

# 地下抗爆结构动力分析的基本方法

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## 摘 要

本文建立了地下抗爆结构动力分析的广义变分原理, 相应的导出地下结构弹塑性分析的广义泛涵, 给出非保守体系的广义变分原理, 从而提出了地下抗爆结构动力分析的基本理论。文中还对地下圆形抗爆结构进行了动力计算和试验对比。

**关键词** 抗爆结构 广义变分原理 动力分析 弹塑性分析

## 一、前 言

地下抗爆结构的分析计算, 涉及到不同的动荷载种类和各种各样的结构形式, 动力分析是十分复杂的。本文根据地下抗爆工程结构常遇到的实际受力状态, 应用 Lagrange 乘子法, 1. 建立了两端边界均为完全约束的地下平面双层结构动力分析的广义泛涵; 2. 导出了地下结构弹塑性分析的广义泛涵; 3. 对于非保守体系的各种情况, 考虑了非保守力的作用, 给出了非保守体系的广义变分原理。从而, 在理论上建立了地下抗爆结构动力分析的基本方法, 可广泛的应用于平面结构的弹性分析和弹塑性分析。最后本文给出了非保守体系地下圆形结构的动力分析实例, 并与试验结果进行了对比。

## 二、地下平面结构抗爆分析的基本理论

### 1. 平面曲杆的基本关系

对于双层复合结构, 在结构的轴线上取一长度  $ds = R d\theta$  的微段作为隔离体, 如图1所示。根据力的平衡条件, 可以得到下面的平衡方程:

$$\frac{\partial N_1}{\partial s} - \frac{Q_1}{R} - m_1 \frac{\partial^2 u_1}{\partial t^2} + q_u(t) - p_u(t) = 0 \quad (2.1)$$

$$\frac{\partial^2 M_1}{\partial s^2} + \frac{N_1}{R} - m_1 \frac{\partial^2 w_1}{\partial t^2} - k w_1 + q_w(t) - p_w(t) = 0 \quad (2.2)$$

$$\frac{\partial N_2}{\partial s} - \frac{Q_2}{R} - m_2 \frac{\partial^2 u_2}{\partial t^2} + p_u(t) = 0 \quad (2.3)$$

$$\frac{\partial^2 M_2}{\partial s^2} + \frac{N_2}{R} - \bar{m}_2 \frac{\partial^2 w_2}{\partial t^2} + p_w(t) = 0 \quad (2.4)$$

上式中  $M, N, Q$  为结构的弯矩、轴力和剪力;  $w, u$  为结构的径向和切向位移;  $q_w, q_u$  为作用在结构上的径向荷载和切向荷载;  $k$  是结构周围介质的弹性地基系数;  $p_w, p_u$  为两层结构之间的径向和切向相互作用力;  $R$  是结构轴线的曲率半径;  $\bar{m}$  是结构单位长度的质量。

根据几何关系, 有如下几何的方程:

设结构的轴向应变为  $\varepsilon$ , 则有

$$\varepsilon_i = \frac{\partial u_i}{\partial s} - \frac{w_i}{R} \quad (2.5)$$

设结构的截面转角为  $\varphi$ , 则有  $\varphi_i = \frac{\partial w_i}{\partial s} + \frac{u_i}{R}$  (2.6)

设结构轴线变形后曲率的改变量为  $\mathcal{K}$ , 则有  $\mathcal{K}_i = \frac{1}{\rho_i} - \frac{1}{R_i} = \frac{\partial \varphi_i}{\partial s}$  (2.7)

结构截面内力与变位之间有下面的关系:

$$M_i = -E_i J_i \mathcal{K}_i = -E_i J_i \left( \frac{\partial^2 w_i}{\partial s^2} + \frac{1}{R} \frac{\partial u_i}{\partial s} \right) \quad (2.8)$$

$$N_i = E_i F_i \varepsilon_i = E_i F_i \left( \frac{\partial u_i}{\partial s} - \frac{w_i}{R} \right) \quad (2.9)$$

$$Q_i = \frac{\partial M_i}{\partial s} = -E_i J_i \left( \frac{\partial^3 w_i}{\partial s^3} + \frac{1}{R} \frac{\partial^2 u_i}{\partial s^2} \right) \quad (2.10)$$

