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弹性矩形薄板受迫振动的功的互等定理法(Ⅱ)——两邻边固定的矩形板*

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摘　要

在本文，我们应用功的互等定理法给出了两邻边固定矩形板在分布和集中谐载作用下固定边弯矩幅值和自由边挠度幅值的分布.

关键词　功的互等定理法　弹性薄板　受迫振动　弯矩幅值　挠度幅值

一、两邻边固定，另两邻边自由在均布简谐干扰力作用下的矩形板

现在让我们研究如图1所示的矩形板．解除两固定边的弯曲约束并以弯矩M_{x0}和M_{y0}代替它们，我们便得到作为一实际系统的如图2所示的矩形板．

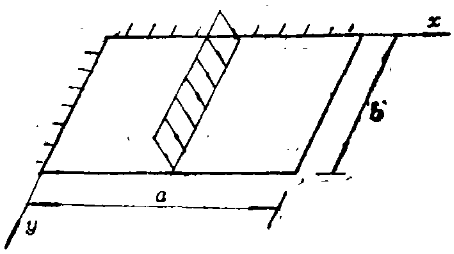

图 1

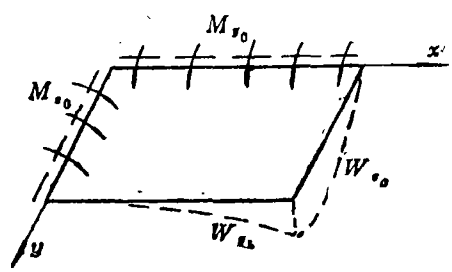

图 2

假设M_{x0}和M_{y0}为

$$\left.\begin{aligned} M_{y0}&=\sum_{m=1,2}^{\infty}A_m\sin k_m x \\ M_{x0}&=\sum_{n=1,2}^{\infty}B_n\sin k_n y \end{aligned}\right\} \tag{1.1}$$

及两自由边的位移幅值为

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$$\left.\begin{aligned} W_{y_b} &= \sum_{m=1,2}^{\infty} C_m \sin k_m x + \frac{x}{a} k \\ W_{x_0} &= \sum_{n=1,2}^{\infty} D_n \sin k_n y + \frac{y}{b} k \end{aligned}\right\} \tag{1.2}$$

在文[1]图1基本系统与本文图2实际系统之间应用功的互等定理，我们得到

$$W(\xi,\ \eta)=\int_0^a\int_0^b qW_1 dxdy+\int_0^a M_{y0}\left(\frac{\partial W_1}{\partial y}\right)_{y=0} dx+\int_0^b M_{x_0}\left(\frac{\partial W_1}{\partial x}\right)_{x=0} dy$$

$$-\int_0^b (V_{1x})_{x=a} W_{xa} dy-\int_0^a (V_{1y})_{y=b} W_{yb} dx+(R_1)_{\substack{x=a\\y=b}} K \tag{1.3}$$

当$\lambda<k_m^2$和$\lambda<k_n^2$时，式(1.3)转换成为

$$W(\xi,\eta)=\frac{4q}{\pi D}\sum_{m=1,3}^{\infty}\frac{1}{m}\left\{\frac{1}{\alpha_m^2-\beta_m^2}\left[\frac{\text{ch}\alpha_m\left(\frac{b}{2}-\eta\right)}{\alpha_m^2\text{ch}\alpha_m\frac{b}{2}}-\frac{\text{ch}\beta_m\left(\frac{b}{2}-\eta\right)}{\beta_m^2\text{ch}\beta_m\frac{b}{2}}\right]+\frac{1}{\alpha_m^2\beta_m^2}\right\}\sin k_m\xi$$

$$\left(\text{或}\ \frac{4q}{\pi D}\sum_{n=1,3}^{\infty}\frac{1}{n}\left\{\frac{1}{\alpha_n^2-\beta_n^2}\left[\frac{\text{ch}\alpha_n\left(\frac{a}{2}-\xi\right)}{\alpha_n^2\text{ch}\alpha_n\frac{a}{2}}-\frac{\text{ch}\beta_n\left(\frac{a}{2}-\xi\right)}{\beta_n^2\text{ch}\beta_n\frac{a}{2}}\right]+\frac{1}{\alpha_n^2\beta_n^2}\right\}\sin k_n\eta\right)$$

$$+\sum_{m=1,2}^{\infty}\frac{A_m}{(\alpha_m^2-\beta_m^2)D}\left[-\frac{\text{sh}\alpha_m(b-\eta)}{\text{sh}\alpha_m b}+\frac{\text{sh}\beta_m(b-\eta)}{\text{sh}\beta_n b}\right]\sin k_m\xi$$

$$+\sum_{n=1,2}^{\infty}\frac{B_n}{(\alpha_n^2-\beta_n^2)D}\left[-\frac{\text{sh}\alpha_n(a-\xi)}{\text{sh}\alpha_n a}+\frac{\text{sh}\beta_n(a-\xi)}{\text{sh}\beta_n a}\right]\sin k_n\eta$$

$$+\sum_{n=1,2}^{\infty}\frac{D_n}{\alpha_n^2-\beta_n^2}\left\{\frac{[\alpha_n^2-k_n^2(2-\mu)]}{\text{sh}\alpha_n a}\text{sh}\alpha_n\xi-\frac{[\beta_n^2-k_n^2(2-\mu)]}{\text{sh}\beta_n a}\text{sh}\beta_n\xi\right\}\sin k_n\eta$$

$$+\sum_{m=1,2}^{\infty}\frac{C_m}{\alpha_m^2-\beta_m^2}\left\{\frac{[\alpha_m^2-k_m^2(2-\mu)]}{\text{sh}\alpha_m b}\text{sh}\alpha_m\eta-\frac{[\beta_m^2-k_m^2(2-\mu)]}{\text{sh}\beta_m b}\text{sh}\beta_m\eta\right\}\sin k_m\xi$$

$$-\sum_{m=1,2}^{\infty}\frac{2\lambda^2}{m\pi}k\left\{\frac{1}{\alpha_m^2-\beta_m^2}\left[\frac{\text{sh}\alpha_m\eta}{\alpha_m^2\text{sh}\alpha_m b}-\frac{\text{sh}\beta_m\eta}{\beta_m^2\text{sh}\beta_m b}\right]\right.$$

$$\left.+\frac{\eta}{\alpha_m^2\beta_m^2 b}\right\}\cos m\pi\sin k_m\xi+\left(\frac{\xi}{a}\right)\left(\frac{\eta}{b}\right)k \tag{1.4}$$

当$\lambda>k_m^2$和$\lambda>k_n^2$时，我们只需分别以$i\beta_m'$和$i\beta_n'$代替β_m和β_n，便可得相应的表达式$W(\xi,\eta)$，因而它在这里被省略了.

