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二维振荡流动中污染云团的收缩*

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摘　　要

当垂向扩散时间尺度与流动的周期相当时，在转流过程中，污染云团将会出现收缩[3]。这时水平剪切分散导数将会出现负值奇性。本文根据作者两维延迟扩散方程[7]：

$$\frac{\partial\overline{C}}{\partial t}+\bar{u}\frac{\partial\overline{C}}{\partial x}+\bar{v}\frac{\partial\overline{C}}{\partial y}=\overline{K}_{xx}\frac{\partial^2\overline{C}}{\partial x^2}+\overline{K}_{yy}\frac{\partial^2\overline{C}}{\partial y^2}+\int_0^\infty\left[\frac{\partial}{\partial\tau}D_{xx}\frac{\partial^2}{\partial x^2}\right.$$

$$\left.+\frac{\partial}{\partial\tau}(D_{xy}+D_{yx})\frac{\partial^2}{\partial x\partial y}+\frac{\partial}{\partial\tau}D_{yy}\frac{\partial^2}{\partial y^2}\right]\overline{C}(x-X,y-Y,t-\tau)d\tau$$

其中 $\bar{u}(t),\bar{v}(t)$为深度平均水平速度。导出$X(t,\tau),Y(t,\tau)$坐标位移，$D_{ij}(t,\tau)$为剪切扩散导数的方程。一般情况下，$\frac{\partial D_{ij}(t,\tau)}{\partial\tau}$是正的。不存在奇异性。但在转流的初期。记忆函数$D_{ij}(t,\tau)$就有可能是负的。本文给出了$D_{ij}$和$X$、$Y$的解析表示式。

关键词　剪切分散理论　记忆特性　振荡流动　污染云团收缩

一、引　　言

剪切分散是污染物扩散的主要原因之一，一般剪切分散导数要比湍流扩散导数大2～3个量级，比分子扩散那就更大了。然而正如Taylor[1]所指出的这种扩散率只有在污染颗粒经历断面的所有速度才能达到。在周期性的流动中，这一条件往往不能满足。比如周期太短，水道太短或断面太大等情况下，剪切分散效应就不能发挥最大作用。这样的例子有，在小动脉中。这种剪切分散效应是很大的。而在大的动脉中，剪切分散效应就受到限制[3]。同理在窄的和浅的河口和在宽而深的河口情况下，剪切分散就有很大差别。

Chatwin[3](1975) 指出在周期性的流动中，前半周期，污染云团澎胀得非常快，而在后半周期只稍有澎大。在这种条件下，剪切分散导数会出现负值（奇性）。即在这种周期性的流动中，污染云团会发生收缩或示踪质的聚合。而根据一般变导数扩散方程往往不能解释这一象现[2]。Smith根据含对流记忆项的延迟扩散方程较好地处理了这种情况下纵向分散问题[5]。

在宽阔的水域、如河口，海湾和浅海区域，一般采用二维分散的理论来处理污染物的扩散问题。但由于潮汐和波作用影响，以及水深h不满足$h<\sqrt{DT}$的条件、其中D是湍流扩散

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导数（层流时是分子扩散导数）．T是周期，因而也会发生Chatwin所指出的那种污染云团在垂向的收缩现象

本文根据作者提出的新的解析分析式[7]：

$$\left.\begin{aligned}&C-\bar{C}=\sum_{m}\int_0^{\infty} l^m_{ij}(z,t,\tau)\frac{\partial^m}{\partial x^i\partial y^j}\bar{C}(x-X,y-Y,t-\tau)d\tau\\&i+j=m,\ i,j\geqslant 0\end{aligned}\right\}$$

采用二维延迟扩散方程从理论上证明了 $\frac{\partial D_{ij}}{\partial\tau}$ 在 τ 很大时会出现负值，从而可以解释在周期性流动中所出现污染云团的收缩现象．

二、水平与垂向分散方程

对于二维非定常污染物扩散，我们一般要寻求下列方程的解：

$$\frac{\partial C}{\partial t}+u\frac{\partial C}{\partial x}+v\frac{\partial C}{\partial y}=K_{xx}\frac{\partial^2 C}{\partial x^2}+K_{yy}\frac{\partial^2 C}{\partial y^2}+K_{zz}\frac{\partial^2 C}{\partial z^2}-q \tag{2.1}$$

其中 $u=u(z,t)$，$v=v(z,t)$，q 是源项．为了使我们的解能适用于分析早期的分散过程，根据刘宇陆[7]中的建议，采用(1.1)式来近似，将(1.1)式代入(2.1)式并取深度平均，则得到如下表达式：

$$\begin{aligned}&\frac{\partial\bar{C}}{\partial t}+\bar{u}\frac{\partial\bar{C}}{\partial x}+\bar{v}\frac{\partial\bar{C}}{\partial y}-\bar{K}_{xx}\frac{\partial^2\bar{C}}{\partial x^2}-\bar{K}_{yy}\frac{\partial^2\bar{C}}{\partial y^2}+\sum_{m=1}^{\infty}\int_0^{\infty}\Big[l^m_{ij}(u-\bar{u})\frac{\partial^{m+1}}{\partial x^{i+1}\partial y^j}\\&+l^m_{ij}(v-\bar{v})\frac{\partial^{m+1}}{\partial x^i\partial y^{j+1}}-l^m_{ij}(K_{xx}-\bar{K}_{xx})\frac{\partial^{m+2}}{\partial x^{i+2}\partial y^j}-l^m_{ij}(K_{yy}-\bar{K}_{yy})\frac{\partial^{m+2}}{\partial x^i\partial y^{j+2}}\Big]\\&\cdot\bar{C}(x-X(\tau,t),\ y-Y(\tau,t),\ t-\tau)d\tau=\bar{q}(x,y,t)\end{aligned} \tag{2.2}$$

