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加肋圆柱壳在轴压作用下的屈曲和后屈曲*

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摘　要

本文讨论完善和非完善的，纵向加肋和正交加肋圆柱壳在轴压作用下的屈曲和后屈曲性态。依据文[1]提供的圆柱薄壳屈曲的边界层理论及其分析方法，给出了加肋圆柱壳在轴压作用下的屈曲和后屈曲理论分析。本文同时讨论肋骨与壳板材料不同时对加肋圆柱壳屈曲和后屈曲性态的影响。

关键词　结构稳定性　屈曲　后屈曲　加肋圆柱壳

一、引　言

在近代结构工程中广泛采用加肋的方法以提高圆柱薄壳的屈曲强度。因此，弄清加肋圆柱壳在轴压作用下的屈曲和后屈曲性态具有十分重要的意义。

60年代以来发表了大量的研究论文和报告[2~34]。早期的工作通常采用等效圆柱壳方法以简化研究[2,3]，但正如Tennyson(1976)[19]所指出，实验表明纵向加肋圆柱壳在轴压作用下的屈曲载荷并不总是和等效柱壳相一致。大量研究表明，有些实验结果与线性理论预测值符合较好[5,6,10~16]，而有些却较差[7,8]。影响加肋圆柱壳屈曲载荷的因素是很多的，如加载偏心，肋骨几何尺寸和布置，边界条件和非线性前屈曲效应．以及初始几何缺陷等等．singer及其合作者对各种影响因素进行了详细的实验研究和分析[9~16]，作出了重要贡献。研究表明，实验数据对线性理论的偏差一般在0.71～1.29的范围内[14]。倘若进一步考虑初始几何缺陷的影响，实验与理论之间的误差可降到10%之内[17]。此种情况自然要比未加肋圆柱壳的情况好得多。但是已有的理论分析还不足以解释全部实验现象。尽管对于加肋圆柱壳的初始后屈曲性态和缺陷敏感度已作过一些研究[23~25]，但是，对于加肋圆柱壳完整的后屈曲分析则较少见到发表，特别是对于非完善加肋圆柱壳。

正如作者在文[1]中所指出，为了对圆柱薄壳的屈曲和后屈曲性态有足够的认识和满意的解释，必须同时考虑非线性前屈曲、大挠度和初始几何缺陷的影响。对于加肋圆柱壳则还必须同时考虑肋骨几何和材料特性的影响。

本文的目的在于讨论完善和非完善的，纵向加肋和正交加肋圆柱壳在轴压作用下的屈曲和后屈曲性态。边界支承为固支的．初挠度的形式取作和圆柱壳非对称屈曲模态一致。本文

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同时讨论肋骨与壳板材料不同时对加肋圆柱壳屈曲和后屈曲性态的影响。

二、基 本 方 程

设加肋圆柱壳的半径为R，长度为L，壳板厚度为t，受到轴向压缩。设纵肋和环肋截面积分别为A_1，A_2；惯性矩分别为I_1，I_2；扭矩分别为J_1，J_2；偏心距分别为e_1，e_2；肋间间距分别为d_1，d_2；纵肋骨、环肋骨和壳板的弹性模数分别为E_1，E_2和E。取坐标系如图1所示，并以W^*和W分别表示初始的和附加的挠度，以F表示应力函数，即使

$$N_x=\partial^2F/\partial y^2,\quad N_y=\partial^2F/\partial x^2,\quad N_{xy}=-\partial^2F/\partial x\partial y \tag{2.1}$$

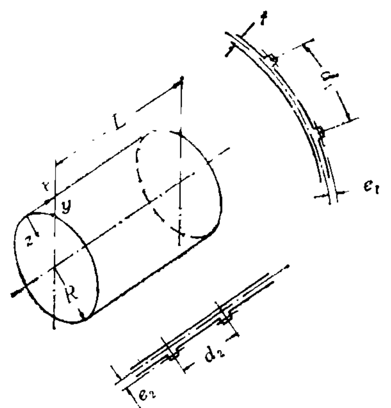


图1 加肋圆柱壳几何参数及坐标系

那么，加肋圆柱壳大挠度方程可表为如下形式

$$L_1^*W-L_2^*F-F_{,xx}/R=L^*(W+W^*,\ F) \tag{2.2}$$

$$L_3^*F+L_2W+W_{,xx}/R=-L^*(W+W^*,\ W)/2 \tag{2.3}$$

其中算子

$$\left.\begin{aligned}
&L_1^*=D_x\partial^4/\partial x^4+2(D_{xy}+2D_k)\partial^4/\partial x^2\partial y^2+D_y\partial^4/\partial y^4\\
&L_2^*=f_{xy}\partial^4/\partial x^4+(f_x+f_y)\partial^4/\partial x^2\partial y^2+f_{yx}\partial^4/\partial y^4\\
&L_3^*=\frac{1}{B_y}\frac{\partial^4}{\partial x^4}+2\left(\frac{1}{B_{xy}}+\frac{1}{2Gt}\right)\frac{\partial^4}{\partial x^2\partial y^2}+\frac{1}{B_x}\frac{\partial^4}{\partial y^4}\\
&L^*=\frac{\partial^2}{\partial x^2}\frac{\partial^2}{\partial y^2}-2\frac{\partial^2}{\partial x\partial y}\frac{\partial^2}{\partial x\partial y}+\frac{\partial^2}{\partial y^2}\frac{\partial^2}{\partial x^2}
\end{aligned}\right\} \tag{2.4}$$

式中抗弯、抗扭及耦合刚度在附录中给出。假定边界支承为固支的，那么边界条件为

$$x=0,\ L;\quad W=W_{,x}=0 \tag{2.5a}$$

$$\int_0^{2\pi R}N_xdy+P=0 \tag{2.5b}$$

闭合条件为

$$\int_0^{2\pi R}\frac{\partial V}{\partial y}dy=0 \tag{2.6}$$

单位端部缩短为

$$\frac{\Delta_x}{L}=-\frac{1}{2\pi RL}\int_0^{2\pi R}\int_0^L\frac{\partial U}{\partial x}dxdy \tag{2.7}$$