执行相应的边界条件，对于$\lambda<k_m^2$和$\lambda<k_n^2$，我们得到

$$\frac{2q[1-(-1)^m]}{m\pi D}\left(-\frac{\text{th}\alpha_m\frac{b}{2}}{\alpha_m}+\frac{\text{th}\beta_m\frac{b}{2}}{\beta_m}\right)+\frac{A_m}{D}(\alpha_m\text{cth}\alpha_m b-\beta_m\text{cth}\beta_m b)$$

$$+\frac{4\lambda}{aD}\sum_{n=1,2}^{\infty}\frac{\beta_n k_m k_n}{K_{mn}}-\frac{4\lambda}{a}\sum_{n=1,2}^{\infty}\frac{D_n k_m k_n}{K_{mn}}[k_m^2+k_n^2(2-\mu)]\cos m\pi$$

$$+C_m\left\{\frac{\alpha_m[\alpha_m^2-k_n^2(2-\mu)]}{\mathrm{sh}\alpha_m b}-\frac{\beta_m[\beta_m^2-k_n^2(2-\mu)]}{\mathrm{sh}\beta_m b}\right\}$$

$$-\frac{2}{m\pi}k\left[\frac{2\lambda}{b}+\lambda^2\left(\frac{1}{\alpha_m\mathrm{sh}\alpha_m b}-\frac{1}{\beta_m\mathrm{sh}\beta_m b}\right)+\frac{2\lambda^3}{\alpha_m^2\beta_m^2 b}\right]\cos m\pi=0 \tag{1.5}$$

$$\frac{2q[1-(-1)^n]}{n\pi D}\left(-\frac{\mathrm{th}\alpha_n\frac{a}{2}}{\alpha_n}+\frac{\mathrm{th}\beta_n\frac{a}{2}}{\beta_n}\right)+\frac{4\lambda}{bD}\sum_{m=1,2}^{\infty}\frac{A_m k_m k_n}{K_{mn}}$$

$$+\frac{B_n}{D}(\alpha_n\mathrm{cth}\alpha_n a-\beta_n\mathrm{cth}\beta_n a)+D_n\left\{\frac{\alpha_n[\alpha_n^2-k_n^2(2-\mu)]}{\mathrm{sh}\alpha_n a}-\frac{\beta_n[\beta_n^2-k_n^2(2-\mu)]}{\mathrm{sh}\beta_n a}\right\}$$

$$-\frac{4\lambda}{b}\sum_{m=1,2}^{\infty}\frac{C_m k_m k_n}{K_{mn}}[k_n^2+k_m^2(2-\mu)]\cos n\pi$$

$$-\frac{2}{n\pi}k\left[\frac{2\lambda}{a}+\lambda^2\left(\frac{1}{\alpha_n\mathrm{ch}\alpha_n a}-\frac{1}{\beta_n\mathrm{sh}\beta_n a}\right)+\frac{2\lambda^3}{\alpha_n^2\beta_n^2 a}\right]\cos n\pi=0 \tag{1.6}$$

$$\frac{2q[1-(-1)^m]}{m\pi D}\left\{\frac{[\alpha_m^2-k_m^2(2-\mu)]}{\alpha_m}\mathrm{th}\alpha_m\frac{b}{2}-\frac{[\beta_m^2-k_m^2(2-\mu)]}{\beta_m}\mathrm{th}\beta_m\frac{b}{2}\right\}$$

$$+\frac{A_m}{D}\left\{\frac{\alpha_m}{\mathrm{sh}\alpha_m b}[\alpha_m^2-k_m^2(2-\mu)]-\frac{\beta_m}{\mathrm{sh}\beta_m b}[\beta_m^2-k_m^2(2-\mu)]\right\}$$

$$-\frac{4\lambda}{aD}\sum_{n=1,2}^{\infty}\frac{B_n k_m k_n}{K_{mn}}[k_n^2+k_m^2(2-\mu)]\cos n\pi+\frac{4\lambda}{a}\sum_{n=1}^{\infty}\frac{D_n k_m k_n}{K_{mn}}[\lambda^2(2-\mu)$$

$$+(1-\mu)^2k_m^2k_n^2]\cos n\pi\cos m\pi+C_m\{\alpha_m[\alpha_m^2-k_m^2(2-\mu)]^2\mathrm{cth}\alpha_m b-\beta_m[\beta_m^2$$

$$-k_m^2(2-\mu)]^2\,\mathrm{cth}\beta_m b\}-\frac{2\lambda^2}{m\pi}k\left\{\frac{\alpha_m^2-k_m^2(2-\mu)]}{\alpha_m}\mathrm{cth}\alpha_m b\right.$$

$$\left.-\frac{[\beta_m^2-k_m^2(2-\mu)]}{\beta_m}\mathrm{cth}\beta_m b-\frac{2\lambda(2-\mu)}{\alpha_m^2\beta_m^2 b}k_m^2\right\}\cos m\pi=0 \tag{1.7}$$

$$\frac{2q[1-(-1)^n]}{n\pi D}\left\{\frac{[\alpha_n^2-k_n^2(2-\mu)]}{\alpha_n}\mathrm{th}\alpha_n\frac{a}{2}-\frac{[\beta_n^2-k_n^2(2-\mu)]}{\beta_n}\mathrm{th}\beta_n\frac{a}{2}\right\}$$

$$-\frac{4\lambda}{bD}\sum_{m=1,2}^{\infty}\frac{A_m k_m k_n}{K_{mn}}[k_m^2+k_n^2(2-\mu)]\cos m\pi+\frac{B_n}{D}\left\{\frac{\alpha_n}{\mathrm{sh}\alpha_n a}[\alpha_n^2-k_n^2(2-\mu)]\right.$$