其中 $l^m_{ij}=l^m_{ij}(z,t,\tau)$，$\bar{u}=\bar{u}(t)$，$\bar{v}=\bar{v}(t)$ 为深度平均速度，X，Y 为污染云团的中心位移．将(1.1)式代入(2.1)式，并利用(2.2)式，而可得到l^m_{ij}满足的方程（精确到二阶）：

$$\left.\begin{aligned}&\frac{\partial}{\partial t}l^1_{1,0}+\frac{\partial}{\partial\tau}l^1_{1,0}-\frac{\partial}{\partial z}K_{zz}\frac{\partial}{\partial z}l^1_{1,0}=0\\&K\mathbf{n}\cdot\nabla l^1_{1,0}|_{\partial A}=0,\ l^1_{1,0}|_{\tau=0}=\bar{u}-u\end{aligned}\right\} \tag{2.3}$$

$$\left.\begin{aligned}&\frac{\partial}{\partial t}l^1_{0,1}+\frac{\partial}{\partial\tau}l^1_{0,1}-\frac{\partial}{\partial z}K_{zz}\frac{\partial}{\partial z}l^1_{0,1}=0\\&K\mathbf{n}\cdot\nabla l^1_{0,1}|_{\partial A}=0,\ l^1_{0,1}|_{\tau=0}=\bar{v}-v\end{aligned}\right\} \tag{2.4}$$

$$\left.\begin{aligned}&\frac{\partial}{\partial t}l^2_{2,0}+\frac{\partial}{\partial\tau}l^2_{2,0}-\frac{\partial}{\partial z}K_{zz}\frac{\partial}{\partial z}l^2_{2,0}=\Big[\frac{\partial X}{\partial t}+\frac{\partial X}{\partial\tau}\Big]l^1_{1,0}+\overline{l^1_{1,0}u}-l^1_{1,0}u\\&K\mathbf{n}\cdot\nabla l^2_{2,0}|_{\partial A}=0,\ l^2_{2,0}|_{\tau=0}=K_{xx}-\bar{K}_{xx}\end{aligned}\right\} \tag{2.5}$$

$$\left.\begin{aligned}&\frac{\partial}{\partial t}l^2_{0,2}+\frac{\partial}{\partial\tau}l^2_{0,2}-\frac{\partial}{\partial z}K_{zz}\frac{\partial}{\partial z}l^2_{0,2}=\Big[\frac{\partial Y}{\partial t}+\frac{\partial Y}{\partial\tau}\Big]l^1_{0,1}+\overline{l^1_{0,1}v}-l^1_{0,1}v\\&K\mathbf{n}\cdot\nabla l^2_{0,2}|_{\partial A}=0,\ l^2_{0,2}|_{\tau=0}=K_{yy}-\bar{K}_{yy}\end{aligned}\right\} \tag{2.6}$$

$$\left.\begin{aligned}&\frac{\partial}{\partial t}l^2_{1,1}+\frac{\partial}{\partial\tau}l^2_{1,1}-\frac{\partial}{\partial z}K_{zz}\frac{\partial}{\partial z}l^2_{1,1}=\Big[\frac{\partial X}{\partial t}+\frac{\partial X}{\partial\tau}\Big]l^1_{0,1}\\&\quad+\Big[\frac{\partial Y}{\partial t}+\frac{\partial Y}{\partial\tau}\Big]l^1_{1,0}+\overline{l^1_{1,0}v}-l^1_{1,0}v+\overline{l^1_{0,1}u}-l^1_{0,1}u\\&K\mathbf{n}\cdot\nabla l^2_{1,1}|_{\partial A}=0,\ l^2_{1,1}=0\end{aligned}\right\} \tag{2.7}$$

一般地(1.1)式的低阶的近似占有主要贡献[4]。因而取$m=1$而得到：

$$\left.\begin{aligned}\frac{\partial}{\partial\tau}D_{xx}&=\overline{l'_{1,0}(\bar{u}-u)}, & \frac{\partial}{\partial\tau}D_{xy}&=\overline{l'_{1,0}(\bar{v}-v)}\\ \frac{\partial}{\partial\tau}D_{yx}&=\overline{l'_{0,1}(\bar{u}-u)}, & \frac{\partial}{\partial\tau}D_{yy}&=\overline{l'_{0,1}(\bar{v}-v)}\end{aligned}\right\}\quad(2.8)$$