引进

$$\left.\begin{aligned}
&\bar{x}=\frac{\pi}{L}x,\quad \bar{y}=\frac{y}{R},\quad \beta=\frac{L}{\pi R},\quad z=\frac{L^2}{Rt}\sqrt{1-\nu^2},\quad \varepsilon=\frac{\pi^2R}{L^2}\sqrt[4]{\frac{D_xD_y}{B_xB_y}}\\
&(\overline{W},\ \overline{W}^*)=(W,\ W^*)\varepsilon\sqrt[4]{\frac{B_xB_y}{D_xD_y}},\quad \overline{F}=\frac{F}{\sqrt{D_xD_y}}\varepsilon^2,\quad \nu_y=-\frac{B_y}{B_{xy}}
\end{aligned}\right|$$

$$\left.\begin{aligned}&\gamma_{11}=\frac{D_{xy}+2D_k}{D_x},\quad \gamma_{12}^2=\frac{D_y}{D_x},\quad \gamma_{21}=B_y\left(\frac{1}{B_{xy}}+\frac{1}{2Gt}\right),\quad \gamma_{22}^2=\frac{B_y}{B_x}\\&(\gamma_{331},\ \gamma_{332},\ \gamma_{23},\ \gamma_{33},\ \gamma_{32})=(f_x,f_y,f_{xy},f_x+f_y,f_{yx})\sqrt[4]{B_xB_y/D_xD_y}\\&\lambda_P=\frac{P}{4\pi\sqrt[4]{D_xD_yB_xB_y}},\qquad \delta_P=\frac{\varDelta_x/L}{2R^{-1}\sqrt[4]{D_xD_y/B_xB_y}}\\&\lambda_0=\frac{P}{2\pi Et^2/\sqrt{3(1-\nu^2)}},\qquad \delta_0=\frac{\varDelta_x/L}{t/R\sqrt{3(1-\nu^2)}}\end{aligned}\right\}\tag{2.8}$$

那么，方程(2.2)、(2.3)可化为如下无量纲形式（略去字母上的“－”号）

$$\varepsilon^2L_1W-\varepsilon\gamma_{12}L_2F-\gamma_{12}F_{,xx}=\gamma_{12}\beta^2L(W+W^*,\ F)\tag{2.9}$$

$$L_3F+\varepsilon\gamma_{22}L_2W+\gamma_{22}W_{,xx}=-\gamma_{22}\beta^2L(W+W^*,\ W)/2\tag{2.10}$$

其中

$$\left.\begin{aligned}&L_1=\frac{\partial^4}{\partial x^4}+2\gamma_{11}\beta^2\frac{\partial^4}{\partial x^2\partial y^2}+\gamma_{12}^2\beta^4\frac{\partial^4}{\partial y^4}\\&L_2=\gamma_{23}\frac{\partial^4}{\partial x^4}+\gamma_{33}\beta^2\frac{\partial^4}{\partial x^2\partial y^2}+\gamma_{32}\beta^4\frac{\partial^4}{\partial y^4}\\&L_3=\frac{\partial^4}{\partial x^4}+2\gamma_{21}\beta^2\frac{\partial^4}{\partial x^2\partial y^2}+\gamma_{22}^2\beta^4\frac{\partial^4}{\partial y^4}\\&L=\frac{\partial^2}{\partial x^2}\frac{\partial^2}{\partial y^2}-2\frac{\partial^2}{\partial x\partial y}\frac{\partial^2}{\partial x\partial y}+\frac{\partial^2}{\partial y^2}\frac{\partial^2}{\partial x^2}\end{aligned}\right\}\tag{2.11}$$

边界条件化为

$$x=0,\ \pi;\ W=W_{,x}=0\tag{2.12a}$$

$$\frac{1}{2\pi}\int_0^{2\pi}\beta^2\frac{\partial^2F}{\partial y^2}dy+2\lambda_P\varepsilon=0\tag{2.12b}$$

闭合条件化为

$$\int_0^{2\pi}\left[\left(\frac{\partial^2F}{\partial x^2}-\nu_y\beta^2\frac{\partial^2F}{\partial y^2}\right)-\varepsilon\gamma_{22}\left(\gamma_{23}\frac{\partial^2W}{\partial x^2}+\gamma_{332}\beta^2\frac{\partial^2W}{\partial y^2}\right)+\gamma_{22}W\right.$$
$$\left.-\frac{1}{2}\gamma_{22}\beta^2\left(\frac{\partial W}{\partial y}\right)^2-\gamma_{22}\beta^2\frac{\partial W}{\partial y}\frac{\partial W^*}{\partial y}\right]dy=0\tag{2.13}$$

单位端部缩短化为

$$\delta_P=-\frac{1}{4\pi^2\gamma_{22}}\varepsilon^{-1}\int_0^{2\pi}\int_0^{\pi}\left[\left(\gamma_{22}^2\beta^2\frac{\partial^2F}{\partial y^2}-\nu_y\frac{\partial^2F}{\partial x^2}\right)-\varepsilon\gamma_{22}\left(\gamma_{331}\frac{\partial^2W}{\partial x^2}+\gamma_{32}\beta^2\frac{\partial^2W}{\partial y^2}\right)\right.$$
$$\left.-\frac{1}{2}\gamma_{22}\left(\frac{\partial W}{\partial x}\right)^2-\gamma_{22}\frac{\partial W}{\partial x}\frac{\partial W^*}{\partial x}\right]dxdy\tag{2.14}$$

式(2.9)至(2.14)即为边缘固定支承，完善和非完善加肋圆柱壳在轴压作用下屈曲问题的控制方程。

当$\varepsilon<1$时，方程(2.9)，(2.10)即为边界层型方程。按文[1]，可用奇异摄动法来构造其渐近解。

三、渐 近 解

设方程(2.9)、(2.10)的解可表为

$$\left.\begin{aligned}W&=w(x,y,\varepsilon)+\tilde{W}(x,\xi,y,\varepsilon)+\hat{W}(x,\zeta,y,\varepsilon)\\F&=f(x,y,\varepsilon)+\tilde{F}(x,\xi,y,\varepsilon)+\hat{F}(x,\zeta,y,\varepsilon)\end{aligned}\right\}\tag{3.1}$$