$$\left.-\frac{\beta_n}{\mathrm{sh}\beta_n a}[\beta_n^2-k_n^2(2-\mu)]\right\}+D_n\{\alpha_n[\alpha_n^2-k_n^2(2-\mu)]^2\mathrm{cth}\alpha_n a$$

$$-\beta_n[\beta_n^2-k_n^2(2-\mu)]^2\mathrm{cth}\beta_n a\}+\frac{4\lambda}{D}\sum_{m=1,2}^{\infty}\frac{C_m k_m k_n}{K_{mn}}[\lambda^2(2-\mu)$$

$$+(1-\mu)^2k_m^2k_n^2]\cos m\pi\cos n\pi-\frac{2\lambda^2}{n\pi}k\left\{\frac{[\alpha_n^2-k_n^2(2-\mu)]}{\alpha_n}\mathrm{cth}\alpha_n a\right.$$

$$\left.-\frac{[\beta_n^2-k_n^2(2-\mu)]}{\beta_n}\mathrm{cth}\beta_n a-\frac{2\lambda(2-\mu)}{\alpha_n^2\beta_n^2 a}k_n^2\right\}\cos n\pi=0 \tag{1.8}$$

$$\frac{2q}{aD}\sum_{m=1,2}^{\infty}[1-(-1)^m]\left(\frac{\operatorname{th}\alpha_m\frac{b}{2}}{\alpha_m}-\frac{\operatorname{th}\beta_m\frac{b}{2}}{\beta_m}\right)\cos m\pi+\sum_{m=1,2}^{\infty}\frac{A_mk_m}{D}\left(\frac{\alpha_m}{\operatorname{sh}\alpha_m b}\right.$$

$$\left.-\frac{\beta_m}{\operatorname{sh}\beta_n b}\right)\cos m\pi+\sum_{n=1,2}^{\infty}\frac{B_nk_n}{D}\left(\frac{\alpha_n}{\operatorname{sh}\alpha_n a}-\frac{\beta_n}{\operatorname{sh}\beta_n a}\right)\cos n\pi$$

$$+\sum_{n=1,2}^{\infty}D_nk_n\{\alpha_n[\alpha_n^2-k_n^2(2-\mu)]\operatorname{cth}\alpha_n a-\beta_n[\beta_n^2-k_n^2(2-\mu)]\operatorname{cth}\beta_n a\}\cos n\pi$$

$$+\sum_{m=1,2}^{\infty}C_mk_m\{\alpha_m[\alpha_m^2-k_m^2(2-\mu)]\operatorname{cth}\alpha_m b-\beta_m[\beta_m^2-k_m^2(2-\mu)]\operatorname{cth}\beta_m b\}\cos m\pi$$

$$+k\left\{\frac{2\lambda}{ab}-\frac{2\lambda^2}{a}\sum_{m=1,2}^{\infty}\left[\left(\frac{\operatorname{cth}\alpha_m b}{\alpha_m}-\frac{\operatorname{cth}\beta_m b}{\beta_m}\right)+\frac{2\lambda}{\alpha_m^2\beta_m^2 b}\right]\right\}=0 \tag{1.9}$$

而对于$\lambda>k_m^2$和$\lambda>k_n^2$，有

$$\frac{2q[1-(-1)^m]}{m\pi D}\left(-\frac{\operatorname{th}\alpha_m\frac{b}{2}}{\alpha_m}+\frac{\operatorname{tg}\beta_m'\frac{b}{2}}{\beta_m'}\right)+\frac{A_m}{D}(\alpha_m\operatorname{cth}\alpha_m b-\beta_m'\operatorname{ctg}\beta_m' b)$$

$$+\frac{4\lambda}{aD}\sum_{n=1,2}^{\infty}\frac{B_nk_mk_n}{K_{mn}}-\frac{4\lambda}{a}\sum_{n=1,2}^{\infty}\frac{D_nk_mk_n}{K_{mn}}[k_m^2+k_n^2(2-\mu)]\cos m\pi$$

$$+C_m\left\{\frac{\alpha_m[\alpha_m^2-k_m^2(2-\mu)]}{\operatorname{sh}\alpha_m b}+\frac{\beta_m'[\beta_m'^2+k_m^2(2-\mu)]}{\sin\beta_m' b}\right\}-\frac{2k}{n\pi}\left[\frac{2\lambda}{a}\right.$$

$$\left.+\lambda^2\left(\frac{1}{\alpha_m\operatorname{sh}\alpha_m a}+\frac{1}{\beta_m'\sin\beta_m' a}\right)-\frac{2\lambda^3}{\alpha_m^2\beta_m'^2 b}\right]\cos m\pi=0 \tag{1.10}$$

$$\frac{2q[1-(-1)^n]}{n\pi D}\left(-\frac{\operatorname{th}\alpha_n\frac{a}{2}}{\alpha_n}+\frac{\operatorname{tg}\beta_n'\frac{a}{2}}{\beta_n'}\right)+\frac{4\lambda}{bD}\sum_{m=1,2}^{\infty}\frac{A_mk_mk_n}{K_{mn}}+\frac{B_n}{D}(\alpha_n\operatorname{cth}\alpha_n a$$

$$-\beta_n'\operatorname{ctg}\beta_n' a)+D_n\left\{\frac{\alpha_n[\alpha_n^2-k_n^2(2-\mu)]}{\operatorname{sh}\alpha_n a}+\frac{\beta_n'[\beta_n'^2+k_n^2(2-\mu)]}{\sin\beta_n' a}\right\}$$

$$-\frac{4\lambda}{b}\sum_{m=1,2}^{\infty}\frac{C_mk_mk_n}{K_{mn}}[k_n^2+k_m^2(2-\mu)]\cos n\pi-\frac{2k}{n\pi}\left[\frac{2\lambda}{a}\right.$$

$$\left.+\lambda^2\left(\frac{1}{\alpha_m\operatorname{sh}\alpha_n a}+\frac{1}{\beta_n'\sin\beta_n' a}\right)-\frac{2\lambda^3}{\alpha_n^2\beta_n'^2}\right]\cos n\pi=0 \tag{1.11}$$

