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非定常沿岸波流的有限单元分析（Ⅱ）——二步显含有限单元法*

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摘 要

本文用二步显含有限单元法求解[1]中提出的非定常沿岸波流相互作用的数学模型，并用程序设计语言 Fortran 在计算机上加以实现．从若干计算例子中可明显看出涡旋的出现，这和观察到的现象是一致的．

关键词 有限元法 非定常波流 涡旋

一、基础方程的Galerkin法形式化

若η，h，H，θ及K分别表示水位、水深、波高及波数，而u,v分别表示水流速度的x，y分量，并令$D=\eta+h$，则由[1]已得如下的基础方程

$$\frac{\partial\eta}{\partial t}+\frac{\partial}{\partial x}(uD)+\frac{\partial}{\partial y}(vD)=0 \tag{1.1}$$

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}+g\frac{\partial\eta}{\partial x}=-\frac{1}{\rho}\frac{\partial\tau_l}{\partial y}-\frac{1}{\rho D}\left(\frac{\partial S_{xx}}{\partial x}+\frac{\partial S_{xy}}{\partial y}\right)+\frac{1}{\rho D}\tau_{wx}-\frac{1}{\rho D}\tau_{bx} \tag{1.2}$$

$$\frac{\partial v}{\partial t}+v\frac{\partial v}{\partial y}+u\frac{\partial v}{\partial x}+g\frac{\partial\eta}{\partial y}=-\frac{1}{\rho}\frac{\partial\tau_l}{\partial x}-\frac{1}{\rho D}\left(\frac{\partial S_{xy}}{\partial x}+\frac{\partial S_{yy}}{\partial y}\right)+\frac{1}{\rho D}\tau_{wy}-\frac{1}{\rho D}\tau_{by} \tag{1.3}$$

$$\frac{2}{H}\frac{\partial H}{\partial t}+(u+C_g\cos\theta)\frac{2}{H}\frac{\partial H}{\partial x}+(v+C_g\sin\theta)\frac{2}{H}\frac{\partial H}{\partial y}+\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}-C_g\sin\theta\frac{\partial\theta}{\partial x}+\cos\frac{\partial C_g}{\partial x}+C_g\cos\theta\frac{\partial\theta}{\partial y}+\sin\theta\frac{\partial C_g}{\partial y}+Q=0 \tag{1.4}$$

$$\frac{\partial(K\cos\theta)}{\partial y}-\frac{\partial(K\sin\theta)}{\partial x}=0 \tag{1.5}$$

* 吴家龙推荐．1987年3月27日第一次收到．

其中 K及Q分别表示如下

$$K=\frac{2\pi/T}{(gh)^{1/2}+u\cos\theta+v\sin\theta} \tag{1.6}$$

$$C_g=c\{1+2Kh/\sinh(2Kh)\}/2 \tag{1.7}$$

$$c=\{g\cdot\tanh(Kh)/K\}^{1/2} \tag{1.8}$$

$$Q=\frac{1}{E}\left\{S_{xx}\frac{\partial u}{\partial x}+S_{xy}\left(\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}\right)+S_{yy}\frac{\partial v}{\partial y}\right\} \tag{1.9}$$

其边界条件为

$$S_1:\ q_n=u\cos(n,\ x)+v\cos(n,\ y)=\bar{q}_n \tag{1.10}$$

$$S_2:\begin{cases}\eta=\bar{\eta}\\ H=\bar{H}\\ \theta=\bar{\theta}\end{cases} \tag{1.11}$$

根据以上的边界条件以及Galekin方法，我们可得如下诸式

$$\iint\left(\frac{\partial\eta}{\partial t}+\frac{\partial}{\partial x}(uD)+\frac{\partial}{\partial y}(vD)\right)\delta\eta dS=\int_{S_1}D(q_n-\bar{q}_n)\delta\eta dl \tag{1.12}$$

上式经分部积分后，可得下式

$$\iint\frac{\partial\eta}{\partial t}\delta\eta dS-\iint D\left(u\frac{\partial\delta\eta}{\partial x}+v\frac{\partial\delta\eta}{\partial y}\right)dS=-\int_{S_1}D\bar{q}_n\delta\eta dl \tag{1.13}$$