其中$w(x,y,\varepsilon)$, $f(x,y,\varepsilon)$为壳体正则解；$\tilde{W}(x,\xi,y,\varepsilon)$, $\tilde{F}(x,\xi,y,\varepsilon)$和$\hat{W}(x,\zeta,y,\varepsilon)$, $\hat{F}(x,\zeta,y,\varepsilon)$分别为$x=0$及$x=\pi$端边界层解，且边界层变量

$$\xi=x/\sqrt{\varepsilon},\quad \zeta=(\pi-x)/\sqrt{\varepsilon}\tag{3.2}$$

设正则解和边界层解为如下渐近展开

$$w(x,y,\varepsilon)=\sum_{j=0}\varepsilon^j w_j(x,y),\quad f(x,y,\varepsilon)=\sum_{j=0}\varepsilon^j f_j(x,y)\tag{3.3a}$$

$$\left.\begin{aligned}\tilde{W}(x,\xi,y,\varepsilon)&=\sum_{j=0}\varepsilon^{j+1}\tilde{W}_{j+1}(x,\xi,y)\\\tilde{F}(x,\xi,y,\varepsilon)&=\sum_{j=0}\varepsilon^{j+2}\tilde{F}_{j+2}(x,\xi,y)\end{aligned}\right\}\tag{3.3b}$$

$$\left.\begin{aligned}\hat{W}(x,\zeta,y,\varepsilon)&=\sum_{j=0}\varepsilon^{j+1}\hat{W}_{j+1}(x,\zeta,y)\\\hat{F}(x,\zeta,y,\varepsilon)&=\sum_{j=0}\varepsilon^{j+2}\hat{F}_{j+2}(x,\zeta,y)\end{aligned}\right\}\tag{3.3c}$$

并设屈曲载荷参数渐近展开为

$$2\lambda_P\varepsilon=K_x=\sum_{j=0}\varepsilon^j k_j\tag{3.4}$$

由于纵向加肋和正交加肋圆柱壳在轴压作用下屈曲模态通常为非对称形式，因此，取无量纲初挠度

$$W^*(x,y,\varepsilon)=\varepsilon^2A^*_{11}\sin mx\sin ny=\varepsilon^2\mu A^{(2)}_{11}\sin mx\sin ny\tag{3.5}$$

其中 $\mu=A^*_{11}/A^{(2)}_{11}$为缺陷参数。

将式(3.1)、(3.3)、(3.5)代入方程(2.9)、(2.10)得各级摄动方程，采用文[1]类似的摄动步骤，我们可以分别求得正则解和边界层解。根据式(3.1)我们有

$$\begin{aligned}W=&\varepsilon\Big[A^{(1)}_{00}-A^{(1)}_{00}\Big(\cos\phi\frac{x}{\sqrt{\varepsilon}}+\frac{\alpha}{\phi}\sin\phi\frac{x}{\sqrt{\varepsilon}}\Big)\exp\Big[-\alpha\frac{x}{\sqrt{\varepsilon}}\Big]-A^{(1)}_{00}\Big(\cos\phi\frac{\pi-x}{\sqrt{\varepsilon}}\\&+\frac{\alpha}{\phi}\sin\phi\frac{\pi-x}{\sqrt{\varepsilon}}\Big)\exp\Big[-\alpha\frac{\pi-x}{\sqrt{\varepsilon}}\Big]\Big]+\varepsilon^2\Big[A^{(2)}_{00}+A^{(2)}_{02}\cos 2ny+A^{(2)}_{11}\sin mx\sin ny\\&-(A^{(2)}_{00}+A^{(2)}_{02}\cos 2ny)\Big(\cos\phi\frac{x}{\sqrt{\varepsilon}}+\frac{\alpha}{\phi}\sin\phi\frac{x}{\sqrt{\varepsilon}}\Big)\exp\Big[-\alpha\frac{x}{\sqrt{\varepsilon}}\Big]\\&-(A^{(2)}_{00}+A^{(2)}_{02}\cos 2ny)\Big(\cos\phi\frac{\pi-x}{\sqrt{\varepsilon}}+\frac{\alpha}{\phi}\sin\phi\frac{\pi-x}{\sqrt{\varepsilon}}\Big)\exp\Big[-\alpha\frac{\pi-x}{\sqrt{\varepsilon}}\Big]\Big]\\&+\varepsilon^{5/2}\Big[-A^{(2)}_{11}\frac{m}{\phi}\sin\phi\frac{x}{\sqrt{\varepsilon}}\exp\Big[-\alpha\frac{x}{\sqrt{\varepsilon}}\Big]\sin ny-A^{(2)}_{11}\frac{m}{\phi}(-1)^m\sin\phi\frac{\pi-x}{\sqrt{\varepsilon}}\\&\cdot\exp\Big[-\alpha\frac{\pi-x}{\sqrt{\varepsilon}}\Big]\sin ny\Big]+\varepsilon^3[A^{(3)}_{00}+A^{(3)}_{02}\cos 2ny+A^{(3)}_{11}\sin mx\sin ny]\end{aligned}$$

$$+\varepsilon^4\Big[A_{00}^{(4)}+A_{20}^{(4)}\cos 2mx+A_{02}^{(4)}\cos 2ny+A_{13}^{(4)}\sin mx\sin 3ny+A_{04}^{(4)}\cos 4ny\Big]+\cdots \tag{3.6}$$

$$F=-B_{00}^{(0)}\frac{y^2}{2}+\varepsilon\Big[-B_{00}^{(1)}\frac{y^2}{2}\Big]+\varepsilon^2\Big[-B_{00}^{(2)}\frac{y^2}{2}+B_{11}^{(2)}\sin mx\sin ny+A_{00}^{(1)}\Big(d_{01}^{(2)}\cos\phi\frac{x}{\sqrt{\varepsilon}}$$

$$+d_{10}^{(2)}\sin\phi\frac{x}{\sqrt{\varepsilon}}\Big)\exp\Big[-\alpha\frac{x}{\sqrt{\varepsilon}}\Big]+A_{00}^{(1)}\Big(d_{01}^{(2)}\cos\phi\frac{\pi-x}{\sqrt{\varepsilon}}+d_{10}^{(2)}\sin\phi\frac{\pi-x}{\sqrt{\varepsilon}}\Big)$$

$$\cdot\exp\Big[-\alpha\frac{\pi-x}{\sqrt{\varepsilon}}\Big]\Big]+\varepsilon^3\Big[-B_{00}^{(3)}\frac{y^2}{2}+B_{02}^{(3)}\cos 2ny+(A_{00}^{(2)}+A_{02}^{(2)}\cos 2ny)$$