方程(2.2)的低阶近似形式为：

$$\frac{\partial\bar{C}}{\partial t}+\bar{u}\frac{\partial\bar{C}}{\partial x}+\bar{v}\frac{\partial\bar{C}}{\partial y}-\bar{K}_{xx}\frac{\partial^2\bar{C}}{\partial x^2}-\bar{K}_{yy}\frac{\partial^2\bar{C}}{\partial y^2}-\int_0^{\infty}\left[\frac{\partial}{\partial\tau}D_{xx}\frac{\partial^2}{\partial x^2}\right.$$

$$\left.+\left(\frac{\partial}{\partial\tau}D_{xy}+\frac{\partial}{\partial\tau}D_{yx}\right)\frac{\partial^2}{\partial x\partial y}+\frac{\partial D_{yy}}{\partial\tau}\frac{\partial^2}{\partial y^2}\right]\cdot\bar{C}(x-X,y-Y,t-\tau)d\tau$$

$$=\bar{q}(x,y,t)\quad(2.9)$$

三、记忆函数

由(2.8)式定义的D_{ij}通常被称为记忆函数或分散函数，而(2.8)式的解又取决于(2.3)～(2.7)式的解。一般情况下，湍流扩散导数K_{zz}应是z和t的函数，（层流时K_{zz}为常数），假定K_{zz}具有如下形式[5]：

$$K_{zz}=K_z(z)K_t(t)\quad(3.1)$$

则(2.3)和(2.4)的解可以写成如下形式：

$$\left.\begin{aligned}l'_{1,0}&=-\sum_{m=1}^{\infty}u_m(t-\tau)\exp(-\lambda_m I(t,\tau))\chi_m(z)\\ l'_{0,1}&=-\sum_{m=1}^{\infty}v_m(t-\tau)\exp(-\lambda_m I(t,\tau))\chi_m(z)\end{aligned}\right\}\quad(3.2)$$

其中：

$$\left.\begin{aligned}u_m(t)&=\overline{(u-\bar{u})\chi_m}\\ v_m(t)&=\overline{(v-\bar{v})\chi_m}\\ I(t,\tau)&=\int_0^{\tau}K_t(t-\tau')d\tau'\end{aligned}\quad m=1,2,3,\cdots\right\}\quad(3.3)$$

而特征函数χ_m满足：

$$\left.\begin{aligned}&\frac{d}{dz}K_z\frac{d}{dz}\chi_m+\lambda_m\chi_m=0\\ &K_z\mathbf{n}\cdot\nabla\chi_m|_{\partial A}=0\end{aligned}\right\}\quad(3.4)$$

并且有：

$$\chi_0=1,\quad\overline{\chi_m^2}=1$$

请注意χ_0对应于$\lambda_0=0$，此时表示垂向浓度均匀分布。

由(2.8)式可以得到：

$$\left.\begin{aligned}\frac{\partial}{\partial\tau}D_{xx}&=\sum u_m(t)u_m(t-\tau)\chi_m(z)\exp(-\lambda_m I(t,\tau))\\\frac{\partial}{\partial\tau}D_{xy}&=\sum u_m(t)v_m(t-\tau)\chi_m(z)\exp(-\lambda_m I(t,\tau))\\\frac{\partial}{\partial\tau}D_{xy}&=\sum v_m(t)u_m(t-\tau)\chi_m(z)\exp(-\lambda_m I(t,\tau))\\\frac{\partial}{\partial\tau}D_{yy}&=\sum v_m(t)v_m(t-\tau)\chi_m(z)\exp(-\lambda_m I(t,\tau))\end{aligned}\right\}\tag{3.5}$$

一般情况下，当τ很小时，$\frac{\partial}{\partial\tau}D_{xx}$和$\frac{\partial}{\partial\tau}D_{yy}$均为正的．而当周期较短，而$\tau$又很大时，它们侧有可能为负值．为此假定扩散导数$K_{zz}$为常数．并且有

$$\left.\begin{aligned}u_m&=U\alpha_m\sin(\omega t+\theta_{um})\\v_m&=V\beta_m\sin(\omega t+\theta_{vm})\end{aligned}\right\}\tag{3.6}$$

