偏心圆柱薄壳的分析*

常 明 常琍琍

（太原重型机械学院，1992年11月30日收到）

摘 要

本文推导了偏心圆柱薄壳小挠度时的近似方程，并利用解析方法求解了该方程，得出了偏心圆柱薄壳的应力、位移与偏心距之间的关系．

关键词 偏心圆柱薄壳 小挠度 近似方程

一、引 言

薄壳、薄板在许多行业中有着广泛的应用，而圆柱薄壳由于形状特殊、性能稳定，便于分析计算又倍受重视．对于同心圆柱薄壳，其线性理论已趋于成熟．但在实际应用中往往见到偏心圆柱薄壳，而偏心圆柱薄壳的理论分析迄今尚未见到．本文详细地分析了偏心圆柱薄壳，推导出了它的近似方程，并采用解析方法得出了它的解析解，从而给出了它的应力、位移与偏心距的关系．

二、基本方程及解

偏心圆柱薄壳横截面如图1所示．A为外圆圆心，r_1为外圆半径，B为内圆圆心，r_2为内圆半径，O为AB中点，e为偏心距，柱高L，上、下边界固定，受均布内压p．

在我们所考虑的范围内

$$e<0.1(r_1-r_2)\sim0.2(r_1-r_2) \tag{2.1}$$

$$r_1-r_2\ll r_1,\ r_2 \tag{2.2}$$

外圆方程

$$r_w^2+\frac{e^2}{4}+er_w\cos\theta=r_1^2 \tag{2.3}$$

内圆方程

$$r_n^2+\frac{e^2}{4}-er_n\cos\theta=r_2^2 \tag{2.4}$$

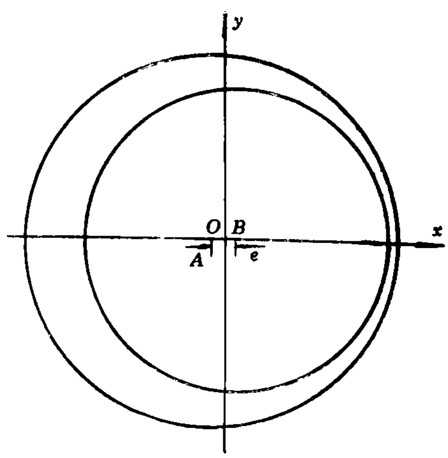

图 1

* 樊大钧推荐．

(2.3)－(2.4)

$$r_w^2-r_n^2+e\cos\theta(r_w+r_n)=r_1^2-r_2^2$$

$$(r_w+r_n)(r_w-r_n)+e\cos\theta(r_w+r_n)=(r_1+r_2)(r_1-r_2) \tag{2.5}$$

记 $h=r_w-r_n$，$h_c=r_1-r_2$ (2.6)

则 h 为偏心圆柱薄壳的厚度，h_c 为无偏心时的厚度。

注意到(2.1)、(2.2)，可得

$$r_w+r_n=r_1+r_2 \tag{2.7}$$

将(2.6)、(2.7)代入(2.5)化简可得

$$h=h_0+h_1e \tag{2.8}$$

式中 $h_0=h_c$

$$h_1=-\cos\theta$$

经过以上分析，偏心圆柱薄壳可视为中面半径 $R=\frac{r_1+r_2}{2}$，厚度 $h=h_0+h_1e$ 的变厚度圆柱薄壳。

如图2所示，以 c 点为原点，以母线为 α 线，以圆周线为 β 线，以法线为 γ 线建立正交曲线坐标系。图2所示面为垂直方向的对称面。

根据壳体理论可得如下方程组。

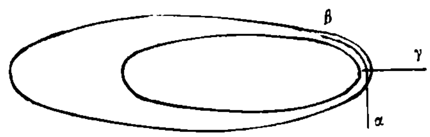

图 2

平衡方程

$$\frac{\partial N_1}{\partial\alpha}+\frac{\partial N_{12}}{\partial\beta}=0 \tag{2.9}$$

$$\frac{\partial N_2}{\partial\beta}+\frac{\partial N_{12}}{\partial\alpha}=0 \tag{2.10}$$

$$-\frac{N_2}{R}+2\frac{\partial^2M_{12}}{\partial\alpha\partial\beta}+\frac{\partial^2M_1}{\partial\alpha^2}+\frac{\partial^2M_2}{\partial\beta^2}+p=0 \tag{2.11}$$