式中  $\rho_i$  为变形后的曲率半径;  $E_i J_i$  为截面抗弯刚度;  $F_i$  为截面面积 ( $i=1, 2$ )。

## 2. 边界条件

曲杆的边界条件, 系指曲杆的端点条件, 对于曲杆的两端点来说, 最多有六个约束条件, 其约束方程为

$$u_i(0, t) - \bar{u}_i(0, t) = 0, \quad w_i(0, t) - \bar{w}_i(0, t) = 0 \quad (2.11 \sim 2.12)$$

$$\varphi_i(0, t) - \bar{\varphi}_i(0, t) = 0, \quad u_i(l, t) - \bar{u}_i(l, t) = 0 \quad (2.13 \sim 2.14)$$

$$w_i(l, t) - \bar{w}_i(l, t) = 0, \quad \varphi_i(l, t) - \bar{\varphi}_i(l, t) = 0 \quad (2.15 \sim 2.16)$$

式中  $u_i(0, t), w_i(0, t), \varphi_i(0, t)$  是曲杆左端待求的切向位移、径向位移和转角;  $u_i(l, t), w_i(l, t), \varphi_i(l, t)$  是曲杆右端待求的切向位移、径向位移和转角; 而  $\bar{u}_i(0, t), \bar{w}_i(0, t), \bar{\varphi}_i(0, t)$  和  $\bar{u}_i(l, t), \bar{w}_i(l, t), \bar{\varphi}_i(l, t)$  为相应的已知值 ( $i=1, 2$ )。

双层结构在接触面上应满足变形协调条件, 即

$$w_1 - w_2 = 0, \quad u_1 - u_2 = 0 \quad (2.17 \sim 2.18)$$

## 3. 地下平面结构抗爆分析的广义变分原理

广义变分原理 I: 在服从几何方程 (2.5)、(2.6)、(2.7) 和内力与变位关系 (2.8)、(2.9)、(2.10), 并在  $t=t_1$  和  $t=t_2$  时  $w_i, u_i$  为已知的条件下, 在所有容许的位移  $w_i, u_i$  中, 其真实解使下述广义泛函取驻值。

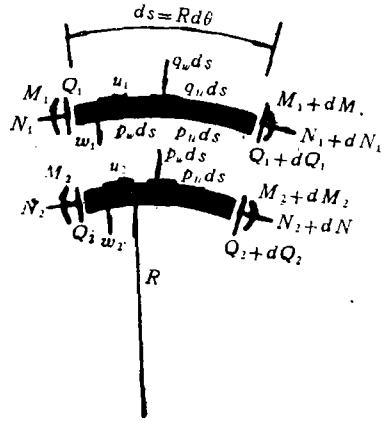


图 1

$$\begin{aligned} \Pi_1 = & \int_{t_1}^{t_2} \left\{ \int_L \frac{1}{2} \bar{m}_i \left[ \left( \frac{\partial w_i}{\partial t} \right)^2 + \left( \frac{\partial u_i}{\partial t} \right)^2 \right] ds - \int_L \frac{E_i J_i}{2} \left( \frac{\partial^2 w_i}{\partial s^2} + \frac{1}{R} \frac{\partial u_i}{\partial s} \right)^2 ds \right. \\ & - \int_L \frac{E_i F_i}{2} \left( \frac{\partial u_i}{\partial s} - \frac{w_i}{R} \right)^2 ds - \int_{L_k} \frac{1}{2} k w_i^2 ds + \int_{L_p} (q_w w_i + q_u u_i) ds \\ & + \int_L p_w (w_1 - w_2) ds + \int_L p_u (u_1 - u_2) ds - N_i(0, t) [u_i(0, t) - \bar{u}_i(0, t)] \\ & - Q_i(0, t) [w_i(0, t) - \bar{w}_i(0, t)] + M_i(0, t) [\varphi_i(0, t) - \bar{\varphi}(0, t)] \\ & + N_i(l, t) [u_i(l, t) - \bar{u}_i(l, t)] + Q_i(l, t) [w_i(l, t) - \bar{w}_i(l, t)] \\ & \left. - M_i(l, t) [\varphi_i(l, t) - \bar{u}_i(l, t)] \right\} dt \end{aligned} \quad (2.19)$$