将(3.6)式代入(3.5)式并积分得到：

$$\left.\begin{aligned}D_{xx}(t,\tau)=&\frac{1}{2}u^2\sum_{m=1}^{\infty}\frac{\alpha_m^2\lambda_m}{\omega^2+\lambda_m^2}\Big\{1-\cos2(\omega t+\theta_{um})-\frac{\omega}{\lambda_m}\sin2(\omega t+\theta_{um})\\&-2\exp(-\lambda_m\tau)\sin(\omega t+\theta_{um})[\sin(\omega t-\omega\tau+\theta_{um})\\&-\frac{\omega}{\lambda_m}\cos(\omega t-\omega\tau)+\theta_{um})]\\D_{xy}=&uv\sum_{m=1}^{\infty}\frac{\alpha_m\beta_m\lambda_m}{\omega^2+\lambda_m^2}\Big\{\sin(\omega t+\theta_{um})\sin(\omega t+\theta_{vm})-\frac{\omega}{\lambda_m}\sin(\omega t+\theta_{um})\\&\cdot\cos(\omega t+\theta_{vm})-\exp(-\lambda_m\tau)\sin(\omega t+\theta_{um})[\mathrm{sim}(\omega t-\omega\tau+\theta_{vm})\\&-\frac{\omega}{\lambda_m}\cos(\omega t-\omega\tau+\theta_{vm})]\\D_{yx}=&uv\sum_{m=1}^{\infty}\frac{\alpha_m\beta_m\lambda_m}{\omega^2+\lambda_m^2}\Big\{\sin(\omega t+\theta_{vm})\sin(\omega t+\theta_{um})-\frac{\omega}{\lambda_m}\sin(\omega t+\theta_{vm})\\&\cdot\cos(\omega t+\theta_{um})-\exp(-\lambda_m\tau)\sin(\omega t+\theta_{vm})[\sin(\omega t-\omega\tau+\theta_{um})\\&-\frac{\omega}{\lambda_m}\cos(\omega t-\omega\tau+\theta_{um})\\D_{yy}=&\frac{1}{2}v^2\sum_{m=1}^{\infty}\frac{\beta_m^2\lambda_m}{\omega^2+\lambda_m^2}\Big\{1-\cos2(\omega t+\theta_{vm})+\frac{\omega}{\lambda_m}\sin2(\omega t+\theta_{vm})\\&-2\exp(-\lambda_m\tau)\sin(\omega t+\theta_{vm})[\sin(\omega t-\omega\tau+\theta_{vm})\\&-\frac{\omega}{\lambda_m}\cos(\omega t-\omega\tau+\theta_{vm})]\end{aligned}\right\}\tag{3.7}$$

注意到(3.7)中 D_{xx} 与 D_{yy} 当不计指数衰减项时的结果与 Holley(1970) 的均匀河口[2]与 Chatwin(1975) 的管道流动结果[3]相一致，实际上(3.7)中τ是污染物释放后的时间度量，那么对应不同τ和 $\frac{\lambda}{\omega}$ 而得到的剪切分散张量D_{ij}如图1、2、3所示，

同样对于方差及相关矩可根据(3.7)式积分得到：

$$\left.\begin{aligned}
&\sigma_{xx}^2-\sigma_{xx}^2(t_0)-\partial\overline{K}_{xx}(t-t_0)=2\int_0^{t-t_0}D_{xx}(t_0+\tau,\tau)d\tau \\
&=u_0^2\sum_{m=1}^{\infty}\frac{\alpha_m^2}{\omega^2+\lambda_m^2}\left\{\lambda_m(t-t_0)-\frac{\lambda_m^2-\omega^2}{\lambda_m^2+\omega^2}+\frac{1}{2}\cos2(\omega t+\theta_{um})\right. \\
&\quad+\frac{1}{2}\cos2(\omega t_0+\theta_{um})-\frac{1}{2}\frac{\lambda_m}{\omega}\sin2(\omega t+\theta_{um})+\frac{1}{2}\frac{\lambda_m}{\omega}\sin2(\omega t_0+\theta_{um}) \\
&\quad+2\frac{\exp(-\lambda_m\tau)}{\omega^2+\lambda_m^2}[\lambda_m\sin(\omega t_0+\theta_{um})-\omega\cos(\omega t_0+\theta_{um})] \\
&\quad\left.\cdot[\lambda_m\sin(\omega t+\theta_{um})+\omega\cos(\omega t+\theta_{um})]\right\} \\
&\sigma_{yy}^2-\sigma_{yy}^2(t_0)-2\overline{K}_{yy}(t-t_0)=2\int_0^{t-t_0}D_{yy}(t_0+\tau,\tau)d\tau \\
&=v_0^2\sum_{m=1}^{\infty}\frac{\beta_m^2}{\omega^2+\lambda_m^2}\left\{\lambda_m(t-t_0)-\frac{\lambda_m^2-\omega^2}{\lambda_m^2+\omega^2}+\frac{1}{2}\cos2(\omega t+\theta_{vm})\right. \\
&\quad+\frac{1}{2}\cos2(\omega t_0+\theta_{vm})-\frac{1}{2}\frac{\lambda_m}{\omega}\sin2(\omega t+\theta_{vm})+\frac{1}{2}\frac{\lambda_m}{\omega}\sin2(\omega t_0+\theta_{vm}) \\
&\quad+2\frac{\exp(-\lambda_m\tau)}{\omega^2+\lambda_m^2}[\lambda_m\sin(\omega t_0+\theta_{vm})-\omega\cos(\omega t_0+\theta_{vm})] \\
&\quad\left.\cdot[\lambda_m\sin(\omega t+\theta_{vm})+\omega\cos(\omega t+\theta_{vm})]\right\} \\
&\sigma_{xy}^2-\sigma_{xy}^2(t_0)=\int_0^{t-t_0}(D_{xy}+D_{yx})d\tau \\
&=u_0v_0\sum_{m=1}^{\infty}\frac{\alpha_m\beta_m}{\omega^2+\lambda_m^2}\left\{\lambda_m(t-t_0)\cos(\theta_{um}-\theta_{vm})-2\frac{\lambda_m^2\sin(\omega t_0+\theta_{vm})\sin(\omega t_0+\theta_{um})}{\omega^2+\lambda_m^2}\right. \\
&\quad+2\frac{\omega^2\cos(\omega t_0+\theta_{vm})\cos(\omega t_0+\theta_{um})}{\omega^2+\lambda_m^2}+\frac{\lambda_m}{2\omega}\sin(2\omega t_0+\theta_{um}+\theta_{vm}) \\
&\quad-\frac{\lambda_m}{2\omega}\sin(2\omega t+\theta_{um}+\theta_{vm})+\frac{1}{2}\cos(2\omega t+\theta_{um}+\theta_{vm}) \\
&\quad-\frac{1}{2}\cos(2\omega t_0+\theta_{um}+\theta_{vm})+\frac{\exp(-\lambda_m\tau)}{\lambda_m^2+\omega^2} \\
&\quad\cdot[(\lambda_m\sin(\omega t+\theta_{um})+\omega\cos(\omega t+\theta_{um}))\cdot(\lambda_m\sin(\omega t_0+\theta_{vm}) \\
&\quad-\omega\cos(\omega t_0+\theta_{vm})+(\lambda_m\sin(\omega t+\theta_{vm})+\omega\cos(\omega t+\theta_{vm})) \\
&\quad\left.\cdot(\lambda_m\sin(\omega t_0+\theta_{um})-\omega\cos(\omega t_0+\theta_{um})))]\right\}
\end{aligned}\right\}\quad(3.8)$$