$$\frac{2q[1-(-1)^m]}{m\pi D}\left\{\frac{[\alpha_m^2-k_m^2(2-\mu)]}{\alpha_m}\operatorname{th}\alpha_m\frac{b}{2}+\frac{[\beta_m'^2+k_m^2(2-\mu)]}{\beta_m^2}\operatorname{tg}\beta_m'\frac{b}{2}\right\}$$

$$+\frac{A_m}{D}\left\{\frac{\alpha_m}{\operatorname{sh}\alpha_m b}[\alpha_m^2-k_m^2(2-\mu)]+\frac{\beta_n'}{\sin\beta_m' b}[\beta_m'^2+k_m^2(2-\mu)]\right\}$$

$$-\frac{4\lambda}{aD}\sum_{n=1,2}^{\infty}\frac{B_nk_mk_n}{K_{mn}}[k_n^2+k_m^2(2-\mu)]\cos n\pi+\frac{4\lambda}{a}\sum_{n=1,2}^{\infty}\frac{D_nk_mk_n}{K_{mn}}[\lambda^2(2-\mu)$$

$$+(1-\mu)^2k_n^2k_n^2]\cos n\pi\cos m\pi+C_m\{\alpha_m[\alpha_m^2-k_m^2(2-\mu)]^2\mathrm{cth}\alpha_m b$$

$$-\beta_m'[\beta_m'^2+k_m^2(2-\mu)]^2\mathrm{ctg}\beta_m'b\}-\frac{2\lambda^2}{m\pi}k\left\{\frac{[\alpha_m^2-k_m^2(2-\mu)]}{\alpha_m}\mathrm{cth}\alpha_m b\right.$$

$$\left.-\frac{[\beta_m'^2+k_m^2(2-\mu)]}{\beta_m'}\mathrm{ctg}\beta_m'b+\frac{2\lambda(2-\mu)k_m^2}{\alpha_m^2\beta_m'^2b}\right\}\cos m\pi=0 \tag{1.12}$$

$$\frac{2q[1-(-1)^n]}{n\pi D}\left\{\frac{[\alpha_n-k_n^2(2-\mu)]}{\alpha_n}\mathrm{th}\alpha_n\frac{a}{2}+\frac{[\beta_n'^2+k_n^2(2-\mu)]}{\beta_n'}\mathrm{tg}\beta_n'\frac{a}{2}\right\}$$

$$-\frac{4\lambda}{bD}\sum_{m=1,2}^{\infty}\frac{A_mk_mk_n}{K_{mn}}[k_m^2+k_n^2(2-\mu)]\cos m\pi+\frac{B_n}{D}\left\{\frac{\alpha_n}{\mathrm{sh}\alpha_n a}[\alpha_n^2-k_m^2(2-\mu)]\right.$$

$$\left.+\frac{\beta_n'}{\sin\beta_n'a}[\beta_n'^2+k_n^2(2-\mu)]\right\}+D_n\{\alpha_n[\alpha_n^2-k_n^2(2-\mu)]^2\mathrm{cth}\alpha_n a-\beta_n'[\beta_n'^2$$

$$+k_n^2(2-\mu)]^2\mathrm{ctg}\beta_n'a\}+\frac{4\lambda}{b}\sum_{m=1,2}^{\infty}\frac{C_mk_mk_n}{K_{mn}}[\lambda^2(2-\mu)$$

$$+(1-\mu)^2k_m^2k_n^2]\cos m\pi\cos n\pi-\frac{2\lambda^2}{n\pi}k\left\{\frac{[\alpha_n^2-k_n^2(2-\mu)]}{\alpha_n}\mathrm{cth}\alpha_n a\right.$$

$$\left.-\frac{[\beta_n'^2+k_n^2(2-\mu)]}{\beta_n'}\mathrm{ctg}\beta_n'a+\frac{2\lambda k_n^2(2-\mu)}{\alpha_n^2\beta_n'^2a}\right\}\cos n\pi=0 \tag{1.13}$$

$$\frac{2q}{aD}\sum_{m=1,2}^{\infty}[1-(-1)^m]\left(\frac{\mathrm{th}\alpha_m\frac{b}{2}}{\alpha_m}-\frac{\mathrm{tg}\beta_m'\frac{b}{2}}{\beta_m'}\right)\cos m\pi$$

$$+\sum_{m=1,2}^{\infty}\frac{A_m}{D}k_m\left(\frac{\alpha_n}{\mathrm{sh}\alpha_m b}-\frac{\beta_m'}{\sin\beta_m'b}\right)\cos m\pi+\sum_{n=1,2}^{\infty}\frac{B_n}{D}k_n\left(\frac{\alpha_n}{\mathrm{sh}\alpha_n a}\right.$$

$$\left.-\frac{\beta_n'}{\sin\beta_n'a}\right)\cos n\pi+\sum_{n=1,2}^{\infty}D_nk_n\{\alpha_n[\alpha_n^2-k_n^2(2-\mu)]\mathrm{ctc}\alpha_n a$$

$$+\beta_m'[\beta_m'^2+k^2(2-\mu)]\mathrm{ctg}\beta_m'a\}\cos n\pi+\sum_{m=1,2}^{\infty}C_mk_m\{\alpha_m[\alpha_m^2-k_m^2(2-\mu)]$$

$$\cdot\mathrm{cth}\alpha_m b+\beta_m'[\beta_m'^2+k_m^2(2-\mu)]\mathrm{ctg}\beta_m'b\}\cos m\pi$$

$$+k\left\{\frac{2\lambda}{ab}-\frac{2\lambda^2}{a}\sum_{m=1,2}^{\infty}\left[\left(\frac{\mathrm{cth}\alpha_m b}{\alpha_m}+\frac{\mathrm{ctg}\beta_m'b}{\beta_m'}\right)-\frac{2\lambda}{\alpha_m^2\beta_m'^2b}\right]\right\}=0 \tag{1.14}$$

作为一个例子，我们计算一方板．令$\mu=1/6$，从每个A_m，B_n，C_m和D_n中取50个系数，由于$A_m=B_n$和$C_m=D_n$，则形成一个101×101阶线性方程组．在M6800微机上进行计算，我们给出如下图表