式中： $w_i, u_i, p_w, p_u, N_i(0, t), Q_i(0, t), M_i(0, t), N_i(l, t), Q_i(l, t), M_i(l, t), u_i(0, t), w_i(0, t), \varphi_i(0, t), u_i(l, t), w_i(l, t), \varphi_i(l, t)$ 在变分时都是独立变量； $L$ 为曲杆结构的全长； $L_k$ 为结构与周围介质的弹性抗力边界； $L_p$ 为外荷载力边界。

下面进行证明：

根据哈密顿原理，在满足条件(2.5)~(2.18)时，体系的泛函为

$$\begin{aligned} \Pi = & \int_{t_1}^{t_2} \left\{ \int_L \frac{1}{2} \bar{m}_i \left[ \left( \frac{\partial w_i}{\partial t} \right)^2 + \left( \frac{\partial u_i}{\partial t} \right)^2 \right] ds - \int_L \frac{E_i J_i}{2} \left( \frac{\partial^2 w_i}{\partial s^2} + \frac{1}{R} \frac{\partial u_i}{\partial s} \right)^2 ds \right. \\ & \left. - \int_L \frac{E_i F_i}{2} \left( \frac{\partial u_i}{\partial s} - \frac{w_i}{R} \right)^2 ds - \int_{L_k} \frac{1}{2} k w_i^2 ds + \int_{L_p} (q_w w_i + q_u u_i) ds \right\} dt \end{aligned} \quad (2.20)$$

我们引入待定的Lagrange乘子 $\mu_1, \mu_2, \lambda_i(i=1, 2, \dots, 12)$ ，将曲杆两端的位移约束条件(2.11)~(2.16)及变形协调条件(2.17)、(2.18)合并到泛函(2.20)中，于是可得到下列无条件变分的泛函

$$\begin{aligned} \Pi_1^* = & \Pi + \int_{t_1}^{t_2} \left\{ \int_L \mu_1 (w_1 - w_2) ds + \int_L \mu_2 (u_1 - u_2) ds + \lambda_1 [u_1(0, t) \right. \\ & - \bar{u}_1(0, t)] + \lambda_2 [w_1(0, t) - \bar{w}_1(0, t)] + \lambda_3 [\varphi_1(0, t) - \bar{\varphi}_1(0, t)] \\ & + \lambda_4 [u_1(l, t) - \bar{u}_1(l, t)] + \lambda_5 [w_1(l, t) - \bar{w}_1(l, t)] + \lambda_6 [\varphi_1(l, t) \\ & - \bar{\varphi}_1(l, t)] + \lambda_7 [u_2(0, t) - \bar{u}_2(0, t)] + \lambda_8 [w_2(0, t) - \bar{w}_2(0, t)] \\ & + \lambda_9 [\varphi_2(0, t) - \bar{\varphi}_2(0, t)] + \lambda_{10} [u_2(l, t) - \bar{u}_2(l, t)] + \lambda_{11} [w_2(l, t) \\ & \left. - \bar{w}_2(l, t)] + \lambda_{12} [\varphi_2(l, t) - \bar{\varphi}_2(l, t)] \right\} dt \end{aligned} \quad (2.21)$$

在对(2.21)进行变分时 $w_i, u_i, u_i(0, t), w_i(0, t), \varphi_i(0, t), u_i(l, t), w_i(l, t), \varphi_i(l, t)$ 及引入的Lagrange乘子均为独立变量；当泛函 $\Pi_1^*$ 达到驻值时，有 $\delta \Pi_1^* = 0$ ，通过连续分部积分的运算，利用几何方程及内力与位移的关系代换，经整理后得到