物理方程

$$N_1=\frac{Eh}{1-\mu^2}\left[\frac{\partial u}{\partial\alpha}+\mu\left(\frac{\partial v}{\partial\beta}+\frac{w}{R}\right)\right] \tag{2.12}$$

$$N_2=\frac{Eh}{1-\mu^2}\left[\left(\frac{\partial v}{\partial\beta}+\frac{w}{R}\right)+\mu\frac{\partial u}{\partial\alpha}\right] \tag{2.13}$$

$$N_{12}=\frac{Eh}{2(1+\mu)}\left(\frac{\partial u}{\partial\beta}+\frac{\partial v}{\partial\alpha}\right) \tag{2.14}$$

$$M_1=-D\left[\frac{\partial^2w}{\partial\alpha^2}+\mu\frac{\partial^2w}{\partial\beta^2}\right] \tag{2.15}$$

$$M_2=-D\left[\frac{\partial^2w}{\partial\beta^2}+\mu\frac{\partial^2w}{\partial\alpha^2}\right] \tag{2.16}$$

$$M_{12}=-D(1-\mu)\frac{\partial^2w}{\partial\alpha\partial\beta} \tag{2.17}$$

式中 E 为弹性模量，μ 为泊松比

$$D=\frac{Eh^3}{12(1-\mu^2)}$$

因为 $h-h_0=-e\cos\theta$ 是个小量，可设

$$(h-h_0)^2=0 \tag{2.18}$$

$$(h-h_0)^3=0 \tag{2.19}$$

由(2.18)得

$$h^2=-h_0^2+2h_0h \tag{2.20}$$

由(2.19)得

$$h^3=-3h_0^2h+3h_0h^2+h_0^3 \tag{2.21}$$

将(2.8)、(2.20)代入(2.21)整理得

$$h^3=h_0^3+3h_0^2h_1e,\quad D=D_0+D_1e \tag{2.22}$$

式中 $D_0=\dfrac{Eh_0^3}{12(1-\mu^2)}$

$D_1=\dfrac{3Eh_0^2h_1}{12(1-\mu^2)}$

设 $N_1=N_{10}+N_{11}e,\ N_2=N_{20}+N_{21}e$

$N_{12}=N_{120}+N_{121}e$

$M_1=M_{10}+M_{11}e,\ M_2=M_{20}+M_{21}e$

$M_{12}=M_{120}+M_{121}e$

$u=u_0+u_1e,\ v=v_0+v_1e$

$w=w_0+w_1e$

将(2.8)、(2.22)以及以上诸式代入(2.9)～(2.17)，比较 e 的零次项、一次项得

$$\frac{\partial N_{10}}{\partial\alpha}+\frac{\partial N_{120}}{\partial\beta}=0 \tag{2.23}$$

$$\frac{\partial N_{20}}{\partial\beta}+\frac{\partial N_{120}}{\partial\alpha}=0 \tag{2.24}$$

$$-\frac{N_{20}}{R}+2\frac{\partial^2\mu_{120}}{\partial\alpha\partial\beta}+\frac{\partial^2M_{10}}{\partial\alpha^2}+\frac{\partial^2M_{20}}{\partial\beta^2}+p=0 \tag{2.25}$$

$$N_{10}=\frac{Eh_0}{1-\mu^2}\left[\frac{\partial u_0}{\partial\alpha}+\mu\left(\frac{\partial v_0}{\partial\beta}+\frac{w_0}{R}\right)\right] \tag{2.26}$$

$$N_{20}=\frac{Eh_0}{1-\mu^2}\left[\left(\frac{\partial v_0}{\partial\beta}+\frac{w_0}{R}\right)+\mu\frac{\partial u_0}{\partial\alpha}\right] \tag{2.27}$$

$$N_{120}=\frac{Eh_0}{2(1+\mu)}\left(\frac{\partial u_0}{\partial\beta}+\frac{\partial v_0}{\partial\alpha}\right) \tag{2.28}$$

$$M_{10}=-D_0\left(\frac{\partial^2w_0}{\partial\alpha^2}+\mu\frac{\partial^2w_0}{\partial\beta^2}\right) \tag{2.29}$$

