(2.10)中，

$$\overline{\partial\phi_L u_i/\partial t},\quad B\rho_L^{-1}[(\bar{U}_i-\bar{V}_i)\overline{\phi_L\phi_S}+\overline{\phi_L\phi_S(u_i-v_i)}]$$

和

$$\nu_L\overline{\partial\phi_L(\partial u_i/\partial x_k+\partial u_k/\partial x_i)}/\partial x_k,\quad \overline{\partial\phi_L u_i u_k}/\partial x_k$$

项与其它项相比，可忽略。根据基本模化式(3.1)，(3.3)和(3.10)可写出液相动量方程的模化形式

$$\frac{\partial}{\partial t}(\bar{\Phi}_L\bar{U}_i)+\frac{\partial}{\partial x_k}(\bar{\Phi}_L\bar{U}_i\bar{U}_k)=\frac{\partial}{\partial x_k}\left[\frac{\nu_t}{\rho_{\Phi L}}\left(\bar{U}_i\frac{\partial\bar{\Phi}_L}{\partial x_k}+\bar{U}_k\frac{\partial\bar{\Phi}_L}{\partial x_i}\right)\right.$$
$$\left.-\bar{\Phi}_L\overline{u_iu_k}\right]-\frac{1}{\rho_L}\left(\bar{\Phi}_L\frac{\partial\bar{P}}{\partial x_i}+\overline{\phi_L\frac{\partial p}{\partial x_i}}\right)+\nu_L\frac{\partial}{\partial x_k}\left[\bar{\Phi}_L\left(\frac{\partial\bar{U}_i}{\partial x_k}+\frac{\partial\bar{U}_k}{\partial x_i}\right)\right]$$
$$-\frac{B}{\rho_L}\left[\bar{\Phi}_L\bar{\Phi}_S(\bar{U}_i-\bar{V}_i)+(\bar{\Phi}_S-\bar{\Phi}_L)\left(\frac{\nu_t}{\sigma_{\Phi S}}-\frac{\nu_t}{\sigma_{\Phi L}}\right)\frac{\partial\bar{\Phi}_L}{\partial x_i}\right]+\bar{\Phi}_L g_i \tag{3.16}$$

类似地，根据基本模化式(3.3)，(3.6)和(3.12)可写出固相动量方程的模化式

$$\frac{\partial}{\partial t}(\bar{\Phi}_S\bar{V}_i)+\frac{\partial}{\partial x_k}(\bar{\Phi}_S\bar{V}_i\bar{V}_k)=\frac{\partial}{\partial x_k}\left[\frac{\nu_t}{\sigma_{\Phi S}}\left(\bar{V}_i\frac{\partial\bar{\Phi}_S}{\partial x_k}+\bar{V}_k\frac{\partial\bar{\Phi}_S}{\partial x_i}\right)\right.$$
$$\left.-\bar{\Phi}_S\overline{v_iv_k}\right]-\frac{1}{\rho_S}\left(\bar{\Phi}_S\frac{\partial\bar{P}}{\partial x_i}+\overline{\phi_S\frac{\partial p}{\partial x_i}}\right)+\nu_S\frac{\partial}{\partial x_k}\left[\bar{\Phi}_S\left(\frac{\partial\bar{V}_i}{\partial x_k}+\frac{\partial\bar{V}_k}{\partial x_i}\right)\right]$$
$$-\frac{B}{\rho_S}\left[\bar{\Phi}_L\bar{\Phi}_S(\bar{V}_i-\bar{U}_i)+(\bar{\Phi}_S-\bar{\Phi}_L)\left(\frac{\nu_t}{\sigma_{\Phi S}}-\frac{\nu_t}{\sigma_{\Phi L}}\right)\frac{\partial\bar{\Phi}_S}{\partial x_i}\right]+\bar{\Phi}_S g_i \tag{3.17}$$