图4、5表示了剪切分散导数和方差，相关矩单一模态下，在不同的τ阶的随时间变 化的关系。

四、污染云团的中心位移

要求得X和Y，就必须得到l_{ii}^2。为此定义如下函数：

$$\left.\begin{aligned}K^{m}_{xy}(t)=\overline{K_{xx}\chi_m},\ u_{mn}=\overline{(u-\bar{u})\chi_m\chi_n}\\K^{m}_{yy}(t)=\overline{K_{yy}\chi_m},\ v_{mn}=\overline{(v-\bar{v})\chi_m\chi_n}\end{aligned}\right\}\tag{4.1}$$

原方程(2.5)～(2.7)是有形式解：

$$\left.\begin{aligned}l^2_{2,0}&=\sum a^{2m}_{2,0}(t,\tau)\exp(-\lambda_m I)\chi_m\\l^2_{0,2}&=\sum a^{m}_{0,2}(t,\tau)\exp(-\lambda_m I)\chi_m\\l^2_{1,1}&=\sum a^{m}_{1,1}(t,\tau)\exp(-\lambda_m I)\chi_m\end{aligned}\right\}\tag{4.2}$$

其中 a^m_{ij}满足：

$$\left.\begin{aligned}&\frac{\partial a^{2m}_{2,0}}{\partial t}+\frac{\partial a^{2m}_{2,0}}{\partial\tau}=\left[\frac{\partial X}{\partial t}+\frac{\partial X}{\partial\tau}-u_{mm}\right]\cdot u_m\\&\qquad-\sum_{m\neq n}u_{mn}u_m\exp[-(\lambda_n-\lambda_m)I]\\&\frac{\partial}{\partial t}a^{2m}_{0,2}+\frac{\partial a^{2m}_{0,2}}{\partial\tau}=\left[\frac{\partial Y}{\partial t}+\frac{\partial Y}{\partial\tau}-v_{mm}\right]v_m\\&\qquad-\sum_{m\neq n}v_{mn}v_n\exp(-(+\lambda_n-\lambda_m)I]\\&\frac{\partial}{\partial t}a^{2m}_{1,1}+\frac{\partial}{\partial\tau}a^{2m}_{1,1}=\left[\frac{\partial X}{\partial t}+\frac{\partial X}{\partial\tau}-u_{mm}\right]v_m+\left[\frac{\partial Y}{\partial t}+\frac{\partial Y}{\partial\tau}-v_{mm}\right]u_m\\&\qquad-\sum_{m\neq n}(u_{mn}v_n+v_{mn}u_n)\exp[-(\lambda_n-\lambda_m)I]\end{aligned}\right\}\tag{4.3}$$

初值条件：

$$a^{2m}_{2,0}|_{\tau=0}=K^m_{xx},\quad a^{2m}_{0,2}|_{\tau=0}=K^m_{yy},\quad a^{2m}_{1,1}|_{\tau=0}=0$$

注意到上式中：$u_m=u_m(t-\tau)$，$v_m=v_m(t-\tau)$

$u_{mn}=u_{mn}(t)$，$v_{mn}=v_{mn}(t)$

(4.3)式的解为：

$$\left.\begin{aligned}a^{2m}_{2,0}(t,\tau)&=K^m_{xx}(t-\tau)+u_m(t-\tau)\left[\int_0^\tau u_{mm}(t-\tau')d\tau'-X\right]\\&\quad+\sum_{m\neq n}u_n(t-\tau)\int_0^\tau u_{mn}(t-\tau)\exp(-(\lambda_n-\lambda_m)I(t-\tau',\tau-\tau'))d\tau'\\a^{2m}_{0,2}(t,\tau)&=K^m_{yy}(t-\tau)+v_m(t-\tau)\left[\int_0^\tau v_{mm}(t-\tau')d\tau'-Y\right]\\&\quad+\sum_{m\neq n}v_n(t-\tau)\int_0^\tau v_{mn}(t-\tau')\exp(-(\lambda_n-\lambda_m)I(t-\tau',\tau-\tau'))d\tau'\\a^{2m}_{1,1}(t,\tau)&=v_m(t-\tau)\left[\int_0^\tau u_{mm}(t-\tau')d\tau'-X\right]+u_m(t-\tau)\left[\int_0^\tau v_{mm}(t-\tau')d\tau'\right.\\&\quad\left.-Y\right]+\sum_{n\neq m}\int_0^\tau[v_n(t-\tau)u_{mn}(t-\tau')+u_n(t-\tau)v_{mn}(t-\tau')]\\&\quad\cdot\exp(-(\lambda_n-\lambda_m)I(t-\tau',\tau-\tau'))d\tau'\end{aligned}\right\}\tag{4.4}$$