表 1 固定边弯矩(单位qa^2)和自由边振幅(单位qa^4/D) ($a/b=1$)

ω/ω_{11}	$x/a(y/b)$ / $M(W)$	0.05	0.25	0.5	0.65	0.75	0.95	1.0
0.0	My_0	0.002594	−0.0400	−0.12897	−0.17194	−0.2104	−0.2934	0.0
	Wy_b	0.000335	0.006527	0.01863	0.02583	0.03023	0.3798	0.03977
0.3	My_0	0.003051	−0.04131	−0.1385	−0.1863	−0.2294	−0.3228	0.0
	Wy_b	0.000370	0.007216	0.02068	0.02875	0.03369	0.4339	0.04439
0.5	My_0	0.004276	−0.04452	−0.1628	−0.2240	−0.2797	−0.4015	0.0
	Wy_b	0.000462	0.009065	0.02624	0.03668	0.04312	0.05451	0.05713
0.8	My_0	0.01951	−0.07968	−0.4562	−0.6826	−0.8956	−1.3787	0.0
	Wy_b	0.001618	0.03212	0.09597	0.1367	0.1625	0.2094	0.2203

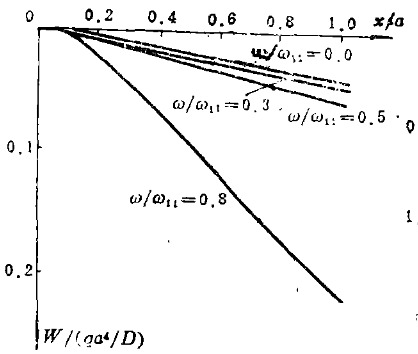

图3 自由边$y=b$振幅曲线

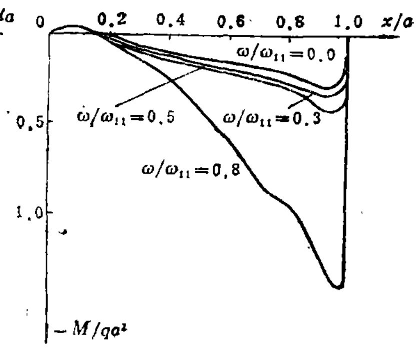

图4 固定边$y=0$弯矩分布曲线

二、两邻边固定，另两邻边自由在一集中简谐干扰力作用下的矩形板

假设，简谐干扰力是一作用在平板上(x_0, y_0)点处的集中载荷$F(x, y)=p\delta(x-x_0, y-y_0)\sin\omega t$，如图5所示。

易知，本节的振幅挠曲面方程与式(1.4)的区别只在于载荷项，因之，将集中载荷项(4.11)[1]替代式(1.4)的均布载荷项之后，便可得相应振幅方程。对边界条件(1.5)～(1.9)(1.10)～(1.14)采用相同方法处理，则得和

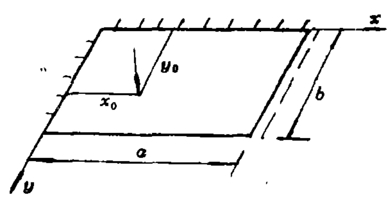

图 5

$$\frac{2p}{aD}\left[-\frac{\mathrm{sh}\alpha_m(b-y_0)}{\mathrm{sh}\beta_n b}+\frac{\mathrm{sh}\beta_m(b-y_0)}{\mathrm{sh}\beta_m b}\right]\sin k_m x_0+\cdots=0 \tag{2.1}$$

$$\frac{2p}{bD}\left[-\frac{\mathrm{sh}\alpha_n(a-x_0)}{\mathrm{sh}\alpha_n a}+\frac{\mathrm{sh}\beta_n(a-x_0)}{\mathrm{sh}\beta_n a}\right]\sin k_n y_0+\cdots=0 \tag{2.2}$$

$$\frac{2p}{aD}\left\{\frac{[\alpha_m^2-k_m^2(2-\mu)]}{\mathrm{sh}\alpha_m b}\mathrm{sh}\alpha_m y_0-\frac{[\beta_m^2-k_m^2(2-\mu)]}{\mathrm{sh}\beta_m b}\mathrm{sh}\beta_m y_0\right\}\sin k_m x_0+\cdots=0 \tag{2.3}$$

$$\frac{2p}{bD}\left\{\frac{[\alpha_n^2-k_n^2(2-\mu)]}{\mathrm{sh}\alpha_n a}\mathrm{sh}\,\alpha_n x_0-\frac{[\beta_n^2-k_n^2(2-\mu)]}{\mathrm{sh}\beta_n a}\mathrm{sh}\beta_n x_0\right\}\sin k_n y_0+\cdots=0 \tag{2.4}$$

$$\frac{2p}{aD}\sum_{m=1,2}^{\infty}k_m\left(\frac{\mathrm{sh}\alpha_m y_0}{\mathrm{sh}\alpha_m b}-\frac{\mathrm{sh}\beta_m y_0}{\mathrm{sh}\beta_m b}\right)\sin k_m x_0\cos m\pi+\cdots=0 \tag{2.5}$$

这里$\lambda<k_m^2$和$\lambda<k_n^2$，和

$$\frac{2p}{aD}\left[-\frac{\mathrm{sh}\alpha_m(b-y_0)}{\mathrm{sh}\alpha_m b}+\frac{\sin\beta_m'(b-y_0)}{\sin\beta_m' b}\right]\sin k_m x_0+\cdots=0 \tag{2.6}$$

$$\frac{2p}{bD}\left[-\frac{\mathrm{sh}\alpha_n(a-x_0)}{\mathrm{sh}\alpha_n a}+\frac{\sin\beta_n'(a-x_0)}{\sin\beta_n' a}\right]\sin k_n y_0+\cdots=0 \tag{2.7}$$

$$\frac{2p}{aD}\left\{\frac{[\alpha_m^2-k_m^2(2-\mu)]}{\mathrm{sh}\alpha_m b}\mathrm{sh}\alpha_m y_0+\frac{[\beta_m'^2+k_m^2(2-\mu)]}{\sin\beta_m' b}\sin\beta_m' y_0\right\}\sin k_m x_0+\cdots=0 \tag{2.8}$$

$$\frac{2p}{bD}\left\{\frac{[\alpha_n^2-k_n^2(2-\mu)]}{\mathrm{sh}\alpha_n a}\mathrm{sh}\alpha_n x_0+\frac{[\beta_n'^2+k_n^2(2-\mu)]}{\sin\beta_n' a}\sin\beta_n' x_0\right\}\sin k_n y_0+\cdots=0 \tag{2.9}$$

$$\frac{2p}{aD}\sum_{m=1,2}^{\infty}k_m\left(\frac{\mathrm{sh}\alpha_m y_0}{\mathrm{sh}\alpha_m b}-\frac{\sin\beta_n' y_0}{\sin\beta_m' b}\right)\sin k_m x_0\cos m\pi+\cdots=0 \tag{2.10}$$