3.3 湍动能K方程的模化

首先，忽略K方程中所有的4次脉动相关项，非稳定项可近似为$\partial(K\bar{\Phi}_L)/\partial t$。第(2)组第1项可忽略$\overline{u_i\partial(\phi_L u_j)/\partial x_j}$相关项可写成

$$\overline{u_i\frac{\partial}{\partial x_j}(\phi_L u_j)}=\frac{\partial}{\partial x_j}\overline{(\phi_L u_i u_j)}-\overline{\phi_L u_j\frac{\partial u_i}{\partial x_j}} \tag{3.18}$$

上式右边第1和第2项已分别由基本模化式(3.7)和(3.13)给出。相关项$\overline{u_i\partial\phi_L/\partial x_j}$可写成

$$\overline{u_i\frac{\partial\phi_L}{\partial x_j}}=\frac{\partial}{\partial x_j}(\overline{\phi_L u_i})-\overline{\phi_L\frac{\partial u_i}{\partial x_j}} \tag{3.19}$$

上式右边第1项已由基本模化式(3.1)给出，第2项较小，可忽略。$\overline{u_i\partial p/\partial x_i}$相关项可写成

$$\overline{u_i\frac{\partial p}{\partial x_i}}=\frac{\partial}{\partial x_i}(\overline{pu_i})-\overline{p\frac{\partial u_i}{\partial x_i}} \tag{3.20}$$

上式右边第1项在湍流扩散中较小，可忽略，第2项已由基本模化式(3.11)给出。$\overline{u_i\partial u_k/\partial x_i}$相关项可写成

$$\overline{u_i\frac{\partial u_k}{\partial x_i}}=\frac{\partial}{\partial x_i}(\overline{u_iu_k})-\overline{u_k\frac{\partial u_i}{\partial x_i}} \tag{3.21}$$

上式右边第1和第2项已由基本模化式(3.3)和(3.12)给出。第(5)大组湍流扩散项中，最后两项可近似成

$$\bar{\Phi}_L\overline{u_i\frac{\partial}{\partial x_k}(u_iu_k)}+\frac{\partial\bar{\Phi}_L}{\partial x_k}\overline{(u_i^2u_k)}\approx-\frac{\partial}{\partial x_k}\left(\bar{\Phi}_L\frac{\nu_t}{\sigma_k}\frac{\partial K}{\partial x_k}\right)$$

相关项$\rho_L^{-1}\overline{\phi_L u_i\partial p/\partial x_i}$可忽略。另外，第6组的$\overline{\phi_L\partial^2(u_iu_i)/\partial x_k^2}$和$\overline{\phi_L u_i\partial^2 u_k/\partial x_k^2}$相关项较小，可忽略。第(7)组的第2项以及第(8)组的第3、5、6和7项均较小，可忽略。因此K方程可模

化成

$$\frac{\partial}{\partial t}(K\bar{\Phi}_L)+\frac{\partial}{\partial x_k}(K\bar{\Phi}_L\bar{U}_k)=P_r+T_{Df}+V_{Df}-\bar{\Phi}_L\varepsilon+A_{Ds} \tag{3.22}$$

式中，P_r，T_{Df}，V_{Df}和A_{Ds}分别表示产生项、湍流扩散项、粘性扩散项和附加耗散项，分别有表达式

$$P_r=\frac{\partial}{\partial x_k}\left(\bar{U}_i\bar{U}_k\frac{\nu_t}{\sigma_{\Phi L}}\frac{\partial\bar{\Phi}_L}{\partial x_i}-\bar{U}_i\overline{\phi_L u_i u_k}-\bar{U}_k\overline{\phi_L u_i^2}\right)$$

$$-\frac{\partial}{\partial x_k}(\bar{\Phi}_L\bar{U}_i)\overline{u_i u_k}+\frac{1}{\rho_L}\frac{\nu_t}{\sigma_{\Phi L}}\frac{\partial\bar{\Phi}_L}{\partial x_i}\frac{\partial\bar{P}}{\partial x_i}$$