$$M_{20}=-D_0\left(\frac{\partial^2w_0}{\partial\beta^2}+\mu\frac{\partial^2w_0}{\partial\alpha^2}\right) \tag{2.30}$$

$$M_{120}=-D_0(1-\mu)\frac{\partial^2w_0}{\partial\alpha\partial\beta} \tag{2.31}$$

$$\frac{\partial N_{11}}{\partial\alpha}+\frac{\partial N_{121}}{\partial\beta}=0 \tag{2.32}$$

$$\frac{\partial N_{21}}{\partial \beta}+\frac{\partial N_{121}}{\partial \alpha}=0 \tag{2.33}$$

$$-\frac{N_{21}}{R}+2\frac{\partial^2 M_{121}}{\partial \alpha \partial \beta}+\frac{\partial^2 M_{11}}{\partial \alpha^2}+\frac{\partial^2 M_{21}}{\partial \beta^2}=0 \tag{2.34}$$

$$N_{11}=\frac{Eh_0}{1-\mu^2}\left[\frac{\partial u_1}{\partial \alpha}+\mu\left(\frac{\partial v_1}{\partial \beta}+\frac{w_1}{R}\right)\right] +\frac{Eh_1}{1-\mu^2}\left[\frac{\partial u_0}{\partial \alpha}+\mu\left(\frac{\partial v_0}{\partial \beta}+\frac{w_0}{R}\right)\right] \tag{2.35}$$

$$N_{21}=\frac{Eh_0}{1-\mu^2}\left[\left(\frac{\partial v_1}{\partial \beta}+\frac{w_1}{R}\right)+\mu\frac{\partial u_1}{\partial \alpha}\right] +\frac{Eh_1}{1-\mu^2}\left[\left(\frac{\partial v_0}{\partial \beta}+\frac{w_0}{R}\right)+\mu\frac{\partial u_0}{\partial \alpha}\right] \tag{2.36}$$

$$N_{121}=\frac{Eh_0}{2(1+\mu)}\left(\frac{\partial u_1}{\partial \beta}+\frac{\partial v_1}{\partial \alpha}\right) +\frac{Eh_1}{2(1+\mu)}\left(\frac{\partial u_0}{\partial \beta}+\frac{\partial v_0}{\partial \alpha}\right) \tag{2.37}$$

$$M_{11}=-D_0\left(\frac{\partial^2 w_1}{\partial \alpha^2}+\mu\frac{\partial^2 w_1}{\partial \beta^2}\right) -D_1\left(\frac{\partial^2 w_0}{\partial \alpha^2}+\mu\frac{\partial^2 w_0}{\partial \beta^2}\right) \tag{2.38}$$

$$M_{21}=-D_0\left(\frac{\partial^2 w_1}{\partial \beta^2}+\mu\frac{\partial^2 w_1}{\partial \alpha^2}\right) -D_1\left(\frac{\partial^2 w_0}{\partial \beta^2}+\mu\frac{\partial^2 w_0}{\partial \alpha^2}\right) \tag{2.39}$$

$$M_{121}=-D_0(1-\mu)\frac{\partial^2 w_1}{\partial \alpha \partial \beta}-D_1(1-\mu)\frac{\partial^2 w_0}{\partial \alpha \partial \beta} \tag{2.40}$$

边界条件

$\alpha=\pm L$处

$$w_0=0,\quad \frac{dw_0}{d\alpha}=0 \tag{2.41}$$

$$v_0=0 \tag{2.42}$$

$$\frac{du_0}{d\alpha}=0 \tag{2.43}$$

$$w_1=0,\quad \frac{dw_1}{d\alpha}=0 \tag{2.44}$$

$$v_1=0 \tag{2.45}$$

$$\frac{du_1}{d\alpha}=0 \tag{2.46}$$

方程(2.23)～(2.31)在边界条件(2.41)～(2.43)下等价于两端固定的无偏心圆柱薄壳，易得

$$u_0=-\frac{\mu}{R}[c_1\sin\lambda\alpha\,\mathrm{ch}\lambda\alpha+c_2\cos\lambda\alpha\,\mathrm{sh}\lambda\alpha] \tag{2.47}$$

$$v_0=0 \tag{2.48}$$

$$w_0=c_3\sin\lambda\alpha\,\mathrm{sh}\lambda\alpha+c_4\cos\lambda\alpha\,\mathrm{ch}\lambda\alpha+\frac{pR^2}{Eh} \tag{2.49}$$