由于(1.2)～(1.3)式中边界项可以不予考虑[2]，因此，我们得到

$$\iint\left(\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}+g\frac{\partial\eta}{\partial x}+\frac{1}{\rho}\frac{\partial\tau_l}{\partial y}+\frac{1}{\rho D}\frac{\partial S_{xy}}{\partial y}+\frac{1}{\rho D}\frac{\partial S_{xx}}{\partial x}-\frac{1}{\rho D}\tau_{wx}+\frac{1}{\rho D}\tau_{bx}\right)\delta udS=0 \tag{1.14}$$

$$\iint\left(\frac{\partial v}{\partial t}+v\frac{\partial v}{\partial y}+u\frac{\partial v}{\partial x}+g\frac{\partial\eta}{\partial y}+\frac{1}{\rho}\frac{\partial\tau_l}{\partial x}+\frac{1}{\rho D}\frac{\partial S_{xy}}{\partial x}+\frac{1}{\rho D}\frac{\partial S_{yy}}{\partial y}-\frac{1}{\rho D}\tau_{wy}+\frac{1}{\rho D}\tau_{by}\right)\delta vdS=0 \tag{1.15}$$

(1.4)式经变量代换

$$w=2\ln H \tag{1.16}$$

后，变成如下的形式

$$\iint\left\{\frac{\partial w}{\partial t}+(u+C_g\cos\theta)\frac{\partial w}{\partial x}+(v+C_g\sin\theta)\frac{\partial w}{\partial y}+\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}-C_g\sin\theta\frac{\partial\theta}{\partial x}+\cos\theta\frac{\partial C_g}{\partial x}+C_g\cos\theta\frac{\partial\theta}{\partial y}+\sin\theta\frac{\partial C_g}{\partial y}+Q\right\}\delta wdS=\int_{S_2}(w-\bar{w})\delta wdl \tag{1.17}$$

其中 $\bar{w}=\ln w$。(1.5)化为

$$\iint\left(a\frac{\partial\theta}{\partial x}+b\frac{\partial\theta}{\partial y}+p\right)\delta\theta dS=\int_{S_2}(\theta-\bar{\theta})\delta\theta dl \tag{1.18}$$

其中

$$\left.\begin{aligned}&a=(gh)^{1/2}\cos\theta+u,\ b=(gh)^{1/2}\sin\theta+v\\&p=\frac{1}{2}\sqrt{\frac{g}{h}}\left(\cos\theta\frac{\partial h}{\partial y}-\sin\theta\frac{\partial h}{\partial x}\right)-\cos\theta\left(\sin\theta\frac{\partial u}{\partial x}-\cos\theta\frac{\partial u}{\partial y}\right)\\&\quad-\sin\theta\left(\sin\theta\frac{\partial v}{\partial x}-\cos\theta\frac{\partial v}{\partial y}\right)\end{aligned}\right\}\tag{1.19}$$

二、有限元的离散化

现在，我们把依赖于时间t的变量η，u，v，w以及θ作如下的离散化

$$\left.\begin{aligned}&\eta=\sum N_iA_i=NA, && \delta\eta=NA^*\\&u=\sum N_iU_i=NU, && \delta u=NU^*\\&v=\sum N_iV_i=NV, && \delta v=NV^*\\&w=\sum N_iW_i=NW, && \delta w=NW^*\\&\theta=\sum N_i\phi_i=N\phi, && \delta\theta=N\phi^*\end{aligned}\right\}\tag{2.1}$$

其中

$$N=N(x,y),\ A=A(t),\ U=U(t),\ V=V(t),\ W=W(t)$$

把(2.1)式分别代入(1.13)，(1.14)～(1.15)、(1.17)以及(1.18)式，经过整理后即可得到下列各式

$$\begin{aligned}&\langle N^TN\rangle\frac{dA}{dt}-\langle N_x^TNUN\rangle A-\langle N_y^TNVN\rangle A\\&\quad-\langle hN_x^TN\rangle U-\langle hN_y^TN\rangle V=0\end{aligned}\tag{2.2}$$