$$-C_k\frac{K}{\varepsilon}\frac{\nu_t}{\sigma_{\Phi L}}\left(\bar{U}_i\frac{\partial\bar{\Phi}_L}{\partial x_k}+\bar{U}_k\frac{\partial\bar{\Phi}_L}{\partial x_i}\right)$$

$$T_{Df}=\frac{\partial}{\partial x_k}\left(\bar{\Phi}_L\frac{\nu_t}{\sigma_{\Phi L}}\frac{\partial K}{\partial x_k}\right)$$

$$V_{Df}=\nu_L\left\{\bar{\Phi}^2_L\frac{\partial^2 K}{\partial x_k^2}+\frac{\partial\bar{\Phi}_L}{\partial x_k}\left(\frac{\partial K}{\partial x_k}+\frac{\partial\overline{u_i u_k}}{\partial x_i}+\nu_t\frac{\partial^2\bar{U}_i}{\partial x_k^2}\right)\right.$$

$$\left.-\frac{\partial}{\partial x_k}\left[\frac{\nu_t}{\sigma_{\Phi L}}\frac{\partial\bar{\Phi}_L}{\partial x_i}\left(\frac{\partial\bar{U}_i}{\partial x_k}+\frac{\partial\bar{U}_k}{\partial x_i}\right)\right]\right\}$$

$$A_{Ds}=-\frac{B}{\rho_L}\left[\bar{\Phi}_L\bar{\Phi}_S(\overline{u_i u_i}-\overline{u_i v_i})+(\bar{\Phi}_L-\bar{\Phi}_S)(\bar{U}_i-\bar{V}_i)\frac{\nu_t}{\sigma_{\Phi L}}\frac{\partial\bar{\Phi}_L}{\partial x_i}\right]+\overline{\phi_L u_i g_i}$$

3.4 能耗率ε方程的模化

首先忽略ε方程中所有四次脉动相关项，第(1)组非稳定项可近似表达成$0.5\partial(\bar{\Phi}_L\varepsilon)/\partial t$；第(2)组相关项$0.5\partial(\bar{\Phi}_L\bar{U}_k\varepsilon)/\partial x_k$比其它各项大，尤其是在大流动雷诺数下，可保留；第(3)组产生项和第(4)组附加产生项可总起来用$0.5C_{\varepsilon1}P_r\varepsilon/K$来模化，$C_{\varepsilon1}\approx1.44$。第(5)大组所有项可模化成$0.5\partial[\bar{\Phi}_L(\nu_t/\sigma_\varepsilon)\partial\varepsilon/\partial x_k]/\partial x_k$，$\sigma_\varepsilon\approx1.3$；第(6)大组可模化成$0.5[\nu_L\partial(\bar{\Phi}_L\partial\varepsilon/\partial x_k)/\partial x_k-C_{\varepsilon2}\bar{\Phi}_L\varepsilon^2/K]$，$C_{\varepsilon2}\approx1.92$；第(7)大组可模化成$-0.5C_{\varepsilon3}\bar{\Phi}_L A_{Ds}\varepsilon/K$，$C_{\varepsilon3}\approx1.2$。这样，$\varepsilon$方程可模化成

$$\frac{\partial}{\partial t}(\varepsilon\bar{\Phi}_L)+\frac{\partial}{\partial x_k}(\varepsilon\bar{\Phi}_L\bar{U}_k)=\frac{\partial}{\partial x_k}\left[\bar{\Phi}_L\left(\nu_L+\frac{\nu_t}{\sigma_\varepsilon}\right)\frac{\partial\varepsilon}{\partial x_k}\right]$$

$$+[C_{\varepsilon1}P_r-\bar{\Phi}_L(C_{\varepsilon2}\varepsilon+C_{\varepsilon3}A_{Ds})]\frac{\varepsilon}{K} \tag{3.23}$$