这里$\lambda>k_n^2$和$\lambda>k_n^2$。

这里我们须指出，当p作用在板的角点$(a,\ b)$时，式(2.5)和(2.10)分别变成

$$\frac{2p}{aD}\sum_{m=1,2}^{\infty}k_m\left(\frac{\mathrm{sh}\alpha_m y_0}{\mathrm{sh}\alpha_m b}-\frac{\mathrm{sh}\beta_m y_0}{\mathrm{sh}\beta_m b}\right)\sin k_m x_0\cos m\pi+\cdots=\frac{p\lambda}{(1-\mu)D} \tag{2.11}$$

$$\frac{2p}{aD}\sum_{m=1,2}^{\infty}k_m\left(\frac{\mathrm{sh}\alpha_m y_0}{\mathrm{sh}\alpha_m b}-\frac{\sin\beta_n' y_0}{\sin\beta_m' b}\right)\sin k_m x_0\cos m\pi+\cdots=\frac{p\lambda}{(1-\mu)D} \tag{2.12}$$

作为一例子，我们计算一集中载荷作用于角点的方板，并给出下述图表.

表 2 固定边弯矩(单位p)和自由边振幅(单位pa^2/D) $(a/b=1)$

ω/ω_{11}	$x/a(y/b)$ / $M(W)$	0.05	0.25	0.5	0.65	0.75	0.95	1.0
0.0	My_0	0.01834	0.005074	−0.2373	−0.4279	−0.6366	−1.1745	0.0
	Wy_b	0.001439	0.02876	0.09294	0.1416	0.1769	0.2525	0.2719
0.3	My_0	0.02092	−0.000408	−0.2860	−0.5046	−0.7402	−1.3397	0.0
	Wy_b	0.001634	0.03266	0.1048	0.1585	0.1972	0.2787	0.2995
0.5	My_0	0.02759	−0.01498	−0.4129	−0.7042	−1.009	−1.7679	0.0
	Wy_b	0.002141	0.04277	0.1354	0.2024	0.2496	0.3465	0.3709
0.8	My_0	0.1092	−0.1970	−1.9711	−3.1476	−4.2983	−7.0024	0.0
	Wy_b	0.008333	0.1664	0.5095	0.7395	0.8913	1.1795	1.2486

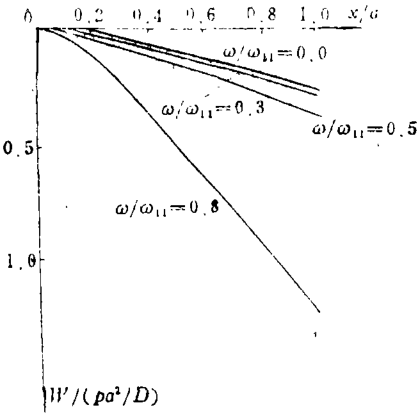

图6 自由边$y=b$振幅曲线

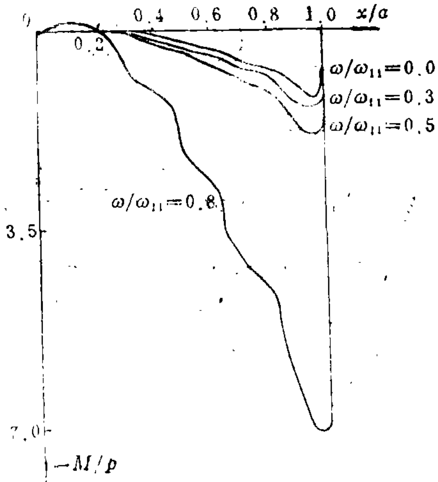

图7 固定边$y=0$弯矩分布曲线

三、讨 论

1. 对于如图 8 所示在简谐均载荷作用下的矩形板，在式(1.4)中令$D_n=k=0$，

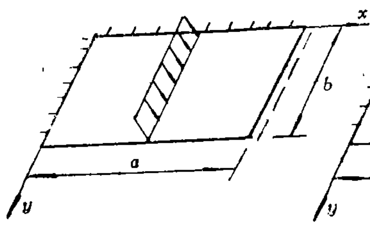

图 8

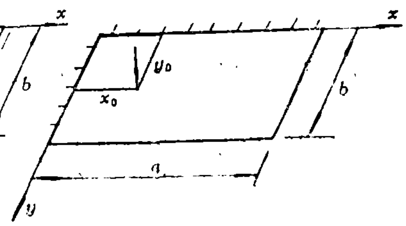

图 9

即可得其振幅方程。在边界条件(1.5)~(1.7)和(1.10)~(1.12)中代入$D_n=k=0$之后，我们即可得其相应的边界条件。经过计算，得下述图表。

表 3 固定边弯矩(单位qa^2)和自由边振幅(单位qa^4/D)