式中 $\lambda=\left[\dfrac{Eh_0}{4R^2D}\right]^{1/4}$

$$c_1=-\frac{4\sin\lambda L\,\mathrm{ch}\lambda L}{\sin 2\lambda L+\mathrm{sh}2\lambda L}\frac{pR^2}{Eh_0\lambda}$$

$$c_2=-\frac{4\cos\lambda L\,\mathrm{sh}\lambda L}{\sin 2\lambda L+\mathrm{sh}2\lambda L}\frac{pR^2}{Eh_0\lambda}$$

$$c_3=-\frac{2(\sin\lambda L\,\mathrm{ch}\lambda L-\cos\lambda L\,\mathrm{sh}\lambda L)}{\sin 2\lambda L+\mathrm{sh}2\lambda L}\frac{pR^2}{Eh_0}$$

$$c_4=-\frac{2(\sin\lambda L\,\mathrm{ch}\lambda L+\cos\lambda L\,\mathrm{sh}\lambda L)}{\sin 2\lambda L+\mathrm{sh}2\lambda L}\frac{pR^2}{Eh_0}$$

将(2.35)～(2.40)、(2.47)～(2.49)代入(2.32)～(2.34)，忽略h/R级小量，整理得

$$\left(\frac{\partial\alpha^2}{\partial\alpha^2}+\frac{1-\mu}{2}\frac{\partial^2}{\partial\beta^2}\right)u_1+\frac{1+\mu}{2}\frac{\partial^2v_1}{\partial\alpha\partial\beta}+\frac{\mu}{R}\frac{\partial w_1}{\partial\alpha}$$
$$=(d_1\sin\lambda\alpha\,\mathrm{ch}\lambda\alpha+d_2\cos\lambda\alpha\,\mathrm{sh}\lambda\alpha)\cos\theta \tag{2.50}$$

$$\frac{1+\mu}{2}\frac{\partial^2u_1}{\partial\alpha\partial\beta}+\left(\frac{\partial^2}{\alpha\beta^2}+\frac{1-\mu}{2}\frac{\partial^2}{\partial\alpha^2}\right)v_1+\frac{1}{R}\frac{\partial w_1}{\partial\beta}$$
$$=(d_3\sin\lambda\alpha\,\mathrm{sh}\lambda\alpha+d_4\cos\lambda\alpha\,\mathrm{ch}\lambda\alpha)\sin\theta \tag{2.51}$$

$$\frac{\mu}{R}\frac{\partial u_1}{\partial\alpha}+\frac{1}{R}\frac{\partial v_1}{\partial\beta}+\frac{w_1}{R^2}+\frac{h_0^2}{12}\nabla^2\nabla^2w_1$$
$$=(d_5\sin\lambda\alpha\,\mathrm{sh}\lambda\alpha+d_6\cos\lambda\alpha\,\mathrm{ch}\lambda\alpha+d_7)\cos\theta \tag{2.52}$$

式中 $d_1=\dfrac{\mu}{Rh_0}[2\lambda^2c_2+\lambda(c_3-c_4)]$

$$d_2=\frac{\mu}{Rh_0}[-2\lambda^2c_1+\lambda(c_3+c_4)]$$

$$d_3=\frac{1}{Rh_0}\left[\frac{c_4}{R}-\frac{\mu^2}{R}\lambda(c_1+c_2)\right]$$

$$d_4=\frac{1}{Rh_0}\left[\frac{c_3}{R}-\frac{\mu^2}{R}\lambda(c_1-c_2)\right]$$

$$d_5=\left(-\frac{1}{R^2h_0}+\frac{3(1-\mu^2)}{R^2h_0}\right)c_3+\frac{\mu^2}{R^2h_0}\lambda(c_1-c_2)$$

$$d_6=\left(-\frac{1}{R^2h_0}+\frac{3(1-\mu^2)}{R^2h_0}\right)c_4+\frac{\mu^2}{R^2h_0}\lambda(c_1+c_2)$$