$$\begin{aligned}&\langle N^TN\rangle\frac{dU}{dt}+\langle N^TNUN_x\rangle U+\langle N^TNVN_y\rangle U+g\langle N^TN_x\rangle A\\&\quad+\frac{1}{2\rho}\left\langle\frac{E}{h}(\sin2\theta N^TN_y+(2\cos^2\theta+1)N^TN_x)\right\rangle W\\&\quad+\frac{f}{2\rho}\left\langle\frac{\sin2\theta}{h}N^TN\right\rangle V+\frac{f}{\rho}\left\langle\frac{1+\cos^2\theta}{h}N^TN\right\rangle U\\&\quad+\frac{1}{\rho}\left\langle\frac{E}{h}\left(\cos2\theta\frac{\partial\theta}{\partial y}-\sin2\theta\frac{\partial\theta}{\partial x}\right)N^T\right\rangle=0\end{aligned}\tag{2.3}$$

$$\begin{aligned}&\langle N^TN\rangle\frac{dV}{dt}+\langle N^TNVN_y\rangle V+\langle N^TNUN_x\rangle V+g\langle N^TN_y\rangle A\\&\quad+\frac{1}{2\rho}\left\langle\frac{E}{h}(\sin2\theta N^TN_x+(2\sin^2\theta+1)N^TN_y)\right\rangle W\\&\quad+\frac{f}{2\rho}\left\langle\frac{\sin2\theta}{h}N^TN\right\rangle U+\frac{f}{\rho}\left\langle\frac{1+\sin^2\theta}{h}N^TN\right\rangle V\\&\quad+\frac{1}{\rho}\left\langle\frac{E}{h}\left(\cos2\theta\frac{\partial\theta}{\partial x}+\sin2\theta\frac{\partial\theta}{\partial y}\right)N^T\right\rangle=0\end{aligned}\tag{2.4}$$

$$\begin{aligned}&\langle N^TN\rangle\frac{dW}{dt}+\langle N^TNUN_x\rangle W+\langle N^TNVN_y\rangle W+\langle C_g(\cos\theta N^TN_x\\&\quad+\sin\theta N^TN_y)\rangle+\left\langle\left(\cos^2\theta+\frac{3}{2}\right)N^TN_x\right\rangle U\end{aligned}$$

$$+\langle\cos\theta\sin\theta N^T N_y\rangle U+\left\langle\left(\sin^2\theta+\frac{3}{2}\right)N^T N_y+\cos\theta\sin\theta N^T N_x\right\rangle V$$

$$+\left\langle\left(-C_g\sin\theta\frac{\partial\theta}{\partial x}+\cos\theta\frac{\partial C_g}{\partial x}+C_g\cos\theta\frac{\partial\theta}{\partial y}\right.\right.$$

$$\left.\left.+\sin\theta\frac{\partial C_g}{\partial y}\right)N^T\right\rangle=0 \tag{2.5}$$

$$G^T\phi=K^T \tag{2.6}$$

(2 2)～(2.6)式还可以写成如下更简单的形式

$$M_{\alpha\beta}\dot{A}-\bar{N}^1_{\alpha\beta\gamma}U_\beta A_\gamma-\bar{N}^2_{\alpha\beta\gamma}V_\beta A_\gamma-\bar{N}^1_{\alpha\beta}U_\beta-\bar{N}^2_{\alpha\beta}V_\beta=0 \tag{2.7}$$

$$M_{\alpha\beta}\dot{U}_\beta+N^1_{\alpha\beta\gamma}U_\beta U_\gamma+N^2_{\alpha\beta\gamma}V_\beta U_\gamma+N^1_{\alpha\beta}A_\beta+P^1_{\alpha\beta}W_\beta$$
$$+R_{\alpha\beta}V_\beta+S^1_{\alpha\beta}U_\beta=\Omega^1_\alpha \tag{2.8}$$

$$M_{\alpha\beta}\dot{V}_\beta+N^2_{\alpha\beta\gamma}V_\beta V_\gamma+N^1_{\alpha\beta\gamma}U_\beta V_\gamma+N^2_{\alpha\beta}A_\beta+P^2_{\alpha\beta}W_\beta$$
$$+R_{\alpha\beta}U_\beta+S^2_{\alpha\beta}V_\beta=\Omega^2_\alpha \tag{2.9}$$

$$M_{\alpha\beta}\dot{W}_\beta+N^1_{\alpha\beta\gamma}U_\beta W_\gamma+N^2_{\alpha\beta\gamma}V_\beta W_\gamma+T_{\alpha\beta}W_\beta+F^1_{\alpha\beta}U_\beta+F^2_{\alpha\beta}V_\beta=\Pi_\alpha \tag{2.10}$$