$$\begin{aligned} \delta \Pi_1^* = & \int_{t_1}^{t_2} \left\{ \int_L \left[ \frac{\partial^2 M_1}{\partial s^2} + \frac{N_1}{R} - \bar{m}_1 \frac{\partial^2 w_1}{\partial t^2} + \mu_1 \right] \delta w_1 ds - \int_{L_k} k w_1 \delta w_1 ds \right. \\ & + \int_{L_p} q_w \delta w_1 ds \left. \right\} dt + \int_{t_1}^{t_2} \left\{ \int_L \left[ \frac{\partial N_1}{\partial s} - \frac{Q_1}{R} - \bar{m}_1 \frac{\partial^2 u_1}{\partial t^2} + \mu_2 \right] \delta u_1 ds + \int_{L_p} q_u \delta u_1 ds \right\} dt \\ & + \int_{t_1}^{t_2} \left\{ \int_L \left[ \frac{\partial^2 M_2}{\partial s^2} + \frac{N_2}{R} - \bar{m}_2 \frac{\partial^2 w_2}{\partial t^2} - \mu_1 \right] \delta w_2 ds \right\} dt + \int_{t_1}^{t_2} \left\{ \int_L \left[ \frac{N_2}{s} - \frac{Q_2}{R} - \bar{m}_2 \frac{\partial^2 u_2}{\partial t^2} \right. \right. \\ & \left. \left. - \mu_2 \right] \delta u_2 ds \right\} dt + \int_{t_1}^{t_2} \left[ \int_L (w_1 - w_2) \delta \mu_1 ds \right] dt + \int_{t_1}^{t_2} \left[ \int_L (u_1 - u_2) \delta \mu_2 ds \right] dt \\ & + \int_{t_1}^{t_2} [\lambda_1 + N_1(0, t)] \delta u_1(0, t) dt + \int_{t_1}^{t_2} [\lambda_2 + Q_1(0, t)] \delta w_1(0, t) dt \end{aligned}$$

$$\begin{aligned}
& + \int_{t_1}^{t_2} [\lambda_3 - M_1(0, t)] \delta \varphi_1(0, t) dt + \int_{t_1}^{t_2} [\lambda_4 - N_1(l, t)] \delta u_1(l, t) dt \\
& + \int_{t_1}^{t_2} [\lambda_5 - Q_1(l, t)] \delta w_1(l, t) dt + \int_{t_1}^{t_2} [\lambda_6 + M_1(l, t)] \delta \varphi_1(l, t) dt \\
& + \int_{t_1}^{t_2} [\lambda_7 + N_2(0, t)] \delta u_2(0, t) dt + \int_{t_1}^{t_2} [\lambda_8 + Q_2(0, t)] \delta w_2(0, t) dt \\
& + \int_{t_1}^{t_2} [\lambda_9 - M_2(0, t)] \delta \varphi_2(0, t) dt + \int_{t_1}^{t_2} [\lambda_{10} - N_2(l, t)] \delta u_2(l, t) dt \\
& + \int_{t_1}^{t_2} [\lambda_{11} - Q_2(l, t)] \delta w_2(l, t) dt + \int_{t_1}^{t_2} [\lambda_{12} + M_2(l, t)] \delta \varphi_2(l, t) dt \\
& + \int_{t_1}^{t_2} [u_1(0, t) - \bar{u}_1(0, t)] \delta \lambda_1 dt + \int_{t_1}^{t_2} [w_1(0, t) - \bar{w}_1(0, t)] \delta \lambda_2 dt \\
& + \int_{t_1}^{t_2} [\varphi_1(0, t) - \bar{\varphi}_1(0, t)] \delta \lambda_3 dt + \int_{t_1}^{t_2} [u_1(l, t) - \bar{u}_1(l, t)] \delta \lambda_4 dt \\
& + \int_{t_1}^{t_2} [w_1(l, t) - \bar{w}_1(l, t)] \delta \lambda_5 dt + \int_{t_1}^{t_2} [\varphi_1(l, t) - \bar{\varphi}_1(l, t)] \delta \lambda_6 dt \\
& + \int_{t_1}^{t_2} [u_2(0, t) - \bar{u}_2(0, t)] \delta \lambda_7 dt + \int_{t_1}^{t_2} [w_2(0, t) - \bar{w}_2(0, t)] \delta \lambda_8 dt \\
& + \int_{t_1}^{t_2} [\varphi_2(0, t) - \bar{\varphi}_2(0, t)] \delta \lambda_9 dt + \int_{t_1}^{t_2} [u_2(l, t) - \bar{u}_2(l, t)] \delta \lambda_{10} dt \\
& + \int_{t_1}^{t_2} [w_2(l, t) - \bar{w}_2(l, t)] \delta \lambda_{11} dt + \int_{t_1}^{t_2} [\varphi_2(l, t) - \bar{\varphi}_2(l, t)] \delta \lambda_{12} dt
\end{aligned} \tag{2.22}$$