由(4.4)式可以得到对高阶剪切分散的贡献是：

$$l^2_{2,0}\overline{(\bar{u}-u)} \text{ 和 } l^2_{0,2}\overline{(\bar{v}-v)} \tag{4.5}$$

根据文献[5]的建议，决定X Y最理想是使得(4.5)等于零，从而方程 (2.2) 式的低阶截断(1.1)式更为精确. 于是由(4.4)可以得到:

$$\left.\begin{aligned} l^2_{2,0}\overline{(\bar{u}-u)} &= 0 \\ l^2_{0,2}\overline{(\bar{v}-v)} &= 0 \end{aligned}\right\} \tag{4.6}$$

导出:

$$\left.\begin{aligned} X(t,\tau) &= \left(\frac{\partial}{\partial\tau}D_{xx}\right)^{-1}\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\Big\{u_m(t)u_n(t-\tau)\exp(-\lambda_m I) \\ &\quad\cdot\int_0^{\tau}u_{mn}(t-\tau')\exp[-(\lambda_n-\lambda_m)I(t-\tau',\tau-\tau')]d\tau'\Big\} \\ Y(t,\tau) &= \left(\frac{\partial}{\partial\tau}D_{yy}\right)^{-1}+\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\Big\{v_m(t)v_n(t-\tau)\exp(-\lambda_m I) \\ &\quad\cdot\int_0^{\tau}v_{mn}(t-\tau)\exp[-(\lambda_n-\lambda_m)I(t-\tau',\tau-\tau']d\tau'\Big] \end{aligned}\right\} \tag{4.7}$$

五、结 论 和 讨 论

本文引入考虑污染物云团释放初期的记忆特性的剪切分散函数. 分析了非定常流动中剪切分散导数和污染物云团中心位移的特性. 得到一般二维非定常流动中，考虑记忆特性的剪切分散导数，由第二节的结论可知：剪切扩散导数的确存在有负值，这种负值的存在与一般的浓度分布的扩散描述相矛盾. 而与现场观察的结果又相符合[2][3]. 这说明延迟分散方程(1.1)对这种现象的分析是有效的，此外本文的结果在一维情况所简化下，与 Smith 的结果[4][6]是完全一致.

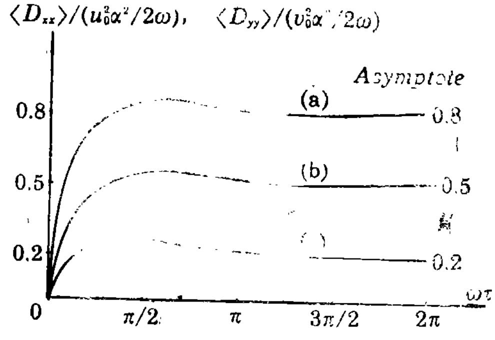

图1 弥散系数时间周期平精值随排污时间$\omega\tau$的变化规律

(a) $\frac{\lambda_a}{\omega}=0.5$ (b) $\frac{\lambda}{\omega}=1$ (c) $\frac{\lambda}{\omega}=2$