$$d_7=-\frac{p}{Eh_0^2}$$

因为u_1，w_1是θ的偶函数，v_1是θ的奇函数，并注意到(2.50)～(2.52)的特点，可设

$$u_1=u_{10}+u_{12}\cos\theta \tag{2.53}$$

$$v_1=v_{11}\sin\theta \tag{2.54}$$

$$w_1=w_{10}+w_{12}\cos\theta \tag{2.55}$$

将(2.53)～(2.55)代入(2.50)～(2.52)，整理并比较$\sin\theta$、$\cos\theta$的系数及不含θ的项可得：

$$u_{10}'' + \frac{\mu}{R} w_{10}' = 0 \tag{2.56}$$

$$\frac{\mu}{R} u_{10}' + \frac{1}{R^2} w_{10} + \frac{h_0^2}{12} w_{10}'''' = 0 \tag{2.57}$$

$$u_{12}'' - \frac{1-\mu}{2R^2} u_{12} + \frac{1+\mu}{2R} v_{11}' + \frac{\mu}{R} w_{12}' = d_1 \sin\lambda\alpha \operatorname{ch}\lambda\alpha + d_2 \cos\lambda\alpha \operatorname{sh}\lambda\alpha \tag{2.58}$$

$$-\frac{1+\mu}{2R} u_{12}' - \frac{1}{R^2} v_{11} + \frac{1-\mu}{2} v_{11}'' - \frac{1}{R^2} w_{12} = d_3 \sin\lambda\alpha \operatorname{ch}\lambda\alpha + d_4 \cos\lambda\alpha \operatorname{sh}\lambda\alpha \tag{2.59}$$

$$\frac{\mu}{R} u_{12}' + \frac{1}{R^2} v_{11} + \frac{1}{R^2} w_{12} + \frac{h_0^2}{12} w_{12}'''' = d_5 \sin\lambda\alpha \operatorname{sh}\lambda\alpha + d_6 \cos\lambda\alpha \operatorname{ch}\lambda\alpha + d_7 \tag{2.60}$$

式中 $(\ \ )' = \frac{d}{d\alpha}(\ \ )$

边界条件化为

$\alpha = \pm L$处 $w_{10} = 0$, $\frac{dw_{10}}{d\alpha} = 0$ (2.61)

$$\frac{du_{10}}{d\alpha} = 0 \tag{2.62}$$

$$w_{12} = 0,\ \frac{dw_{12}}{d\alpha} = 0 \tag{2.63}$$

$$v_{11} = 0 \tag{2.64}$$

$$\frac{du_{12}}{d\alpha} = 0 \tag{2.65}$$

在条件(2.61)、(2.62)下求解(2.56)、(2.57)得

$$u_{10} = 0,\ w_{10} = 0 \tag{2.66}$$

联立(2.58)～(2.60)，消去u_{12}，忽略h/R级小量，可得

$$w_{12}'''' + \frac{1-\mu}{1+\mu} \frac{12}{R^2 h_0^2} (w_{12} + v_{11}) + \frac{12}{h_0^2} \frac{\mu(1-\mu)}{1+\mu} v_{11}'' = e_1 \sin\lambda\alpha \operatorname{sh}\lambda\alpha + e_2 \cos\lambda\alpha \operatorname{ch}\lambda\alpha + e_3 \tag{2.67}$$

$$v_{11}'''' = e_4 \sin\lambda\alpha \operatorname{sh}\lambda\alpha + e_5 \cos\lambda\alpha \operatorname{ch}\lambda\alpha \tag{2.68}$$

式中

$$e_1 = \frac{12}{h_0^2} \left(d_5 + \frac{2\mu}{1+\mu} d_3 \right)$$

$$e_2 = \frac{12}{h_0^2} \left(d_6 + \frac{2\mu}{1+\mu} d_4 \right)$$

$$e_3 = \frac{12}{h_0^2} d_7$$

$$e_4 = \frac{1+\mu}{1-\mu} \frac{1}{R} (d_1 - d_2)\lambda - \frac{4}{1-\mu} d_4 \lambda^2 + \frac{1}{R^2} d_3$$

$$e_5 = \frac{1+\mu}{1-\mu} \frac{1}{R} (d_1 + d_2)\lambda + \frac{4}{1-\mu} d_3 \lambda^3 + \frac{1}{R^2} d_4$$

由(2.59)、(2.63)～(2.65)得

$$\alpha=\pm L\text{处，}\ v''_{11}=\frac{2}{1-\mu}[d_3\sin\lambda L\,\mathrm{sh}\lambda L+d_4\cos\lambda L\,\mathrm{ch}\lambda L] \tag{2.69}$$