所以, 由  $\delta \Pi^*$  取驻值的条件, 导出了 Lagrange 乘子的物理意义, 从广义功的角度解释, 是比较容易理解的:

$$\left. \begin{aligned}
\mu_1 &= -\frac{\partial^2 M_1}{\partial s^2} - \frac{N_1}{R} + \bar{m}_1 \frac{\partial^2 w_1}{\partial t^2} + k w_1 - q_w = -p_w \\
\mu_2 &= -\frac{\partial N_1}{\partial s} + \frac{Q_1}{R} + \bar{m} \frac{\partial^2 u_1}{\partial t^2} - q_u = -p_u \\
\mu_1 &= \frac{\partial^2 M_2}{\partial s^2} + \frac{N_2}{R} - \bar{m}_2 \frac{\partial^2 w_2}{\partial t^2} = -p_w \\
\mu_2 &= \frac{\partial N_2}{\partial s} - \frac{Q_2}{R} - \bar{m}_2 \frac{\partial^2 u_2}{\partial t^2} = -p_u \\
\lambda_1 &= -N_1(0, t), \quad \lambda_2 = -Q_1(0, t), \quad \lambda_3 = M_1(0, t) \\
\lambda_4 &= N_1(l, t), \quad \lambda_5 = Q_1(l, t), \quad \lambda_6 = -M_1(l, t) \\
\lambda_7 &= -N_2(0, t), \quad \lambda_8 = -Q_2(0, t), \quad \lambda_9 = M_2(0, t) \\
\lambda_{10} &= N_2(l, t), \quad \lambda_{11} = Q_2(l, t), \quad \lambda_{12} = -M_2(l, t)
\end{aligned} \right\} \tag{2.23}$$

同时, 还导出了平衡方式:

$$\begin{aligned}
\frac{\partial^2 M_1}{\partial s^2} + \frac{N_1}{R} - \bar{m}_1 \frac{\partial^2 w_1}{\partial t^2} - k w_1 + q_w - p_w &= 0 \\
\frac{\partial N_1}{\partial s} - \frac{Q_1}{R} - \bar{m}_1 \frac{\partial^2 u_1}{\partial t^2} + q_u - p_u &= 0
\end{aligned}$$

$$\frac{\partial^2 M_2}{\partial s^2} + \frac{N_2}{R} - \bar{m}_2 \frac{\partial^2 w_2}{\partial t^2} + p_w = 0, \quad \frac{\partial N_2}{\partial s} - \frac{Q_2}{R} - \bar{m}_2 \frac{\partial^2 u_2}{\partial t^2} + p_u = 0$$

位移边界条件和变形协调条件,

$$\begin{aligned} u_i(0,t) - \bar{u}_i(0,t) &= 0, \quad w_i(0,t) - \bar{w}_i(0,t) = 0, \\ \varphi_i(0,t) - \bar{\varphi}_i(0,t) &= 0, \quad u_i(l,t) - \bar{u}_i(l,t) = 0, \\ w_i(l,t) - \bar{w}_i(l,t) &= 0, \quad \varphi_i(l,t) - \bar{\varphi}_i(l,t) = 0, \\ w_1 - w_2 &= 0, \quad u_1 - u_2 = 0 \end{aligned}$$

从上面的结果可见, 由  $\delta \Pi_1^* = 0$  导出了问题的全部方程, 从而广义变分原理得证. 将 (2.23) 式代入 (2.21) 式, 便可得到广义泛函 (2.19) 式.