$$G_{\alpha\beta}\phi_\beta=K_\alpha \tag{2.11}$$

上列各式中出现的诸变量分别定义如下

$$M_{\alpha\beta}=\iint N_\alpha N_\beta dS \tag{2.12}$$

$$N^i_{\alpha\beta\gamma}=\iint N_\alpha N_\beta N_{\gamma,i}dS,\quad \bar{N}^i_{\alpha\beta\gamma}=\iint N_{\alpha,i}N_\beta N_\gamma dS \tag{2.13}$$

$$N^i_{\alpha\beta}=g\iint N_\alpha N_{\beta,i}dS,\quad \bar{N}^i_{\alpha\beta}=\iint hN_{\alpha,i}N_\beta dS \tag{2.14}$$

$$P^1_{\alpha\beta}=\frac{1}{2\rho}\iint\frac{E}{h}N_\alpha[\sin 2\theta N_{\beta,j}+(2\cos^2\theta+1)N_{\beta,i}]dS \tag{2.15}$$

$$P^2_{\alpha\beta}=\frac{1}{2\rho}\iint\frac{E}{h}N_\alpha[\sin 2\theta N_{\beta,i}+(2\sin^2\theta+1)N_{\beta,j}]dS \tag{2.16}$$

$$K_\alpha=-\iint pN_\alpha dS+\int_{S_2}\bar{\theta}N_\alpha dl \tag{2.17}$$

$$G_{\alpha\beta}=\iint N_\alpha(aN_{\beta,i}+aN_{\beta,j})dS-\int_{S_2}N_\alpha N_\beta dl \tag{2.18}$$

$$R_{\alpha\beta}=\frac{f}{2\rho}\iint\frac{\sin 2\theta}{h}N_\alpha N_\beta dS \tag{2.19}$$

$$S^1_{\alpha\beta}=\frac{f}{\rho}\iint\frac{1+\cos^2\theta}{h}N_\alpha N_\beta dS,\quad S^2_{\alpha\beta}=\frac{f}{\rho}\iint\frac{1+\sin^2\theta}{h}N_\alpha N_\beta dS \tag{2.20}$$

$$\Omega^1_\alpha=\frac{1}{\rho}\iint\frac{E}{h}N_\alpha\left(\sin 2\theta\frac{\partial\theta}{\partial x}-\cos 2\theta\frac{\partial\theta}{\partial y}\right)dS \tag{2.21}$$

$$\Omega^2_\alpha=-\frac{1}{\rho}\iint\frac{E}{h}N_\alpha\left(\cos 2\theta\frac{\partial\theta}{\partial x}+\sin 2\theta\frac{\partial\theta}{\partial y}\right)dS \tag{2.22}$$

$$T_{\alpha\beta}=\iint C_g N_\alpha(\cos\theta N_{\beta,i}+\sin\theta N_{\beta,j})dS \tag{2.23}$$

$$F^1_{\alpha\beta}=\iint N_\alpha\left[\left(\cos^2\theta+\frac{3}{2}\right)N_{\beta,i}+\cos\theta\sin\theta N_{\beta,j}\right]dS \tag{2.24}$$

$$F^2_{\alpha\beta}=\iint N_\alpha\left[\left(\sin^2\theta+\frac{3}{2}\right)N_{\beta,j}+\cos\theta\sin\theta N_{\beta,i}\right]dS \tag{2.25}$$

$$\Pi_\alpha=\iint N_\alpha\left(C_g\sin\theta\frac{\partial\theta}{\partial x}-\cos\theta\frac{\partial C_g}{\partial x}-C_g\cos\theta\frac{\partial\theta}{\partial y}-\sin\theta\frac{\partial C_g}{\partial y}\right)dS \tag{2.26}$$

三、二步显含有限元法

为了解含时间变量的方程(2.7)～(2.11)，本文采用选择成块二步显含有限元方法[8] (Selectiue lumping Finite Element Method)。它是目前解这方面问题的一个非常有效的方法。这个过程由二步组成，对于方程(2.7)～(2.11)，有如下两组关系，其中n表示第n个时间步，Δt表示时间间隔，$\bar{M}_{\alpha\beta}$表示块系数。