$$\cdot\Big(d_{01}^{(3)}\cos\phi\frac{x}{\sqrt{\varepsilon}}+d_{10}^{(3)}\sin\phi\frac{x}{\sqrt{\varepsilon}}\Big)\exp\Big[-\alpha\frac{x}{\sqrt{\varepsilon}}\Big]+(A_{00}^{(2)}+A_{02}^{(2)}\cos 2ny)$$

$$\cdot\Big(d_{01}^{(3)}\cos\phi\frac{\pi-x}{\sqrt{\varepsilon}}+d_{10}^{(3)}\sin\phi\frac{\pi-x}{\sqrt{\varepsilon}}\Big)\exp\Big[-\alpha\frac{\pi-x}{\sqrt{\varepsilon}}\Big]\Big]+\varepsilon^{7/2}\Big[A_{11}^{(2)}\frac{m}{\phi}d_{01}^{(7/2)}$$

$$\cdot\cos\phi\frac{x}{\sqrt{\varepsilon}}\exp\Big[-\alpha\frac{x}{\sqrt{\varepsilon}}\Big]\sin ny+A_{11}^{(2)}\frac{m}{\phi}(-1)^m d_{01}^{(7/2)}\cos\phi\frac{\pi-x}{\sqrt{\varepsilon}}$$

$$\cdot\exp\Big[-\alpha\frac{\pi-x}{\sqrt{\varepsilon}}\Big]\sin ny\Big]+\varepsilon^4\Big[-B_{00}^{(4)}\frac{y^2}{2}+B_{02}^{(4)}\cos 2ny+B_{11}^{(4)}\sin mx\sin ny$$

$$+B_{20}^{(4)}\cos 2mx+B_{13}^{(4)}\sin mx\sin 3ny\Big]+\cdots \tag{3.7}$$

式中各系数如文[35]给出。

由式(3.6)可以看出，由于边界层的贡献，即使对于完善壳体，前屈曲挠度亦具有非线性性质。

将式(3.7)代入边界条件(2.12b)，我们得到载荷参数

$$\lambda_P=\lambda_P^{(0)}-\lambda_P^{(2)}(A_{11}^{(2)}\varepsilon)^2+\lambda_P^{(4)}(A_{11}^{(2)}\varepsilon)^4+\cdots \tag{3.8a}$$

其中

$$\lambda_P^{(0)}=\frac{1}{2}\Big\{\frac{\gamma_{22}m^2}{(1+\mu)g_2}\varepsilon^{-1}-\frac{\gamma_{22}g_3}{g_2}\frac{(2+\mu)}{(1+\mu)^2}+\frac{1}{(1+\mu)m^2}\Big[\frac{g_1}{\gamma_{12}}+\frac{\gamma_{22}g_3^2}{g_2}\frac{(1-\mu-\mu^2)}{(1+\mu)^2}\Big]\varepsilon$$

$$+\frac{\mu}{(1+\mu)^2}\frac{g_3}{m^4}\Big[1-\frac{g_3}{(1+\mu)m^2}\varepsilon\Big]\Big[\frac{g_1}{\gamma_{12}}+\frac{\gamma_{22}g_3^2}{g_2}\frac{(4+4\mu+\mu^2)}{(1+\mu)^2}\Big]\varepsilon^2\Big\}$$

$$\lambda_P^{(2)}=\frac{1}{8}\Big\{\frac{1}{2}\Big(\frac{\gamma_{22}^2}{\gamma_{12}\gamma_{22}+\gamma_{32}^2}\Big)\frac{\gamma_{22}m^6(2+\mu)}{g_2^2}\varepsilon^{-1}-\frac{1}{2}\Big(\frac{\gamma_{22}^2}{\gamma_{12}\gamma_{22}+\gamma_{32}^2}\Big)\frac{m^4}{g_2}\Big[\frac{\gamma_{32}}{\gamma_{22}}$$

$$\cdot\frac{(1+\mu)^2+(1+2\mu)}{(1+\mu)}+\frac{\mu(3+\mu)}{(1+\mu)}\frac{\gamma_{22}g_3}{g_2}\Big]-\frac{1}{4}\Big(\frac{\gamma_{12}}{\gamma_{12}\gamma_{22}+\gamma_{32}^2}\Big)m^2(1+2\mu)\varepsilon$$

$$+\frac{\gamma_{22}m^2n^4\beta^4}{g_2}\cdot\frac{g_2(5+11\mu+4\mu^2)+8m^4(1+\mu)(2+\mu)}{g_2(1+\mu)-4m^4}\varepsilon$$

$$-\frac{1}{4}\Big(\frac{\gamma_{22}^2}{\gamma_{12}\gamma_{22}+\gamma_{32}^2}\Big)\frac{m^2g_3}{g_2}\Big[\frac{\gamma_{32}}{\gamma_{22}}\frac{(4+12\mu+15\mu^2+4\mu^3)}{(1+\mu)^2}+\frac{\gamma_{22}g_3}{g_2}\frac{2(4+\mu+2\mu+\mu^3)}{(1+\mu)^2}\Big]\varepsilon\Big\}$$

$$\lambda_P^{(4)}=\frac{1}{128}\Big\{\Big(\frac{\gamma_{22}^2}{\gamma_{12}\gamma_{22}+\gamma_{32}^2}\Big)^2\frac{\gamma_{22}m^{10}(1+\mu)}{g_2^3}\frac{g_{13}(6+6\mu+\mu^2)+g_2(6-\mu^2)(1+\mu)}{g_{13}-g_2(1+\mu)}\varepsilon^{-1}\Big\} \tag{3.8b}$$

将式(3.6)、(3.7)代入(2.14)，我们得到单位端部缩短

$$\delta_P=\left(\gamma_{22}-\frac{4}{\pi}\frac{a}{b}\frac{\nu_y^2}{\gamma_{22}}\varepsilon^{\frac{1}{2}}\right)\lambda_P+\left(\frac{1}{2\pi}\frac{b}{a}\frac{\nu_y^2}{\gamma_{22}^2}\varepsilon^{\frac{1}{2}}\right)\lambda_P^2$$

$$+\frac{1}{16}\left\{m^2\left[(1+2\mu)+\frac{1}{\pi a}\varepsilon^{\frac{1}{2}}\right]\varepsilon+\frac{g_3^2}{m^2}\varepsilon^3\right\}(A_{11}^{(2)}\varepsilon)^2$$

$$+\frac{1}{128}\left\{m^2n^4\beta^4(1+\mu)^2\left[\frac{g_2(1+2\mu)+8m^4(1+\mu)}{g_2(1+\mu)-4m^4}\right]^2\varepsilon^3\right\}(A_{11}^{(2)}\varepsilon)^4+\cdots \tag{3.9}$$