广义变分泛函(2.19), 可以应用于各种支承条件的平面结构, 是问题精确求解或近似求解的基础.

#### 4. 泛函 (2.19) 的退化

当遇到单层平面结构时, 两层之间的作用力  $p_w, p_u$  为零, 两层结构接触面上的变形协调条件亦不存在, 这时广义变分原理的泛函为

$$\begin{aligned} \Pi_{\text{I}} = & \int_{t_1}^{t_2} \left\{ \int_L \frac{1}{2} \bar{m} \left[ \left( \frac{\partial w}{\partial t} \right)^2 + \left( \frac{\partial u}{\partial t} \right)^2 \right] ds - \int_L \frac{EJ}{2} \left( \frac{\partial^2 w}{\partial s^2} + \frac{1}{R} \frac{\partial u}{\partial s} \right)^2 ds \right. \\ & - \int_L \frac{EF}{2} \left( \frac{\partial u}{\partial s} - \frac{w}{R} \right)^2 ds - \int_{L_k} \frac{1}{2} k w^2 ds + \int_{L_p} (q_w \cdot w + q_u \cdot u) ds \\ & - N(0,t)[u(0,t) - \bar{u}(0,t)] - Q(0,t)[w(0,t) - \bar{w}(0,t)] \\ & + M(0,t)[\varphi(0,t) - \bar{\varphi}(0,t)] + N(l,t)[u(l,t) - \bar{u}(l,t)] \\ & \left. + Q(l,t)[w(l,t) - \bar{w}(l,t)] - M(l,t)[\varphi(l,t) - \bar{\varphi}(l,t)] \right\} dt \end{aligned} \quad (2.24)$$

5. 在平面曲杆结构的弹塑性动力分析中, 只要把塑性铰当成外力, 广义泛函(2.19)、(2.24)也同样适用. 例如, 在一维平面波作用下的一个对称无铰拱, 当拱顶出现一个塑性铰时, 其广义泛函为 (取其一半):

$$\begin{aligned} \Pi_{\text{I}} = & \int_{t_1}^{t_2} \left\{ \int_{L/2} \frac{1}{2} \bar{m} \left[ \left( \frac{\partial w}{\partial t} \right)^2 + \left( \frac{\partial u}{\partial t} \right)^2 \right] ds - \int_{L/2} \frac{EJ}{2} \left( \frac{\partial^2 w}{\partial s^2} + \frac{1}{R} \frac{\partial u}{\partial s} \right)^2 ds \right. \\ & - \int_{L/2} \frac{EF}{2} \left( \frac{\partial u}{\partial s} - \frac{w}{R} \right)^2 ds - \int_{L/2} \frac{1}{2} k w^2 ds + \int_{L/2} (q_w \cdot w + q_u \cdot u) ds \\ & - N(0,t)[u(0,t) - \bar{u}(0,t)] - Q(0,t)[w(0,t) - \bar{w}(0,t)] \\ & + M(0,t)[\varphi(0,t) - \bar{\varphi}(0,t)] + N\left(\frac{l}{2}, t\right) \left[ u\left(\frac{l}{2}, t\right) - \bar{u}\left(\frac{l}{2}, t\right) \right] \\ & \left. + M\left(\frac{l}{2}, t\right) \varphi\left(\frac{l}{2}, t\right) \right\} dt \end{aligned} \quad (2.25)$$

式中:  $M(l/2, t)$  为拱顶的塑性弯矩, 当材料和几何尺寸一定时为一个定值.

#### 6. 非保守体系的广义变分原理

上述广义变分原理的泛函  $\Pi_1, \Pi_{\text{I}}, \Pi_{\text{II}}$  只适用于保守体系, 若推广到非保守体系<sup>[3]</sup>, 便有如下的形式

$$\delta \Pi_{\text{I}} + \int_{t_1}^{t_2} \sum_{j=1}^J Q_j \delta q_j dt = 0 \quad (2.26)$$

这里  $Q_j$  为体系的诸非保守力,  $q_j$  为体系的广义坐标 ( $i = \text{I}, \text{II}, \text{III}$ ).