第一步

$$\begin{cases}
\bar{M}_{\alpha\beta}A_\beta^{n+1/2}=\tilde{M}_{\alpha\beta}A_\beta^n+\dfrac{\Delta t}{2}(\bar{N}^i_{\alpha\beta\gamma}U_\beta^nA_\gamma^n+\bar{N}^j_{\alpha\beta\gamma}V_\beta^nA_\gamma^n \\
\qquad +\bar{N}^i_{\alpha\beta}U_\beta^n+\bar{N}^j_{\alpha\beta}V_\beta^n) & (3.1) \\
\bar{M}_{\alpha\beta}U_\beta^{n+1/2}=\bar{M}_{\alpha\beta}U_\beta^n-\dfrac{\Delta t}{2}(N^i_{\alpha\beta\gamma}U_\beta^nU_\gamma^n+N^j_{\alpha\beta\gamma}V_\beta^nU_\gamma^n+N^i_{\alpha\beta}A_\beta^n \\
\qquad +P^1_{\alpha\beta}W_\beta^n+R_{\alpha\beta}V_\beta^n+S^1_{\alpha\beta}U_\beta^n-\Omega^{1n}_\alpha) & (3.2) \\
\bar{M}_{\alpha\beta}V_\beta^{n+1/2}=\bar{M}_{\alpha\beta}V_\beta^n-\dfrac{\Delta t}{2}(N^j_{\alpha\beta\gamma}V_\beta^nV_\gamma^n+N^i_{\alpha\beta\gamma}U_\beta^nU_\gamma^n+N^j_{\alpha\beta}A_\beta^n \\
\qquad +P^2_{\alpha\beta}W_\beta^n+R_{\alpha\beta}U_\beta^n+S^2_{\alpha\beta}V_\beta^n-\Omega^{2n}_\alpha) & (3.3) \\
\bar{M}_{\alpha\beta}W_\beta^{n+1/2}=\tilde{M}_{\alpha\beta}W_\beta^n-\dfrac{\Delta t}{2}(N^i_{\alpha\beta\gamma}U_\beta^nW_\gamma^n+N^j_{\alpha\beta\gamma}V_\beta^nW_\gamma^n \\
\qquad +T_{\alpha\beta}W_\beta^n+F^1_{\alpha\beta}U_\beta^n+F^2_{\alpha\beta}V_\beta^n-\Pi_\alpha^n) & (3.4)
\end{cases}$$

第二步

$$\begin{cases}
\bar{M}_{\alpha\beta}A_\beta^{n+1}=\tilde{M}_{\alpha\beta}A_\beta^n+\Delta t\,(\bar{N}^i_{\alpha\beta\gamma}U_\beta^{n+\frac{1}{2}}A_\gamma^{n+\frac{1}{2}}+\bar{N}^j_{\alpha\beta\gamma}V_\beta^{n+\frac{1}{2}}A_\gamma^{n+\frac{1}{2}} \\
\qquad +\bar{N}^i_{\alpha\beta}U_\beta^{n+\frac{1}{2}}+\bar{N}^j_{\alpha\beta}V_\beta^{n+\frac{1}{2}}) & (3.5) \\
\bar{M}_{\alpha\beta}U_\beta^{n+1}=\bar{M}_{\alpha\beta}U_\beta^n-\Delta t\,(N^i_{\alpha\beta\gamma}U_\beta^{n+\frac{1}{2}}U_\gamma^{n+\frac{1}{2}}+N^j_{\alpha\beta\gamma}V_\beta^{n+\frac{1}{2}}U_\gamma^{n+\frac{1}{2}} \\
\qquad +N^i_{\alpha\beta}A_\beta^{n+\frac{1}{2}}+P^1_{\alpha\beta}W_\beta^{n+\frac{1}{2}}+R_{\alpha\beta}V_\beta^{n+\frac{1}{2}}+S^1_{\alpha\beta}U_\beta^{n+\frac{1}{2}}-\Omega^{1n}_\alpha) & (3.6) \\
\bar{M}_{\alpha\beta}V_\beta^{n+1}=\bar{M}_{\alpha\beta}V_\beta^n-\Delta t\,(N^j_{\alpha\beta\gamma}V_\beta^{n+\frac{1}{2}}V_\gamma^{n+\frac{1}{2}}+N^i_{\alpha\beta\gamma}U_\beta^{n+\frac{1}{2}}V_\gamma^{n+\frac{1}{2}} \\
\qquad +N^j_{\alpha\beta}A_\beta^{n+\frac{1}{2}}+P^2_{\alpha\beta}W_\beta^{n+\frac{1}{2}}+R_{\alpha\beta}U_\beta^{n+\frac{1}{2}}+S^2_{\alpha\beta}V_\beta^{n+\frac{1}{2}}-\Omega^{2n}_\alpha) & (3.7) \\
\bar{M}_{\alpha\beta}W_\beta^{n+1}=\tilde{M}_{\alpha\beta}W_\beta^n-\Delta t\,(N^i_{\alpha\beta\gamma}U_\beta^{n+\frac{1}{2}}W_\gamma^{n+\frac{1}{2}}+N^j_{\alpha\beta\gamma}V_\beta^{n+\frac{1}{2}}W_\gamma^{n+\frac{1}{2}} \\
\qquad +T_{\alpha\beta}W_\beta^{n+\frac{1}{2}}+F^1_{\alpha\beta}U_\beta^{n+\frac{1}{2}}+F^2_{\alpha\beta}V_\beta^{n+\frac{1}{2}}-\Pi_\alpha^n) & (3.8)
\end{cases}$$