ω/ω_1	$x/a(y/b)$ / $M_{(W)}$	0.05	0.15	0.35	0.5	0.7	0.9	0.95
0.0	Mx_0	−0.001310	−0.018193	−0.061843	−0.080336	−0.097757	−0.10109	−0.17079
	My_0	−0.001705	−0.017578	−0.059031	−0.077283	−0.073877	−0.02424	−0.018396
	Wy_b	0.000146	0.001082	0.003780	0.005107	0.004765	0.001965	0.001003
0.3	Mx_0	−0.001265	−0.018875	−0.065911	−0.086459	−0.10655	−0.11124	−0.18884
	My_0	−0.001703	−0.018193	−0.062673	−0.082619	−0.07902	−0.036351	−0.019473
	Wy_b	0.000161	0.001201	0.004208	0.005693	0.005312	0.002186	0.001116
0.5	Mx_0	−0.001156	−0.020472	−0.075557	−0.10105	−0.12769	−0.135812	−0.23271
	My_0	−0.001699	−0.019627	−0.071269	−0.09525	−0.091211	−0.041359	−0.02203
	Wy_b	0.000200	0.001489	0.005248	0.007117	0.006638	0.002725	0.001390
0.8	Mx_0	−0.000416	−0.030643	−0.138725	−0.19756	−0.27042	−0.30429	−0.53597
	My_0	−0.001687	−0.028660	−0.126945	−0.17759	−0.17098	−0.07415	−0.03874
	Wy_b	0.000466	0.003482	0.012418	0.016934	0.015784	0.006435	0.003278

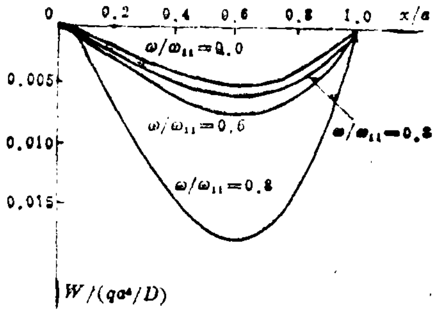

图10 自由边振幅曲线

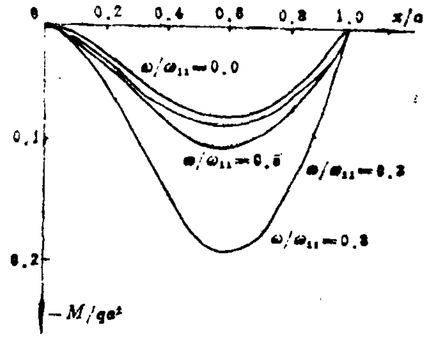

图11 固定边$y=0$弯矩分布曲线

2．对于如图9所示在简谐集中载荷作用下的矩形板，将文[1]式(4.11)和$D_n=k=0$分别代入式(1.4)中的载荷项和其它项，即得振幅方程．在边界条件(2.1)～(2.3)和(2.6)～(2.8)中，令$D_n=k=0$，即得相应的边界条件．对于简谐集中载荷作用于中点的方板，有如下图表．

表 4 固定边弯矩(单位p)和自由边振幅(单位pa^2/D)

ω/ω_{11}	$x/a(y/o)$ $M(W)$	0.05	0.15	0.35	0.5	0.7	0.9	0.95
0.0	Mx_0	−0.000262	−0.023445	−0.121036	−0.17647	−0.17692	−0.12934	−0.13735
	My_0	0.002196	−0.021213	−0.12325	−0.17202	−0.14190	−0.05056	−0.02533
	Wy_b	0.000107	0.000986	0.004051	0.00571	0.005306	0.002124	0.001078
0.3	Mx_0	−0.000694	−0.02542	−0.12808	−0.18833	−0.19435	−0.14661	−0.16499
	My_0	0.002434	−0.022576	−0.13086	−0.18299	−0.15217	−0.05471	−0.02743
	My_b	0.000132	0.001170	0.004720	0.00663	0.006164	0.002472	0.001255
0.5	Wx_0	−0.001798	−0.03002	−0.14396	−0.21573	−0.23607	−0.19598	−0.23390
	My_0	0.003020	−0.02562	−0.14817	−0.20807	−0.17580	−0.06429	−0.03228
	My_b	0.000196	0.001128	0.006376	0.008904	0.008283	0.003332	0.001693
0.8	Wx_0	−0.01027	−0.05889	−0.23685	−0.36620	−0.51805	−0.53466	−0.74004
	My_0	0.007199	−0.04285	−0.25110	−0.35960	−0.32083	−0.12371	−0.06236
	My_b	0.000667	0.004973	0.018357	0.025317	0.02357	0.009535	0.004847

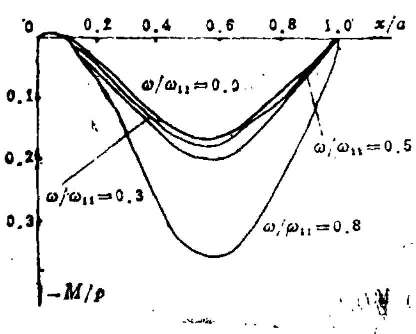

图12 固定边$y=0$弯矩分布曲线

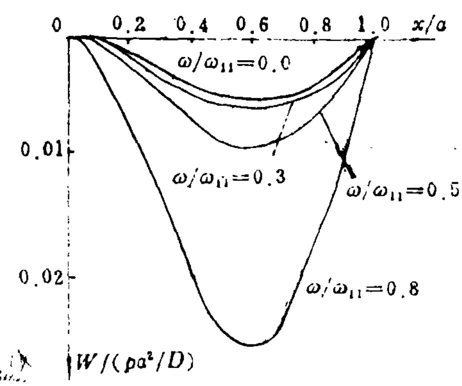

图13 自由边振幅曲线

3. 对于图14所示在简谐均布载荷作用下的矩形板，在式(1.4)中令$C_m=D_n=k=0$，即得其振幅方程．在边界条件(1.5)～(1.6)和(6.10)～(6.11)中令$C_m=k=0$，即得相应的边