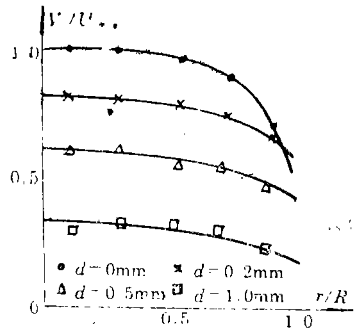
变量$\bar{U}$，$\bar{V}$，$\bar{P}$，$\bar{\Phi}_L$，$\bar{\Phi}_S$，K和ε可通过方程(2.12)，(3.14)～(3.17)，(3.22)和(3.23)所构成的方程组进行求解。

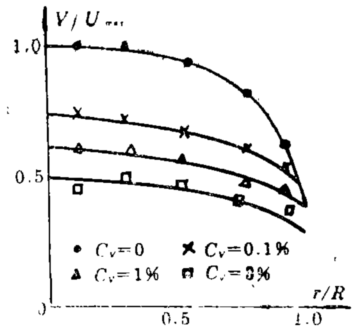
四、数 值 算 例

为了估价本湍流模型，并将数值结果与最近的湍管流实验结果进行比较，我们进行了一雷诺数为5.6×10^4，颗粒尺寸均匀，颗粒与流体密度比为2.65的沙水管湍流数值模拟。数值方法采用Partarkar[11]所给的计算方法。

图1给出了颗粒尺寸对轴向平均速度的影响（固相浓度$C_V=1\%$），数值结果表明当颗

粒尺寸小于1.0mm时，预测结果与实验结果比较一致；图2给出了固相体积分数对轴向平均速度的影响（颗粒直径$d=0.5$mm），数值结果表明当颗粒尺寸小于0.05时，预测结果与实验结果比较一致。图中实线为计算结果，$d=0$和$C_V=0$的颗粒速度定为水流速度。





—— 理论值　　●×△□ 实验值

图1　颗粒尺寸对轴向平均速度的影响　　图2　固相体积分数对轴向平均速度的影响

五、结　　论

本文给出了一个在目前比较精确的固液两相湍流模型。应用此湍流模型对一管湍流中沙水流动进行了数值预测，从数值结果和试验结果的比较可知，当固相体积浓度小于5%及颗粒直径小于1.0mm时，两种结果比较一致。

附　　录

Ⅰ. 湍动能K方程

$$[K\bar{\Phi}_{L,t}+(K\bar{\Phi}_L)_{,t}+\bar{U}_{i,t}\overline{\phi_L u_i}+\bar{U}_i\,\overline{u_i\phi_{L,t}}+\overline{u_i(\phi_L u_i)_{,t}}]^{(1)}+[K(\bar{\Phi}_L\bar{U}_k)_{,k}+(K\bar{\Phi}_L\bar{U}_k)_{,k}]^{(2)}$$

$$=-[(\bar{\Phi}_L\bar{U}_i)_{,k}\overline{u_iu_k}+\bar{U}_{i,k}\overline{\phi_L u_iu_k}]^{(3)}-[\rho_L^{-1}\bar{P}_{,i}\overline{\phi_L u_i}+\bar{U}_i\overline{u_i(\phi_L u_k)_{,k}}+\bar{U}_k\overline{u_i(\phi_L u_i)_{,k}}+(\bar{U}_i\bar{U}_k)_{,k}\overline{\phi_L u_i}+\bar{U}_i\bar{U}_k\overline{u_i\phi_{L,k}}+\overline{u_i(\phi_L u_iu_k)_{,k}}]^{(4)}$$
$$-[\rho_L^{-1}(\bar{\Phi}_L\overline{u_ip_{,i}}+\overline{\phi_L u_ip_{,i}})+\bar{\Phi}_L\overline{u_i(u_iu_k)_{,k}}+\bar{\Phi}_{L,k}\overline{u_iu_iu_k}]^{(5)}+\nu_L[\bar{\Phi}_L K_{,kk}+(\bar{U}_{i,k}+\bar{U}_{k,i})\overline{u_i\phi_{L,k}}+\bar{\Phi}_{L,k}(K_{,k}+\overline{u_iu_{k,i}})+\overline{\phi_L(u_iu_i)_{k}}+\bar{U}_{i,kk}\overline{\phi_L u_i}+\overline{u_i(u_{i,k}+u_{k,i})\phi_{L,k}}]^{(6)}-(\bar{\Phi}_L\varepsilon+\nu_L\overline{\phi_L u_{i,k}u_{i,k}})^{(7)}-\{B\rho_L^{-1}[\bar{\Phi}_L\bar{\Phi}_S\overline{u_i(u_i-v_i)}+(\bar{\Phi}_S-\bar{\Phi}_L)(\bar{U}_i-\bar{V}_i)\overline{\phi_L u_i}+\bar{\Phi}_L\overline{\phi_S u_i(u_i-v_i)}+(\bar{U}_i-\bar{V}_i)\overline{\phi_L\phi_S u_i}+\overline{\phi_L\phi_S u_i(u_i-v_i)}]-\overline{\phi_L u_i}g_i\}^{(8)}$$