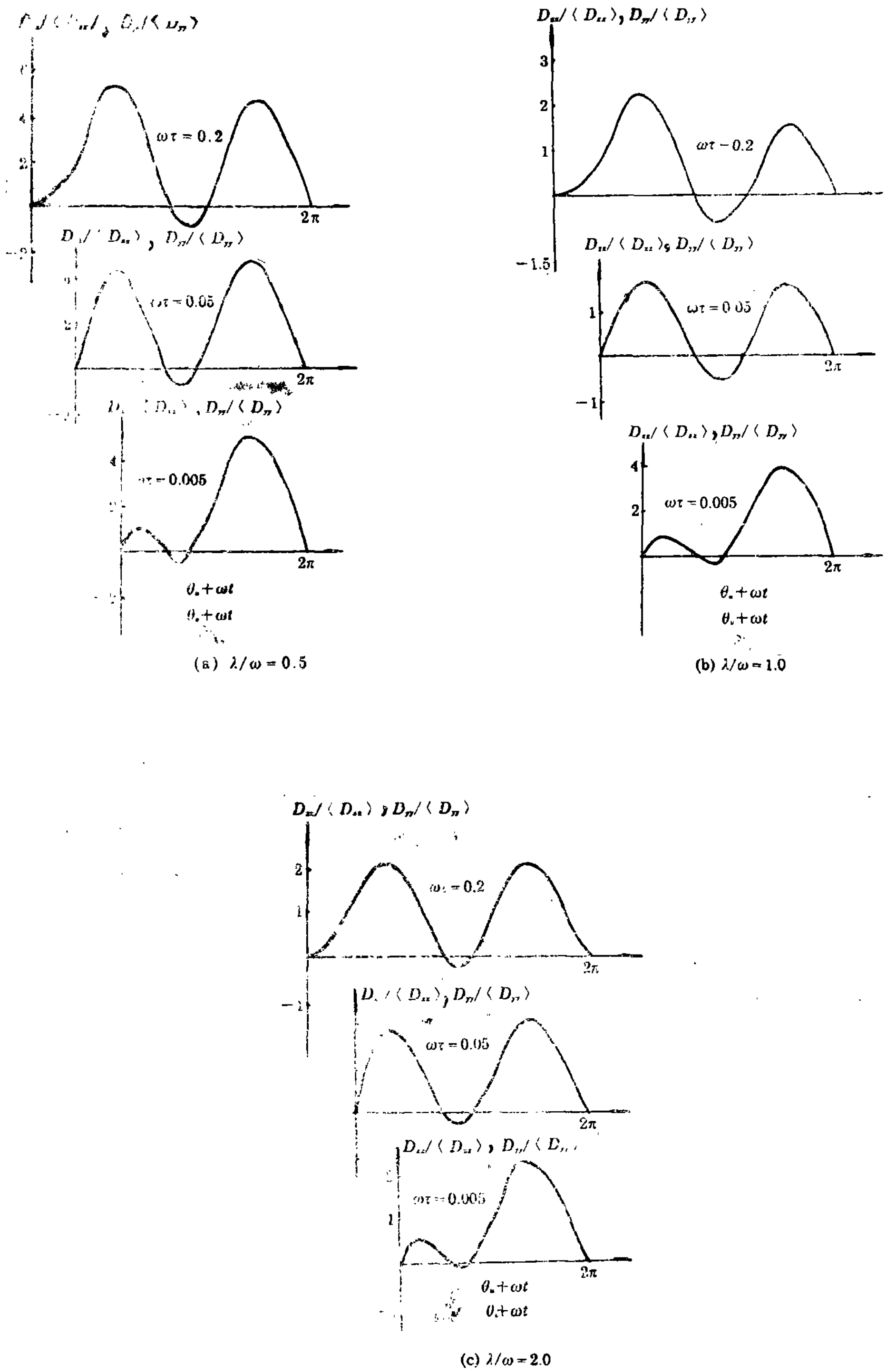

图2 在正弦型振荡流动中，二维剪切离散系数在不同的排污时间下的变化规律

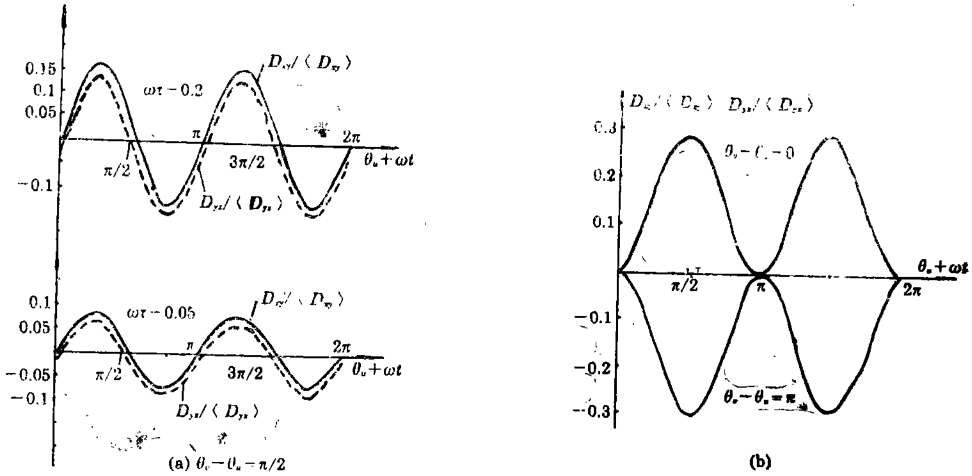

图3 在不同速度相位差下，相关系数D_{xy}，D_{yx}的变化规律($\lambda/\omega=0.5$)