式中摄动参数

$$A_{11}^{(2)}\varepsilon=\overline{w}_m-\left\{\frac{1}{16}\left(\frac{\gamma_{22}^2}{\gamma_{12}\gamma_{22}+\gamma_{32}^2}\right)\frac{m^4(1+\mu)}{n^2\beta^2g_2}\varepsilon^{-1}-\frac{1}{32}\left(\frac{\gamma_{22}^2}{\gamma_{12}\gamma_{22}+\gamma_{32}^2}\right)\frac{m^2}{\gamma_{22}n^2\beta^2}\right.$$

$$\left.\cdot\left[\frac{\gamma_{32}}{\gamma_{22}}(1+2\mu)-2\frac{\gamma_{22}g_3}{g_2}\right]+\frac{2\nu_y}{\gamma_{22}}\lambda_P^{(2)}\right\}\overline{w}_m^3+\cdots \tag{3.10}$$

及最大无量纲挠度

$$\overline{w}_m=W_m\sqrt[4]{B_xB_y/D_xD_y}+2\nu_y\lambda_P^{(0)}/\gamma_{22} \tag{3.11}$$

式(3.8)至(3.11)中 g_1，g_2，g_3，g_{13} 由下式确定:

$$\left.\begin{aligned}&g_1=(m^4+2\gamma_{11}m^2n^2\beta^2+\gamma_{12}^2n^4\beta^4),\quad g_2=(m^4+2\gamma_{21}m^2n^2\beta^2+\gamma_{22}^2n^4\beta^4)\\&g_3=(\gamma_{23}m^4+\gamma_{33}m^2n^2\beta^2+\gamma_{32}n^4\beta^4),\quad g_{13}=(m^4+18\gamma_{21}m^2n^2\beta^2+81\gamma_{22}^2n^4\beta^4)\end{aligned}\right\} \tag{3.12}$$

式(3.8)表征加肋圆柱壳后屈曲载荷一挠度关系. 在式(3.8)中令 $W^*/t=0$(即 $\mu=0$)，取 $W_m=0$，我们得到完善加肋圆柱壳在轴压作用下的屈曲载荷. 但由(3.11)式，此时 $\overline{w}_m\neq0$，故 $A_{11}^{(2)}\varepsilon\neq0$. 因此，本文得到的屈曲载荷与小挠度解是有差别的. 这种差别主要是由于边界层效应引起的. 从理论上讲，只有当壳体足够长时，小参数 ε 趋于零，此时，边界层效应方可完全忽略. 事实上，计算表明，当几何参数 $Z>1000$ 时，边界层效应只有不大的影响，在设计分析中可以忽略.

四、数值结果与讨论

作为数值分析例子，表1给出边缘固定支承，外加肋圆柱壳屈曲载荷比较. 比较发现，本文计算结果较之文[10，12，13，16]计算结果更接近实验数据，且当计及不大的初挠度时，计算结果与实验结果变得相当吻合.

图2为纵向加肋圆柱壳屈曲载荷随几何参数 Z 的变化曲线.

图3为纵向外加肋和内加肋圆柱壳屈曲载荷之比随几何参数 Z 的变化曲线.

图示表明，在轴压作用下外加肋圆柱壳较之内加肋圆柱壳具有较高的屈曲载荷. 算例表明，当肋骨与壳板材料相同时($E_1/E=1.0$)，外加肋圆柱壳屈曲载荷较之内加肋圆柱壳屈曲载荷，在 $Z=200$ 时高出80%，而在 $Z=4000$ 时仅高出20%. 当肋骨与壳板材料不同时，对于 $E_1/E=0.75$，会使加肋圆柱壳屈曲载荷明显降低，在常用范围内($Z=10^2\sim10^3$)，对外加肋圆柱壳屈曲载荷降低8～15%，对内加肋圆柱壳屈曲载荷降低2～10%. 反之，对于 $E_1/E=1.25$，会使加肋圆柱壳屈曲载荷提高，并使外加肋与内加肋圆柱壳屈曲载荷之比最高接近2倍.

图4为纵向加肋和正交加肋圆柱壳在轴压作用下屈曲时的缺陷敏感度曲线. 其中 $\lambda^*=\lambda_P$(非完善壳)$/\lambda_P$(完善壳). 可以看出，加肋圆柱壳与未加肋圆柱壳相类似，其缺陷敏感度也与

表1　　纵向加肋圆柱壳屈曲载荷比较($E=73600$MPa, $\nu=0.3$)

文献	L (mm)	R (mm)	t (mm)	$\frac{A_1}{d_1t}$	$-\frac{e_1}{t}$	$\frac{I_1}{d_1t^3}$	$\frac{G_1J_1}{d_1D}$	Singer 及其合作者 P_{exp} (kN)	P_{exp}/P_{cr} 简支	P_{exp}/P_{cr} 固支	本文 W^*/t	P_{cr} (kN)	$\frac{P_{exp}}{P_{cr}}$
[16]	110.0	120.1	0.257	0.700	3.95	2.80	—	46.11(1,9)*	0.94	0.51	0.0	45.75(1,10)	1.008
	110.0	120.1	0.253	0.700	3.97	2.83	—	48.07(1,9)	1.01	0.55	0.0	44.28(1,10)	1.086
	154.0	120.1	0.253	0.700	3.95	2.78	—	33.35(1,8)	0.86	0.47	0.0	35.77(1,10)	0.93
											0.03	33.52(1,10)	0.995
	154.0	120.1	0.253	0.700	3.94	2.75	—	35.46(1,8)	0.91	0.50	0.0	35.71(1,10)	0.993
	130.0	120.1	0.252	0.680	3.87	2.59	—	35.12(1,9)	0.93	0.51	0.0	37.86(1,10)	0.93
											0.03	35.25(1,10)	0.996
	130.0	120.1	0.254	0.590	3.42	1.68	—	39.49(1,9)	1.09	0.62	0.0	33.44(1,10)	1.18
[13]	130.0	120.1	0.251	0.698	3.97	2.809	8.513	42.18	0.66	0.64	0.0	45.20(1,10)	0.93
											0.02	42.95(1,10)	0.982
	215.0	120.1	0.249	0.799	4.48	4.236	10.450	46.04	0.72	0.68	0.0	38.54(1,8)	1.19
[12]	150.0	120.1	0.237	0.220	1.61	0.092	0.98	19.77(2,10)	0.84	0.83	0.0	20.19(2,14)	0.979
	150.0	120.1	0.234	0.440	2.70	0.713	4.36	27.47(1,9)	0.73	0.71	0.0	24.03(1,10)	1.14
[10]	139.7	101.6	0.197	0.506	1.72	0.2466	—	14.47	1.04	0.697	0.0	14.59(1,10)	0.992
	139.7	101.6	0.281	0.330	1.92	0.0679	—	22.42	0.91	0.703	0.0	27.98(1,9)	0.80
											0.20	22.23(1,9)	1.008
	139.7	101.6	0.259	0.235	1.07	0.0248	—	17.01	0.88	0.731	0.0	21.44(1,9)	0.79
											0.20	17.24(1,9)	0.987