在上列各式中，$\tilde{M}_{\alpha\beta}$定义如下

$$\tilde{M}_{\alpha\beta}=eM_{\alpha\beta}+(1-e)M_{\alpha\beta} \tag{3.9}$$

其中　e是块参数，在0和1之间取值。

四、数值计算过程及几个例子

至今，我们已把[1]中提出的数学模型用Galekin法加以离散化，并用二步显含有限元法求解，得出二步方程组(3.1)～(3.8)。以下，我们再把整个数值计算过程说明如下：

（1）　规定边界条件及其初值；

（2）　由边界条件(1.10)～(1.11)以及初值θ_0，U_0，V_0，解非线性方程(2.11)式，然后把所得出的结果代入(1.6)式，从而求得波数K；

（3）　由θ，K及初值A_0，U_0，V_0，$W_0(H_0)$，通过(2.12)～(2.26)各式，可求得(3.1)～(3.8)各式的系数；

（4）　求解二步显含有限元方程(3.1)～(3.8)；

（5）　加上时间增量Δt，然后转向(2)

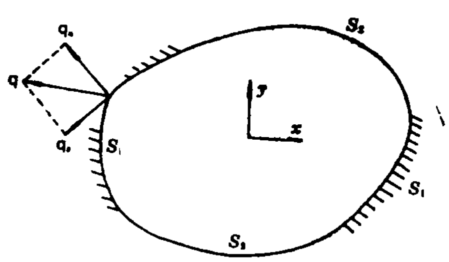

图1　边界条件定义图

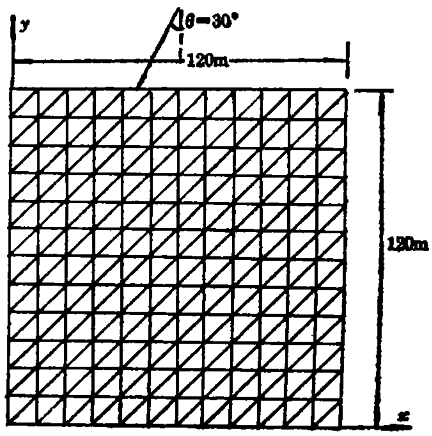

图2　有限元网格图

为使数据输入简单起见，本文假定底面是个倾面，海湾是个矩形（见图2），海水从上面入口进入。初值、边界条件以及有关的物理常数分别如表1～3所示。

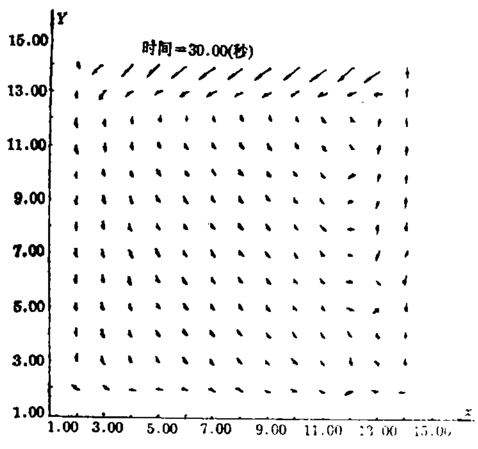

图　3

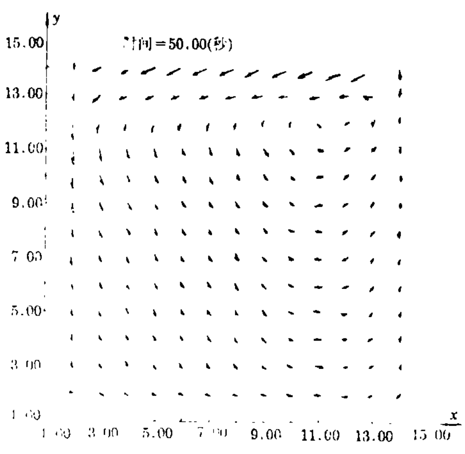

图　4

本例采用三角形单元，共包括169个节点及288个单元。图2～5分别表示30秒，50秒，70秒及90秒后水流速度场。图4～5的右上角明显地出现涡旋。

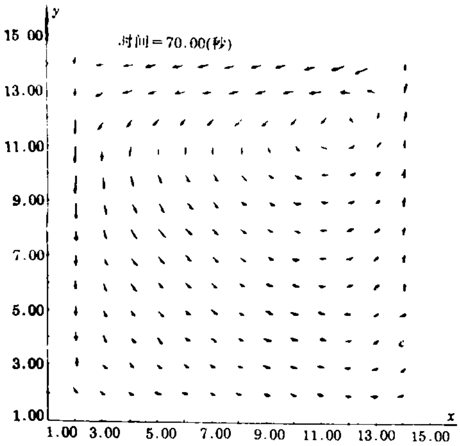

图 5

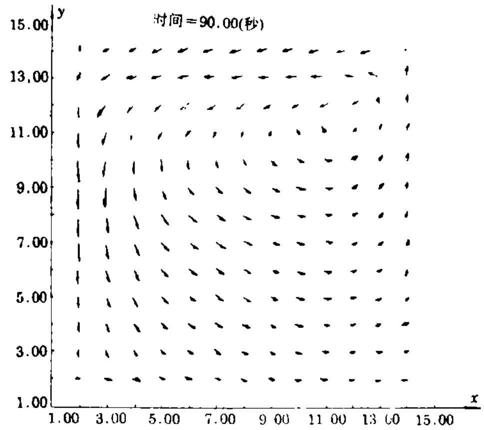

图 6

表1 边值条件

底线1	$u=0$
底线2	$v=0$
底线3	$u=0$
外 海	$H_0=2m$

表2 初值

$A_0=0,\ U_0=0,\ V_0=0$