式中，为了书写简单，约定$\partial(\ )/\partial t\equiv(\ )_{,t}$，$\partial(\ )/\partial x_i\equiv(\ )_{,i}$，$\partial^2(\ )/\partial x_i\partial x_j\equiv(\ )_{,ij}$，湍动能$K\equiv\overline{u_iu_i}/2$，耗散$\varepsilon\equiv\nu_L\overline{u_{i,k}u_{i,k}}$。此方程由8大组组成，第(1)组为非稳定项，第(2)组为对流项，第(1)组为产生项，第(4)组为附加产生项和转移项，第(5)组为湍流扩散项，第(6)组为粘性扩散项，第(7)组为耗散项，第(8)组为附加耗散项。

Ⅱ. 能耗率ε方程

$$\{0.5[\varepsilon\bar{\Phi}_{L,t}+(\varepsilon\bar{\Phi}_L)_{,t}]+\nu_L[\bar{\Phi}_{L,l}\overline{u_iu_{i,l}}+\bar{\Phi}_{L,l}\overline{u_{i,l}u_{i,t}}+\bar{U}_{i,t}\overline{\phi_{L,l}u_{i,l}}+\bar{U}_{i,lt}\overline{\phi_L u_{i,l}}+\bar{U}_{i,l}\overline{\phi_{L,t}u_{i,l}}+\bar{U}_i\overline{\phi_{L,lt}u_{i,l}}+\overline{u_{i,l}(\phi_L u_i)_{,lt}}]\}^{(1)}+\{0.5[\varepsilon(\bar{\Phi}_L\bar{U}_k)_{,k}+(\varepsilon\bar{\Phi}_L\bar{U}_k)_{,k}]+\nu_L[(\bar{\Phi}_L\bar{U}_i)_{,lk}\overline{u_ku_{i,l}}+(\bar{\Phi}_L\bar{U}_i)_{,k}\overline{u_{k,l}u_{i,l}}+(\bar{\Phi}_L\bar{U}_i)_{,lk}\overline{u_ku_{i,l}}]\}^{(2)}$$

$$=-\nu_L\bar{\Phi}_L\overline{u_{i,l}(u_iu_k)_{,lk}}^{(3)}-\nu_L[(\bar{U}_i\bar{U}_k)_{,l}\overline{\phi_{L,k}u_{i,l}}+\bar{U}_i\bar{U}_k\overline{\phi_{L,lk}u_{i,l}}+(\bar{U}_i\bar{U}_k)_{,k}\overline{\phi_{L,l}u_{i,l}}+(\bar{U}_i\bar{U}_k)_{,lk}\overline{\phi_L u_{i,l}}+\bar{U}_{i,l}\overline{u_{i,l}(\phi_L u_k)_{,k}}+\bar{U}_i\overline{u_{i,l}(\phi_L u_k)_{,lk}}+\bar{U}_{i,k}\overline{u_{i,l}(\phi_L u_k)_{,l}}+\bar{U}_{i,lk}\overline{\phi_L u_ku_{i,l}}+\bar{U}_{k,l}\overline{u_{i,l}(\phi_L u_i)_{,l}}+\bar{U}_{k,l}\overline{u_{i,l}(\phi_L u_i)_{,l}}+\overline{u_{i,l}(\phi_L u_iu_k)_{,lk}}+\rho_L^{-1}(\bar{\Phi}_{L,l}\overline{p_{,i}u_{i,l}}+\bar{P}_{,il}\overline{\phi_L u_{i,l}}+\bar{P}_{,i}\overline{\phi_{L,l}u_{i,l}}+\overline{p_{,i}\phi_L u_{i,l}})]^{(4)}-\nu_L[\bar{\Phi}_{L,l}\overline{u_{i,l}(u_iu_k)_{,k}}$$