* 括号内的值为屈曲模态(m, n)

壳体几何尺寸有关。计算还表明：

短壳对初始缺陷表现不敏感；

对于中等长度壳体，缺陷敏感度随几何参数Z增加而减弱；

内加肋圆柱壳对初始缺陷的敏感度大体与未加肋圆柱壳相同，而外加肋圆柱壳则更为敏感一点；

加强肋较之加弱肋，壳体对初始缺陷敏感度相对弱一点；

肋骨与壳板材料不同对壳体缺陷敏感度影响不大。

图 5 显示肋骨扭转刚度对纵向加肋圆柱壳屈曲载荷的影响。其中 $\bar{\lambda}=\lambda_P(G_1J_1/d_1D=C)/\lambda_P(G_1J_1/d_1D=0)$。

算例表明，肋骨扭转刚度对内加肋圆柱壳屈曲载荷具有更大的影响。肋骨扭转刚度$G_1J_1/d_1D=10.0$较之$G_1J_1/d_1D=0.0$，在$Z=200$时，内加肋圆柱壳屈曲载荷高出42%，外加肋圆柱壳屈曲载荷只高出27%，而在$Z=4000$时，内加肋圆柱壳屈曲载荷高出12%，外加肋圆柱壳屈曲载荷高出11%，两者已相当接近。

图 6 为肋骨扭转刚度或强弱不同时，纵向加肋圆柱壳后屈曲载荷一缩短曲线。可以看

出：

肋骨扭转刚度增加，不仅屈曲载荷增加，而且具有相对高的后屈曲最小载荷；

加弱肋较之加强肋，屈曲载荷，后屈曲最小载荷都明显低得多。

图 7 为肋骨与壳板材料不同时，加肋圆柱壳在轴压作用下的后屈曲载荷—缩短曲线比较。计算表明：

外加肋圆柱壳较之内加肋圆柱壳具有相对高的屈曲载荷，亦具有相对高的后屈曲最小载荷；

通常后屈曲最小载荷随几何参数Z的增加而降低；

随着几何参数Z的增加，外加肋和内加肋圆柱壳的屈曲载荷，后屈曲最小载荷之间的差别逐渐减小。因此，当Z较大时，内外加肋圆柱壳在轴压作用下，实验的屈曲载荷差别不大，后屈曲最小载荷的差别也不大[28]。

肋骨与壳板材料不同对后屈曲最小载荷的影响较之对屈曲载荷的影响更大。

图 8 为典型的非完善、纵向加肋和正交加肋圆柱壳在轴压作用下后屈曲载荷—缩短曲线和载荷—挠度曲线。如同未加肋圆柱壳受轴压作用，初始几何缺陷显著地降低屈曲载荷和后屈曲最小载荷。

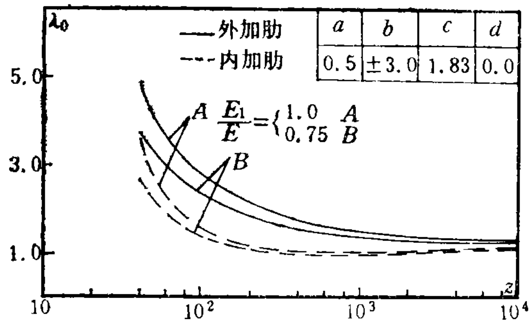


图2　纵向加肋圆柱壳屈曲载荷曲线

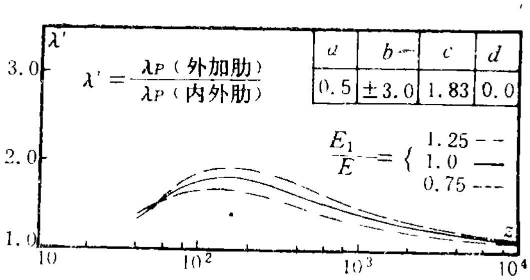
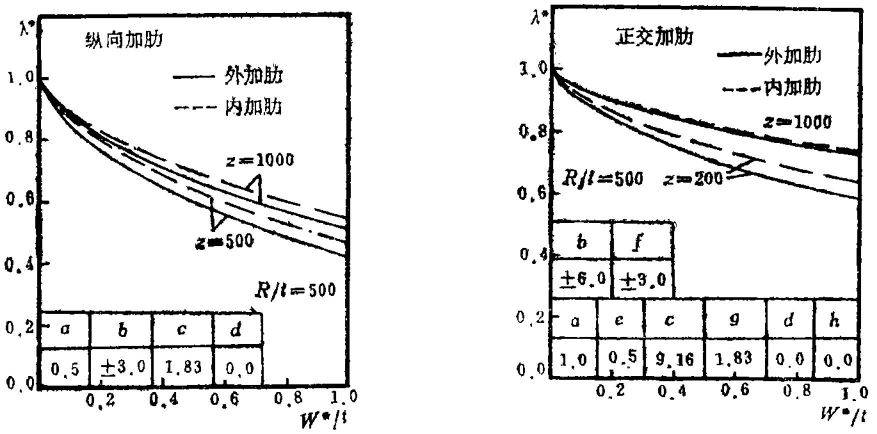