### 三、抗爆结构动力分析基本理论的应用

#### 1. 问题的提出及计算图

设地面下 $h_0$ 深处,有一足够长的圆形隧道受到地面平面冲击波的作用,取图2所示的计算简图

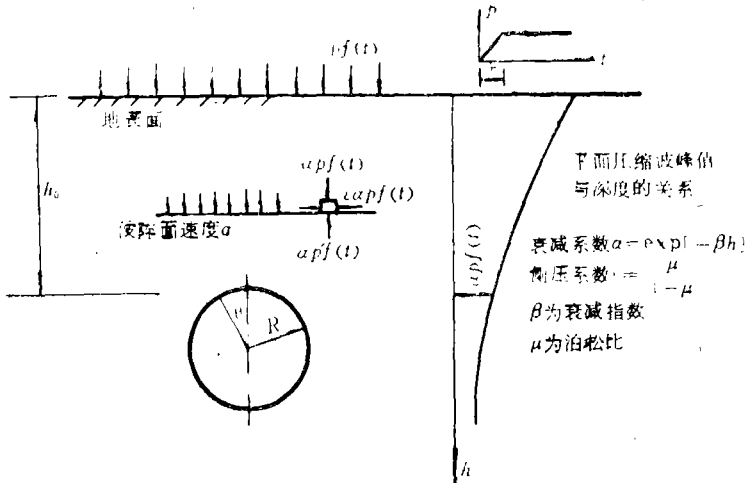


图 2

#### 2. 结构周边荷载的确定

根据:(1)、一维平面波在传播过程中,介质质点应力与介质质点速度之间的关系;(2)、波在结构表面上的压力协调条件;(3)、结构表面上质点速度与结构变形速度的协调条件,可得到作用在结构表面上的径向荷载和切向荷载

在 $0 \sim \pi$ 之间

$$q_w = 2\alpha p_f(t)(\cos^2\theta + \varepsilon \sin^2\theta) - \alpha \rho \dot{w}(\theta, t)$$

$$q_u = 2\alpha p_f(1 - \varepsilon) \sin\theta \cos\theta$$

#### 3. 结构位移的选取

利用圆形结构的特点,用三角函数表示结构的位移,并例设结构在变形过程中轴线长度保持不变,则

$$w(\theta, t) = \sum_{m=1}^{\infty} q_m \cos m\theta, \quad u(\theta, t) = \sum_{m=1}^{\infty} \frac{1}{2} q_m \sin m\theta$$

式中: $q_m$ 为广义坐标,只与时间有关。

#### 4. 结构运动微分方程的建立

在忽略结构剪力位能的情况下,将广义力和位移的表达式代入(2.24)中,可得广义泛函为

$$\Pi_I = \int_{t_1}^{t_2} \left\{ \int_L \frac{1}{2} m \left[ \left( \frac{\partial w}{\partial t} \right)^2 + \left( \frac{\partial u}{\partial t} \right)^2 \right] ds - \int_L \frac{EJ}{2} \left( \frac{\partial^2 w}{\partial s^2} + \frac{1}{R} \frac{\partial u}{\partial s} \right)^2 ds \right.$$

$$\begin{aligned}
 & - \int_{L_h} \frac{1}{2} k w^2 ds + \int_{L_p} (q_w \cdot w + q_u \cdot u) ds \} dt \\
 & = \int_{t_1}^{t_2} \left\{ \frac{\pi m R}{2} \sum_{m=1}^{\infty} \left( 1 + \frac{1}{m^2} \right) \dot{q}_m^2 - \frac{\pi E J}{2 R^3} \sum_{m=1}^{\infty} (1 - m^2)^2 q_m^2 \right. \\
 & \quad - k R \sum_{m=1}^{\infty} \left( \frac{\pi - \alpha_c}{2} - \frac{1}{4m} \sin 2m\alpha_c \right) q_m^2 + k R \sum_{\substack{m=1 \\ (m+n)}}^{\infty} \sum_{\substack{n=1 \\ (m+n)}}^{\infty} \left[ \frac{\sin(m+n)\alpha_c}{\alpha(m+n)} \right. \\
 & \quad \left. + \frac{\sin(m-n)\alpha_c}{\alpha(m-n)} \right] q_m q_n + 4 R p f(t) \sum_{m=1}^{\infty} H_m q_m \left. \right\} dt
 \end{aligned}$$