$$
\begin{aligned}
&+\bar{\Phi}_{L,k}\overline{u_{i,l}(u_iu_k)_{,l}}+\bar{\Phi}_{L,lk}\overline{u_iu_ku_{i,l}}+\rho_L^{-1}\bar{\Phi}_L\overline{p_{,lk}u_{,il}}]^{(5)}+\nu_L\{0.5(\bar{\Phi}_L\varepsilon_{,k})_{,k}+\nu_L[\bar{\Phi}_{L,k}\overline{u_{i,l}u_{k,il}}\\
&+\bar{\Phi}_{L,l}\overline{u_{i,l}u_{i,kk}}-\bar{\Phi}_L\overline{u_{i,lk}u_{i,lk}}+\bar{U}_{i,lkk}\overline{\phi_Lu_{i,l}}+\overline{u_{i,l}(\phi_Lu_{i,k}+\phi_Lu_{k,i})_{,lk}}]\}^{(6)}-\nu_LB\rho_L^{-1}\{[(\bar{\Phi}_S\\
&-\bar{\Phi}_L)(\bar{U}_i-\bar{V}_i)]_{,l}\overline{\phi_Lu_{i,l}}+(\bar{\Phi}_S-\bar{\Phi}_L)(\bar{U}_i-\bar{V}_i)\overline{\phi_{L,l}u_{i,l}}+(\bar{U}_i-\bar{V}_i)_{,l}\overline{\phi_L\phi_Su_{i,}}\\
&+\bar{\Phi}_L\bar{\Phi}_S\overline{u_{i,l}(u_i-v_i)}_{,l}+(\bar{U}_i-\bar{V}_i)\overline{(\phi_L\phi_S)_{,l}u_{i,l}}+(\bar{\Phi}_L\bar{\Phi}_S)_{,l}\overline{u_{i,l}(u_i-v_i)}+(\bar{\Phi}_S-\bar{\Phi}_L)_{,l}\\
&\cdot\overline{\phi_Lu_{i,l}(u_i-v_i)}+(\bar{\Phi}_S-\bar{\Phi}_L)\overline{u_{i,l}[\phi_L(u_i-v_i)]_{,l}}+\overline{u_{i,l}[\phi_L\phi_S(u_i-v_i)]_{,l}}-B^{-1}\rho_L\overline{\phi_{L,l}u_{i,l}}g_i\}^{(7)}
\end{aligned}
$$

式中，此方程由 7 大组组成，第(1)组为非稳定项，第(2)组为对流项，第(3) 组为产生项，第(4) 组为附加产生项，第(5)组为湍流扩散项，第(6)组为耗散项，第(7)组为附加耗散项。

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A K-ε Two-Equation Turbulence Model for the Solid-Liquid Two-Phase Flow

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Abstract

A two-equation turbulence model has been developed for predicting two-phase flow. The two equations describe the conservation of turbulence kinetic energy and dissipation rate of that energy for the incompressible carrier fluid in a two-phase flow. The continuity, the momentum, K and ε equations are modeled. In this model, the solid-liquid slip velocities, the particle-particle interactions and the interactions between two phases are considered. The sandy water pipe turbulent flows are successfully predicted by this turbulence model.

Key words solid-liquid two-phase, $K-\varepsilon$ two-equation, turbulence model