图3　纵向外加肋和内加肋屈曲载荷之比曲线



(a)　　　　(b)

图4　加肋圆柱壳缺陷敏感度曲线

* 图2～8中 $a=A_1/d_1t$，$b=e_1/t$，$c=I_1/d_1t^3$，$d=G_1J_1/d_1D$，$e=A_2/d_2t$，$f=e_2/t$，$g=I_2/d_2t^3$，$h=G_2J_2/d_2D$。

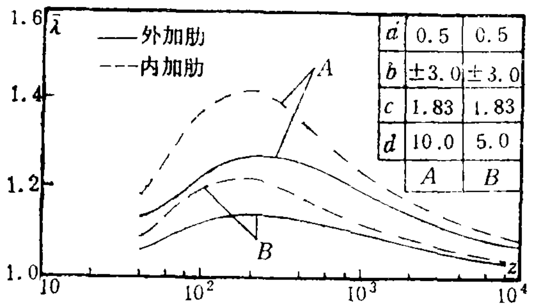


图5 肋骨扭转刚度对纵向加肋圆柱壳屈曲载荷影响

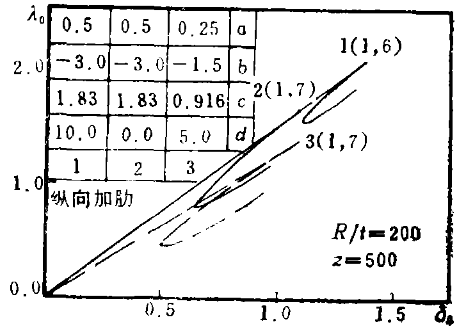
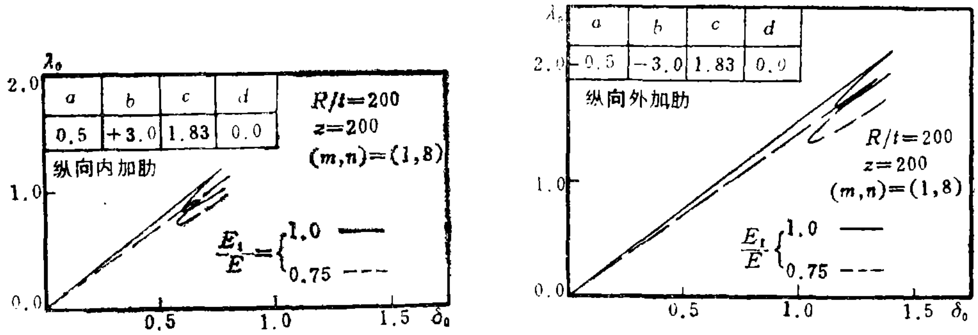


图6 肋骨几何参数对加肋圆柱壳载荷—缩短曲线影响



(a) (b)

图7 肋骨与壳板材料不同时，加肋圆柱壳载荷—缩短曲线比较

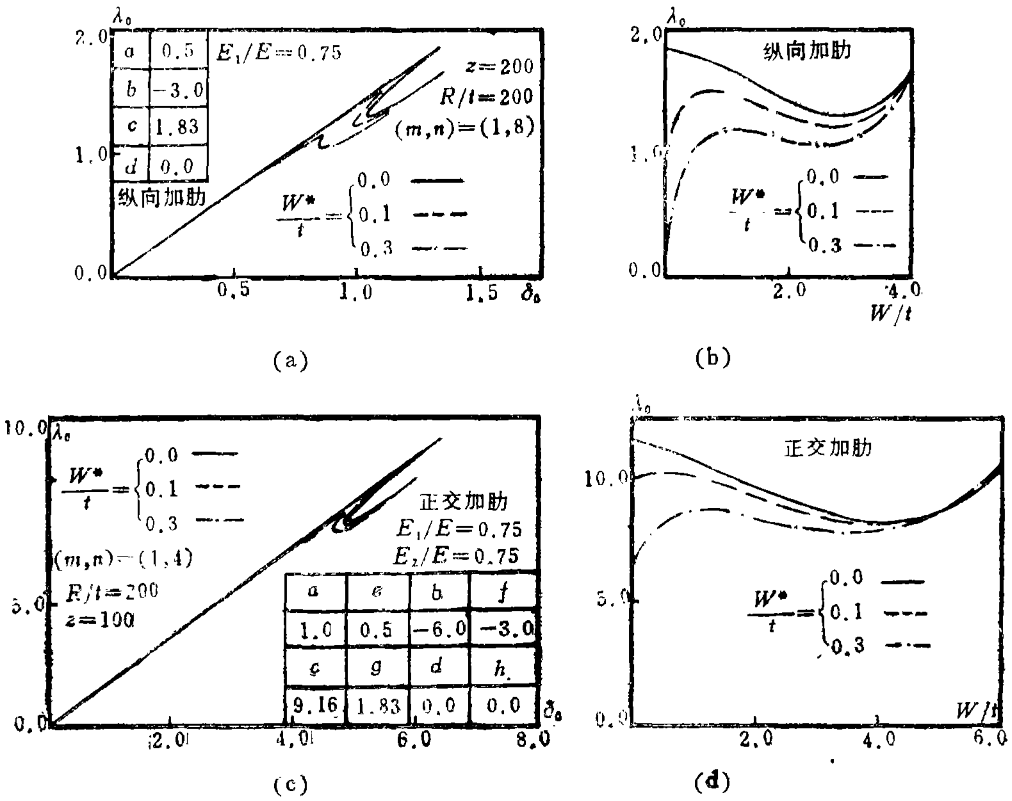


图8 非完善加肋圆柱壳后屈曲平衡路径

五、结　语

本文依据圆柱薄壳屈曲的边界层理论，同时考虑非线性前屈曲，大挠度，初始几何缺陷，以及肋骨几何和材料特性对加肋圆柱壳屈曲和后屈曲性态的影响。根据计算所得结果，我们可以得出：

（1） 外加肋圆柱壳相对未加肋圆柱壳，虽然其对初始缺陷较为敏感，但由于加肋圆柱壳本身的缺陷程度较小，且其具有较高的后屈曲最小载荷，因此，屈曲载荷的实验值只有较小的离散度。