由(2.64)、(2.68)、(2.69)得

$$v_{11}=c\alpha^2+d-\frac{e_4}{4\lambda^4}\sin\lambda\alpha\,\mathrm{sh}\lambda\alpha-\frac{e_5}{4\lambda^4}\cos\lambda\alpha\,\mathrm{ch}\lambda\alpha \tag{2.70}$$

式中 $$c=-\frac{e_5}{4\lambda^2}\sin\lambda L\,\mathrm{sh}\lambda L+\frac{e_4}{4\lambda^2}\cos\lambda L\,\mathrm{ch}\lambda L$$

$$d=\frac{e_4}{4\lambda^4}\sin\lambda L\,\mathrm{sh}\lambda L+\frac{e_5}{4\lambda^4}\cos\lambda L\,\mathrm{ch}\lambda L-cL^2$$

将(2.70)代入(2.67)，并在条件(2.63)下求解得

$$\begin{aligned}w_{12}=&f_1\sin\lambda_1\alpha\,\mathrm{sh}\lambda_1\alpha+f_2\cos\lambda_1\alpha\,\mathrm{ch}\lambda_1\alpha\\&+f_3\sin\lambda\alpha\,\mathrm{sh}\lambda\alpha+f_4\cos\lambda\alpha\,\mathrm{ch}\lambda\alpha\\&+f_5\alpha^2+f_6\end{aligned} \tag{2.71}$$

式中 $$\lambda_1=\left[\frac{1-\mu}{1+\mu}\frac{3}{R^2h_0^2}\right]^{1/4}$$

$$f_3=\frac{1}{4(\lambda_1^4-\lambda^4)}\left[e_1+\frac{1-\mu}{1+\mu}\frac{3}{R^2h_0^2\lambda^4}e_4-\frac{6}{h_0^2}\frac{\mu(1-\mu)}{1+\mu}\frac{1}{\lambda^2}e_5\right]$$

$$f_4=\frac{1}{4(\lambda_1^4-\lambda^4)}\left[e_2-\frac{1-\mu}{1+\mu}\frac{3}{R^2h_0^2\lambda^4}e_5+\frac{6}{h_0^2}\frac{\mu(1-\mu)}{1+\mu}\frac{1}{\lambda^2}e_4\right]$$

$$f_5=-c$$

$$f_6=\frac{1+\mu}{1-\mu}\frac{R^2h_0^2}{12}e_3-2\mu R^2c-d$$

$$f_1=\frac{\Delta_1}{\Delta},\quad f_2=\frac{\Delta_2}{\Delta}$$

$$\Delta=-\lambda(\sin_2\lambda_1L+\mathrm{sh}_2\lambda_1L)$$

$$\Delta_1=\lambda[-\sin\lambda_1L\,\mathrm{ch}\lambda_1L+\cos\lambda_1L\,\mathrm{sh}\lambda_1L]g_1-\cos\lambda_1L\,\mathrm{ch}\lambda_1Lg_2$$

$$\Delta_2=-\lambda[\cos\lambda_1L\,\mathrm{sh}\lambda_1L+\sin\lambda_1L\,\mathrm{ch}\lambda_1L]g_1+\sin\lambda_1L\,\mathrm{sh}\lambda_1Lg_2$$

$$g_1=-f_3\sin\lambda L\,\mathrm{sh}\lambda L-f_4\cos\lambda L\,\mathrm{ch}\lambda L-f_5L^2-f_6$$

$$\begin{aligned}g_2=&-f_3[\lambda\cos\lambda L\,\mathrm{sh}\lambda L+\lambda\sin\lambda L\,\mathrm{ch}\lambda L]\\&-f_4[-\lambda\sin\lambda L\,\mathrm{ch}\lambda L+\lambda\cos\lambda L\,\mathrm{sh}\lambda L]\\&-2f_5L\end{aligned}$$

将(2.70)、(2.71)代入(2.60)整理得

$$\begin{aligned}u'_{12}=&s_1\sin\lambda_1\alpha\,\mathrm{sh}\lambda_1\alpha+s_2\cos\lambda_1\alpha\,\mathrm{ch}\lambda_1\alpha+s_3\sin\lambda\alpha\,\mathrm{sh}\lambda\alpha\\&+s_4\cos\lambda\alpha\,\mathrm{ch}\lambda\alpha+s_5\end{aligned} \tag{2.72}$$