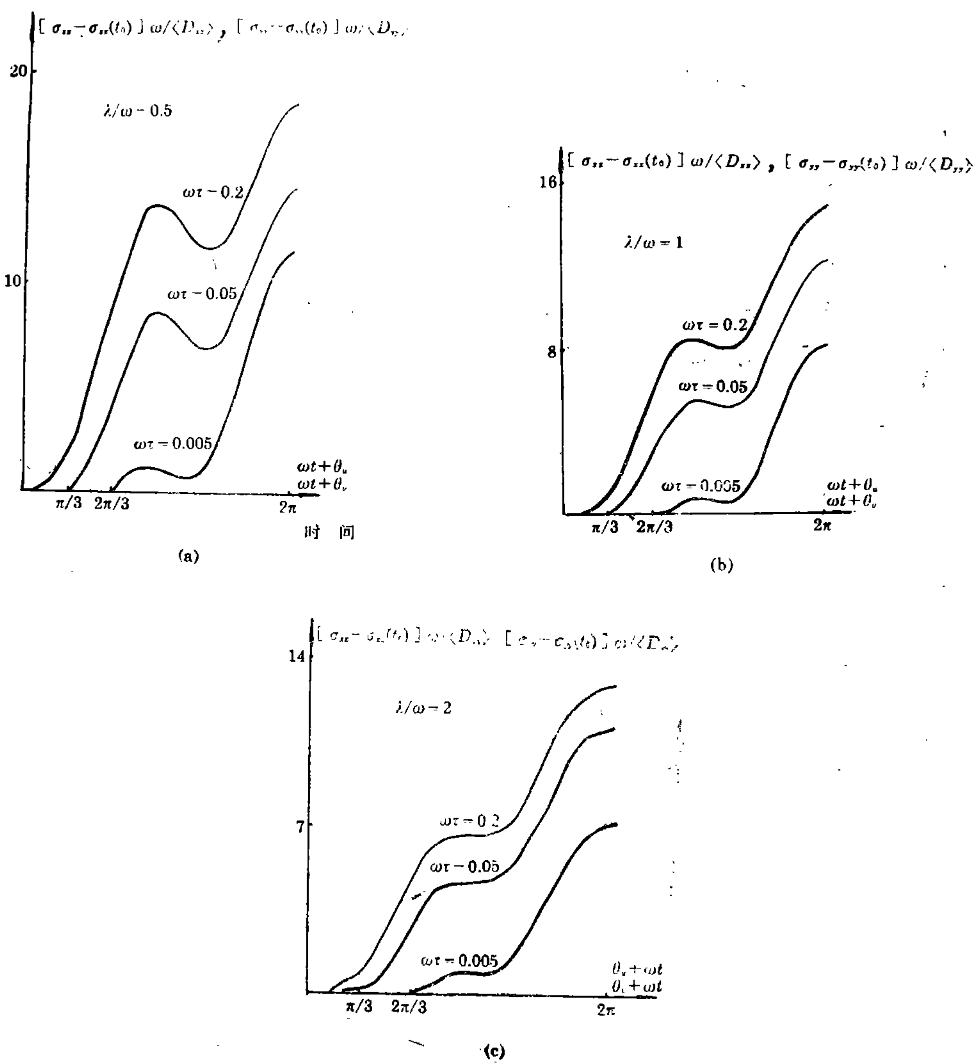

图4 单一模态在不同的排污时间下，x和y方向方差的变化曲线

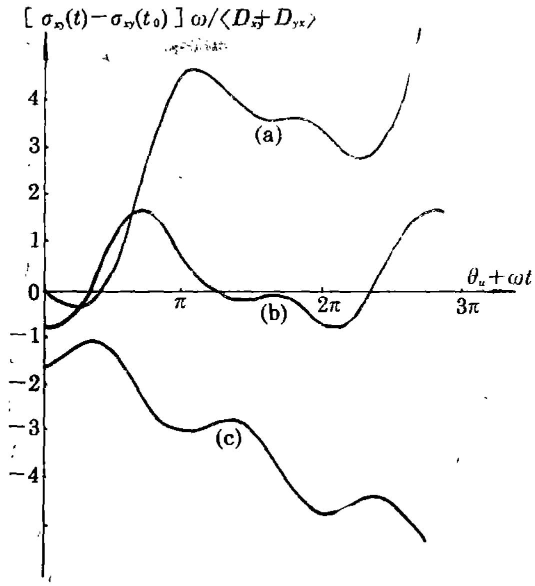

图5 相关矩随时间的变化 ($\lambda/\omega=0.5$, $a=1$, $\alpha=1$, $\omega t=0.2$
(a) $\theta_v-\theta_u=0$, (b) $\theta_v-\theta_u=\frac{\pi}{2}$, (c) $\theta_v-\theta_u=\pi$)

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Contaminant Contraction in Two-dimensional Oscillatory Flows

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Abstract

If the vertically-mixing time is comparable with that of period of oscillatory current, the contaminant contraction may occur. The coefficient of shear dispersion is negative (singulurity). According to the two-dimensional delay-diffusion equation derived by the author:

$$\frac{\partial\bar{C}}{\partial t}+\bar{u}\frac{\partial\bar{C}}{\partial x}+\bar{v}\frac{\partial\bar{C}}{\partial y}=K_{xx}\frac{\partial^2\bar{C}}{\partial x^2}+K_{yy}\frac{\partial^2\bar{C}}{\partial y^2}+\int_0^\infty\left[\frac{\partial}{\partial\tau}D_{xx}\frac{\partial^2}{\partial x^2}+\frac{\partial}{\partial\tau}(D_{xy}+D_{yx})\frac{\partial^2}{\partial x\partial y}+\frac{\partial}{\partial\tau}D_{yy}\frac{\partial^2}{\partial y^2}\right]\bar{C}(x-X,x-Y,t-\tau)d\tau$$

where $u(t)$, $v(t)$ are verticay-averaged velocities, the equations for $X(t,\tau)$, $Y(t,\tau)$, central displacements, dispersion tensor, had been derived. D_{ij} is positive when τ is small. If the τ is large, the memory functions may be negative. Also the expressions for and X,Y had been obtained.

Key words shear dispersion, oscillatory flow, memory function, contaminant contraction