（2） 某些加弱肋圆柱壳，尽管初始缺陷较小，但其后屈曲最小载荷较低，因此，仍有可能在较低的轴向载荷作用下发生屈曲。

（3） 肋骨与壳板材料不同尽管对壳体缺陷敏感度影响不大，但由于显著改变加肋壳的屈曲载荷与后屈曲最小载荷，因此对实验结果会有明显影响。

总之，加肋圆柱壳的缺陷敏感度，屈曲和后屈曲性态主要依赖于壳体和肋骨本身的特性。

附　录

式(2.4)中各刚度定义为

$$D_x=Et^3\left[\frac{1}{12(1-\nu^2)}+\frac{E_1}{E}\frac{I_1}{d_1t^3}+\frac{E_1}{E}\frac{A_1}{d_1t}\left(\frac{e_1}{t}\right)^2-\frac{\left[1+\frac{E_2}{E}\frac{A_2}{d_2t}(1-\nu^2)\right]\left(\frac{E_1}{E}\frac{A_1}{d_1t}\frac{e_1}{t}\right)^2(1-\nu^2)}{\left[1+\frac{E_2}{E}\frac{A_2}{d_2t}(1-\nu^2)\right]\left[1+\frac{E_1}{E}\frac{A_1}{d_1t}(1-\nu^2)\right]-\nu^2}\right]$$

$$D_y=Et^3\left[\frac{1}{12(1-\nu^2)}+\frac{E_2}{E}\frac{I_2}{d_2t^3}+\frac{E_2}{E}\frac{A_2}{d_2t}\left(\frac{e_2}{t}\right)^2-\frac{\left[1+\frac{E_1}{E}\frac{A_1}{d_1t}(1-\nu^2)\right]\left(\frac{E_2}{E}\frac{A_2}{d_2t}\frac{e_2}{t}\right)^2(1-\nu^2)}{\left[1+\frac{E_1}{E}\frac{A_1}{d_1t}(1-\nu^2)\right]\left[1+\frac{E_2}{E}\frac{A_2}{d_2t}(1-\nu^2)\right]-\nu^2}\right]$$

$$D_{xy}=\nu Et^3\left[\frac{1}{12(1-\nu^2)}+\frac{\left(\frac{E_1}{E}\frac{A_1}{d_1t}\frac{e_1}{t}\right)\left(\frac{E_2}{E}\frac{A_2}{d_2t}\frac{e_2}{t}\right)(1-\nu^2)}{\left[1+\frac{E_1}{E}\frac{A_1}{d_1t}(1-\nu^2)\right]\left[1+\frac{E_2}{E}\frac{A_2}{d_2t}(1-\nu^2)\right]-\nu^2}\right]$$

$$D_k=\frac{D}{2}\left[(1-\nu)+\frac{1}{2}\left(\frac{G_1J_1}{d_1D}+\frac{G_2J_2}{d_2D}\right)\right]$$

$$f_x=t\frac{\left[1+\frac{E_2}{E}\frac{A_2}{d_2t}(1-\nu^2)\right]\left(\frac{E_1}{E}\frac{A_1}{d_1t}\frac{e_1}{t}\right)(1-\nu^2)}{\left[1+\frac{E_1}{E}\frac{A_1}{d_1t}(1-\nu^2)\right]\left[1+\frac{E_2}{E}\frac{A_2}{d_2t}(1-\nu^2)\right]-\nu^2}$$

$$f_y=t\frac{\left[1+\frac{E_1}{E}\frac{A_1}{d_1t}(1-\nu^2)\right]\left(\frac{E_2}{E}\frac{A_2}{d_2t}\frac{e_2}{t}\right)(1-\nu^2)}{\left[1+\frac{E_1}{E}\frac{A_1}{d_1t}(1-\nu^2)\right]\left[1+\frac{E_2}{E}\frac{A_2}{d_2t}(1-\nu^2)\right]-\nu^2}$$

$$f_{xy}=-\nu t\frac{\left(\frac{E_1}{E}\frac{A_1}{d_1t}\frac{e_1}{t}\right)(1-\nu^2)}{\left[1+\frac{E_1}{E}\frac{A_1}{d_1t}(1-\nu^2)\right]\left[1+\frac{E_2}{E}\frac{A_2}{d_2t}(1-\nu^2)\right]-\nu^2}$$

$$f_{yx}=-\nu t\frac{\left(\frac{E_2}{E}\frac{A_2}{d_2t}\frac{e_2}{t}\right)(1-\nu^2)}{\left[1+\frac{E_1}{E}\frac{A_1}{d_1t}(1-\nu^2)\right]\left[1+\frac{E_2}{E}\frac{A_2}{d_2t}(1-\nu^2)\right]-\nu^2}$$

$$\frac{1}{B_x}=\frac{1}{Et}\frac{\left[1+\frac{E_1}{E}\frac{A_2}{d_2t}(1-\nu^2)\right](1-\nu^2)}{\left[1+\frac{E_1}{E}\frac{A_1}{d_1t}(1-\nu^2)\right]\left[1+\frac{E_2}{E}\frac{A_2}{d_2t}(1-\nu^2)\right]-\nu^2}$$

$$\frac{1}{B_y}=\frac{1}{Et}\frac{\left[1+\frac{E_1}{E}\frac{A_1}{d_1t}(1-\nu^2)\right](1-\nu^2)}{\left[1+\frac{E_1}{E}\frac{A_1}{d_1t}(1-\nu^2)\right]\left[1+\frac{E_2}{E}\frac{A_2}{d_2t}(1-\nu^2)\right]-\nu^2}$$

$$\frac{1}{B_{xy}}=-\frac{\nu}{Et}\frac{(1-\nu^2)}{\left[1+\frac{E_1}{E}\frac{A_1}{d_1t}(1-\nu^2)\right]\left[1+\frac{E_2}{E}\frac{A_2}{d_2t}(1-\nu^2)\right]-\nu^2}$$

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Buckling and Postbuckling of Stiffened Cylindrical Shells under Axial Compression

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Abstract

Buckling and postbuckling behaviors of perfect and imperfect, stringer and orthotropically stiffened cylindrical shells have been studied under axial compression. Based on the boundary layer theory for the buckling of thin elastic shells suggested in ref. [1], a theoretical analysis is presented. The effects of material properties of stiffeners and skin, which are made of different materials, on the buckling load and postbuckling behavior of stiffened cylindrical shells have also been discussed.

Key words structural stability, buckling, postbuckling, stiffened cylindrical shell