式中： $q_w = 2\alpha p f(t)(\cos^2\theta + \varepsilon \sin^2\theta)$ ，只考虑了保守力； $\alpha_c$ 为结构变形过程中不挤压周围介质的角度，计算中通过迭代确定。

由介质声阻抗 $\alpha\rho$ 引起体系的耗散函数为

$$D = \int_0^{2\pi} \frac{1}{2} \alpha \rho \dot{w}^2 \cdot R d\theta = \sum_{m=1}^{\infty} \frac{1}{2} \pi R \alpha \rho \dot{q}_m^2$$

于是，体系的非保守力为  $Q_m = \frac{\partial D}{\partial \dot{q}_m} = \pi R \alpha \rho \dot{q}_m$

将上述结果代入 (2.26) 式，可得到结构运动的微分方程

$$a_m \ddot{q}_m + d \dot{q}_m + c_{mm} q_m + \sum_{\substack{n=1 \\ (n+m)}}^{\infty} e_{mn} q_n = G_m$$

式中， $a_m = \frac{\pi m R}{2} \left( 1 + \frac{1}{m^2} \right)$ ， $d = \frac{1}{2} \pi R \alpha \rho$ ，

$$c_{mm} = \frac{\pi E J}{2 R^3} (1 - m^2)^2 + k R \left( \frac{\pi - \alpha_c}{2} - \frac{1}{4m} \sin 2m\alpha_c \right)$$

$$e_{mn} = -k R \left[ \frac{\sin(m+n)\alpha_c}{2(m+n)} + \frac{\sin(m-n)\alpha_c}{2(m-n)} \right]$$

$$G_m = 2 R p f(t) H_m$$

$$\begin{aligned}
 H_m = & \int_0^{\pi} \exp[-\beta(h_0 + R(1 - \cos\theta))] \\
 & \cdot \left[ \cos m\theta (\cos^2\theta + \varepsilon \sin^2\theta) \right. \\
 & \left. + \frac{\sin m\theta}{m} (1 - \varepsilon) \sin\theta \cos\theta \right] d\theta
 \end{aligned}$$

### 5. 计算结果

运动微分方程的求解问题见[4]。现将模爆器中结构模型的试验结果与本文理论结果对比在图3中。

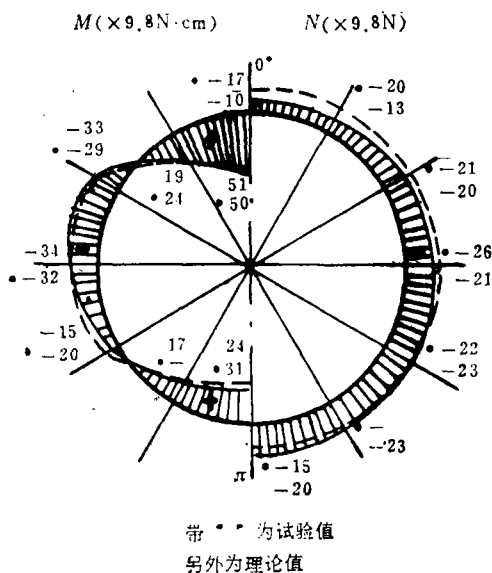


图 3

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## The Fundamental Methods of Dynamical Analysis of the Blast-Resistant Underground Structures

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### Abstract

In this paper, the generalized variational principle of dynamic analysis for the blast-resistant underground structures is established, and the corresponding generalized functionals of elastoplastic analysis for underground structures is derived, and the generalized variational principle of nonconservative system is given, thus the fundamental of dynamical analysis for underground structures to resist blast is proposed. Finally, for the underground cylindrical structure to resist blast, dynamical calculations are made, and compared with the test results.

**Key words** generalized variational principle, dynamical analysis of structure, elastoplastic analysis, blast-resistant underground structure