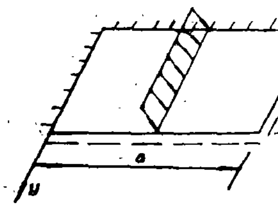

图 14

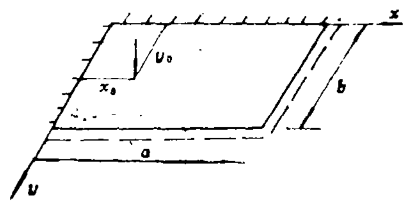

图 15

表5 固定边弯矩(单位qa^2)和中点振幅(单位qa^4/D) $(a/b=2)$

ω/ω_{11} \ $x/a(y/b)$ M		0.15	0.25	0.5	0.75	0.95	W中
0.0	Mx_0	−0.004389	−0.009958	−0.19621	−0.017182	−0.004656	0.000318
	My_0	−0.012661	−0.021356	−0.029417	−0.02612	−0.008577	
0.3	Mx_0	−0.004535	−0.010461	−0.020927	−0.018310	−0.004920	0.000350
	My_0	−0.013380	−0.022952	−0.032226	−0.028345	−0.009105	
0.5	Mx_0	−0.004872	−0.011628	−0.023976	−0.020947	−0.005536	0.000426
	My_0	−0.015054	−0.026703	−0.038922	−0.033625	−0.010349	
0.8	Mx_0	−0.009829	−0.018537	−0.042265	−0.036817	−0.009248	0.000895
	My_0	−0.02505	−0.049559	−0.08108	−0.066514	−0.017941	

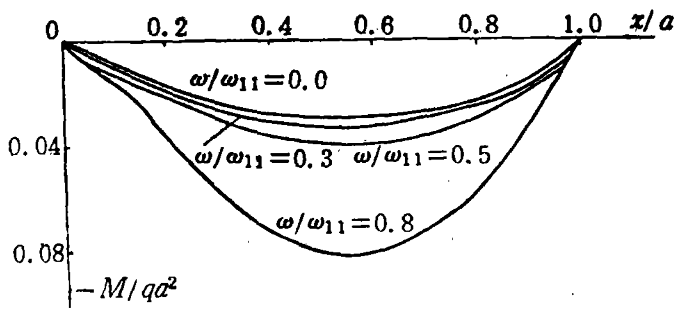

图16 固定边$y=0$弯矩分布曲线$(a/b=2)$

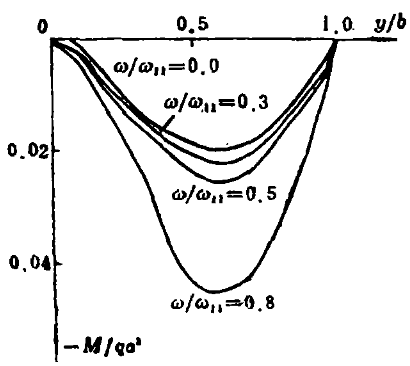

图17 固定边$x=0$弯矩分布曲线$(a/b=2)$

界条件．经过计算，我们得到下述图表．

4. 对于如图15所示在简谐集中载荷作用下的矩形板，在(1.4)中以文[1](4.11)代替其载荷项后并令$C_m=D_n=k=0$，即得相应的振幅方程．在(2.1)～(2.2)和(2.6)～(2.7)中，令$C_m=D_n=k=0$，即得相应的边界条件．对于集中载荷作用于矩形板的中点，我们得到下述图表．

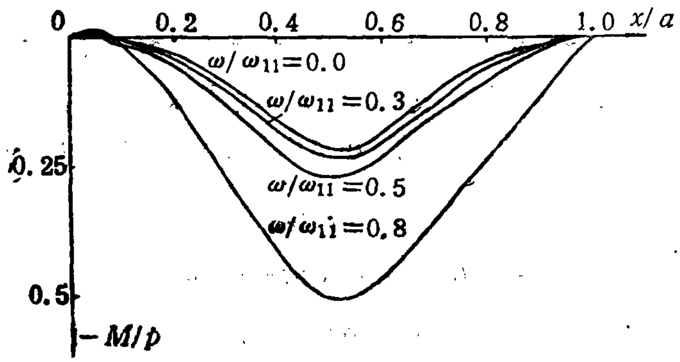

图18 固定边$y=0$弯矩分布曲线($a/b=2$)

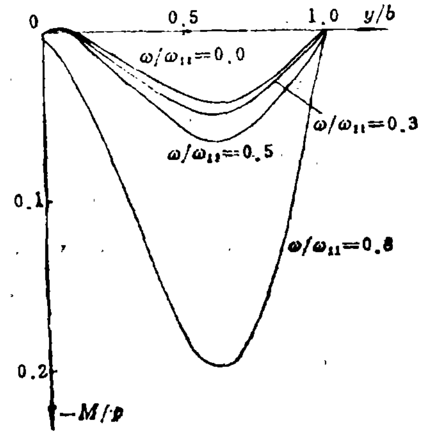

图19 固定边$x=0$弯矩分布曲线($a/b=2$)

表6 固定边弯矩(单位p)和中点振幅(单位pa^2/D) ($a/b=2$)

ω/ω_{11}	$x/a(y/b)$ / M	0.15	0.25	0.5	0.75	0.95	$W_中$
0.0	Mx_0	−0.002239	−0.011818	−0.037457	−0.033256	−0.007753	0.002511
	My_0	−0.019715	−0.063931	−0.213037	−0.072170	−0.008885	
0.3	Mx_0	−0.002871	−0.014157	−0.043857	−0.038827	−0.009052	0.002697
	My_0	−0.023255	−0.072581	−0.23047	−0.082910	−0.010954	
0.5	Mx_0	−0.004428	−0.019849	−0.059341	−0.052296	−0.012193	0.003136
	My_0	−0.031813	−0.093192	−0.27125	−0.110908	−0.016171	
0.8	Mx_0	−0.040854	−0.084813	−0.18719	−0.15681	−0.03645	0.005715
	My_0	−0.067152	−0.20648	−0.50953	−0.27942	−0.052317	

参 考 文 献

[1] 付宝连、李农，弹性矩形薄板受迫振动的功的互等定理(Ⅰ)——四边固定的矩形板和三边固定的矩形板，应用数学和力学，10，8 (1989)，693—714.

The Method of the Reciprocal Theorem of Force Vibration for the Elastic Thin Rectangular Plates (Ⅱ) —— Rectangular Plates with Two Adjacent Clamped Edges

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Abstract

In this paper, applying the method of reciprocal theorem, we give the distributions of the amplitude of bending moments along clamped edges and the amplitudes of deflections along free edges of rectangular plates with two adjacent clamped edges under harmonic distributed and concentrated loads.

Key words the method of reciprocal theorem, force vibration, the amplitude of bending moment, the amplitude of deflection, elastic thin plate