$H(x,y,0)=H_0\exp(y-y_0)$
$\theta_0=30°$

表3 物理常数

$T=8$sec
$f=0.02$
$s=0.025$
$e=0.5$
$\Delta t=0.5$sec

五、结 论

二步显含有限元法已经成功地解决了文[1]提出的非定常沿岸波流相互作用的数学模型。

特别值得指出的是：通过时间间隔的适当选择，其基础方程组中所包含的连续性方程、动量方程、能量方程、色散方程以折射方程能够被联立求解，这就修正了文献[3]中关于所有波、流及运动方程几乎不可能同时求解的结论。

关于这个模型的实际应用，还期待有关单位的合作研究。

致谢 本计算工作是在中国科学院上海分院测试计算中心的Burroughs-6933型计算机上进行的，曾得到大量的计算机时，在此表示衷心的感谢！

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Finite Element Analysis for the Unsteady Nearshore Circulation Due to Wave-Current Interaction (Ⅱ) Two-Step Explicit Finite Element Method

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Abstract

In this paper, the two-step explicit finite element analysis for the numerical model of the unsteady nearshore circulation proposed in Part (Ⅰ) and its realization of Fortran program are presented.

A circulation has been clearly shown in the calcuated wavecurrent velocity field, and it is in good agreement with observations.

key words finite element method, unsteady circulation, circulation