式中 $$s_1=\frac{Rh_0^2f_1}{48\mu\lambda_1^4}$$

$$s_2=\frac{Rh_0^2f_2}{48\mu\lambda_1^4}$$

$$s_3=-\frac{1}{\mu R}\left(f_3-\frac{e_4}{4\lambda^4}\right)+\frac{Rh_0^2f_3}{48\mu\lambda^4}+\frac{R}{\mu}d_5$$

$$s_4=-\frac{1}{\mu R}\left(f_4-\frac{e_5}{4\lambda^4}\right)+\frac{Rh_0^2f_4}{48\mu\lambda^4}+\frac{R}{\mu}d_6$$

$$s_5=-\frac{1}{\mu R}\left[\frac{1+\mu}{1-\mu}\frac{R^2h_0^2}{12}e_3-2\mu R^2c\right]+\frac{R}{\mu}d_7$$

由(2.72)、并注意到$\alpha=0$处$u_{12}=0$可得

$$\begin{aligned}u_{12}=&t_1\sin\lambda_1\alpha\,\mathrm{ch}\lambda_1\alpha+t_2\cos\lambda_1\alpha\,\mathrm{sh}\lambda_1\alpha\\&+t_3\sin\lambda\alpha\,\mathrm{ch}\lambda\alpha+t_4\cos\lambda\alpha\,\mathrm{sh}\lambda\alpha+s_5\alpha\end{aligned}\tag{2.73}$$

式中

$$t_1=\frac{s_1+s_2}{2}\frac{1}{\lambda_1}$$

$$t_2=\frac{s_2-s_1}{2}\frac{1}{\lambda_1}$$

$$t_3=\frac{s_3+s_4}{2}\frac{1}{\lambda}$$

$$t_4=\frac{s_4-s_3}{2}\frac{1}{\lambda}$$

至此，我们得出了位移

$$\begin{aligned}u=&-\frac{\mu}{R}[c_1\sin\lambda\alpha\,\mathrm{ch}\lambda\alpha+c_2\cos\lambda\alpha\,\mathrm{sh}\lambda\alpha]\\&+[t_1\sin\lambda_1\alpha\,\mathrm{ch}\lambda_1\alpha+t_2\cos\lambda_1\alpha\,\mathrm{sh}\lambda_1\alpha+t_3\sin\lambda\alpha\,\mathrm{ch}\lambda\alpha\\&+t_4\cos\lambda\alpha\,\mathrm{sh}\lambda\alpha+s_5\alpha]\cos\theta\cdot e\end{aligned}\tag{2.74}$$

$$v=\left[c\alpha^2+d-\frac{e_4}{4\lambda^4}\sin\lambda\alpha\,\mathrm{sh}\alpha-\frac{e_5}{4\lambda^4}\cos\lambda\alpha\,\mathrm{ch}\lambda\alpha\right]\sin\theta\cdot e\tag{2.75}$$

$$\begin{aligned}w=&c_3\sin\lambda\alpha\,\mathrm{sh}\lambda\alpha+c_4\cos\lambda\alpha\,\mathrm{ch}\lambda\alpha+\frac{pR^2}{Eh_0}\\&+[f_1\sin\lambda_1\alpha\,\mathrm{sh}\lambda_1\alpha+f_2\cos\lambda_1\alpha\,\mathrm{ch}\lambda_1\alpha+f_3\sin\lambda\alpha\,\mathrm{sh}\lambda\alpha\\&+f_4\cos\lambda\alpha\,\mathrm{ch}\lambda\alpha+f_5\alpha^2+f_6]\cos\theta\cdot e\end{aligned}\tag{2.76}$$

将(2.74)～(2.76)代入(2.12)～(2.17)即可得到内力和弯矩，限于篇幅，这里不再列出。

Analysis of the Eccentric Cylindrical Thin Shell

Chang Ming　　Chang Li-li

(Taiyuan Heavy Machinery Institute, Taiyuan)

Abstract

This paper gives the approximate equations of the eccentric cylindrical thin shell under small deflections. By means of the analytical method, we solved the equations. Thus the relations between the stress, the displacement and the eccentricity of the eccentric cylindrical thin shell are obtained.

Key words eccentric cylindrical thin shell, small deflections, approximate equations