力学中单参数变换群的应用*

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摘　要

本文包括无限小形式的变换群用于减少偏微分方程中的自变量，获得相似变量的理论，以及它在力学中具有两个自变量、两个因变量的非线性偏微分方程组中的应用．

一、引　言

1950年Birkhoff[1]提出了用代数方法来减少偏微分方程中自变量数目的方法，1951年Michal[2]给出了在赋范线性空间（normed linear spaces）在连续变换群下的一般不变理论，1952年Morgan[3]对Birkhoff的工作进行了推广，并对任意偏微分方程组利用单参数变换群时，建立了获得自变量和因变量的绝对不变量的一些定理以后，对于力学（特别是流体力学）中的某些偏微分方程在寻求相似性解时得到了应用[4~18]．60年代以后，首先由Lie应用于偏微分方程中的无限小变换群在力学中也得到了应用[14,19~33]．

我们在[33]中，曾给出了两个自变量和一个因变量的偏微分方程应用无限小变换群时的例子，本文将讨论两个自变量和两个因变量的非线性偏微分方程组在无限小变换群下的应用．

二、无限小变换群

对于自变量为x，y，因变量为u，v的非线性偏微分方程组，则无限小变换群为

$$\left.\begin{aligned}x^1&=x+\varepsilon X(x,y,u,v)+O(\varepsilon^2)\\y^1&=y+\varepsilon Y(x,y,u,v)+O(\varepsilon^2)\\u^1&=u+\varepsilon U(x,y,u,v)+O(\varepsilon^2)\\v^1&=v+\varepsilon V(x,y,u,v)+O(\varepsilon^2)\end{aligned}\right\}\tag{2.1}$$

式中　参数ε为一小值；X，Y，U，V为待定函数．

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三、变换前后偏导数之间的关系式

现来导出在群（2.1）变换下一阶、二阶偏导数变换前后的关系式．如

$$u^1_{x^1}=u^1_x x_{x^1}+u^1_y y_{x^1} \tag{3.1}$$

而

$$\left.\begin{aligned}u^1_x&=u_x+\varepsilon[U_x+U_u u_x+U_v v_x]+O(\varepsilon^2)\\ u^1_y&=u_y+\varepsilon[U_y+U_u u_y+U_v v_y]+O(\varepsilon^2)\end{aligned}\right\} \tag{3.2}$$

$$\left.\begin{aligned}x_{x^1}&=1-\varepsilon[X_x+X_u u_x+X_v v_x]+O(\varepsilon^2)\\ y_{x^1}&=-\varepsilon(Y_x+Y_u u_x+Y_v v_x]+O(\varepsilon^2)\end{aligned}\right\} \tag{3.3}$$

将式（3.2），（3.3）式入式(3.1)，得

$$u^1_{x^1}=u_x+\varepsilon[U_x]+O(\varepsilon^2) \tag{3.4}$$

式中

$$\begin{aligned}[][U_x]=&U_x+(U_u-X_x)u_x-Y_x u_y+U_v v_x-X_u u_x^2\\ &-X_v u_x v_x-Y_u u_x u_y-Y_v u_y v_x\end{aligned} \tag{3.5}$$

同理，得

$$u^1_{y^1}=u_y+\varepsilon[U_y]+O(\varepsilon^2) \tag{3.6}$$

$$v^1_{x^1}=v_x+\varepsilon[V_x]+O(\varepsilon^2) \tag{3.7}$$

$$v^1_{y^1}=v_y+\varepsilon[V_y]+O(\varepsilon^2) \tag{3.8}$$

式中 $[U_y]$，$[V_x]$，$[V_y]$ 均可从 $[U_x]$ 中作相应的置换而得，如分别用 Y，X，y，x 置换 $[U_x]$中的X，Y，x，y即得$[U_y]$；用V，v，u置换$[U_x]$中的U，u，v即得$[V_v]$等．

对于二阶偏导数，如

$$u^1_{x^1x^1}=u^1_{x^1x}x_{x^1}+u^1_{x^1y}y_{x^1} \tag{3.9}$$

$$u^1_{x^1y^1}=u^1_{y^1x}x_{x^1}+u^1_{y^1y}y_{x^1} \tag{3.10}$$

将式（3.3）和（3.4）与（3.3）和（3.6）分别代入式（3.9）与（3.10），得

$$u^1_{x^1x^1}=u_{xx}+\varepsilon[U_{xx}]+O(\varepsilon^2) \tag{3.11}$$

$$u^1_{x^1y^1}=u_{xy}+\varepsilon[U_{xy}]+O(\varepsilon^2) \tag{3.12}$$

式中

$$\begin{aligned}[][U_{xx}]=&U_{xx}+(2U_{ux}-X_{xx})u_x-Y_{xx}u_y+2U_{vx}v_x\\ &+(U_{uu}-2X_{ux})u_x^2+U_{vv}v_x^2-2Y_{ux}u_x u_y\\ &+2(U_{uv}-X_{vx})u_x v_x-2Y_{vx}u_y v_x-X_{uu}u_x^2-Y_{uu}u_x^2u_y\\ &-2X_{uv}u_x^2v_x-2Y_{uv}u_x u_y v_x-X_{vv}u_x v_x^2-Y_{vv}u_y v_x^2\\ &+(U_u-2X_x)u_{xx}-2Y_x u_{xy}+U_v v_{xx}-3X_u u_x u_{xx}\\ &-2Y_u u_x u_{xy}-Y_u u_y u_{xx}-2X_v v_x u_{xx}\\ &-X_v u_x v_{xx}-Y_v u_y v_{xx}-2Y_v v_x u_{xy}\end{aligned} \tag{3.13}$$

$$\begin{aligned}[][U_{xy}]=&U_{xy}+(U_{uy}-X_{xy})u_x+(U_{ux}-Y_{xy})u_y+U_{vy}v_x+U_{vx}v_y\\ &-X_{uy}u_x^2+(U_{uu}-Y_{uy}-X_{ux})u_x u_y-Y_{ux}u_y^2-X_{vy}u_x v_x\\ &+(U_{uv}-X_{xv})u_x u_y+(U_{uv}-Y_{vy})u_y v_x-Y_{vx}u_y v_y\\ &+U_{vv}v_x v_y-X_{uu}u_x^2u_y-X_{uv}u_x^2v_y-Y_{uu}u_x u_y^2\end{aligned}$$

$$-Y_{uv}v_x u_y^2-X_{uv}u_x u_y v_x-Y_{uv}u_x u_y v_y-X_{vv}u_x v_x v_y$$
$$-Y_{vv}u_y v_x v_y-X_y u_{xx}+(U_u-X_x-Y_y)u_{xy}-Y_x u_{yy}$$
$$+U_v v_{xy}-X_u u_y u_{xx}-X_v v_y u_{xx}-2X_u u_x u_{xy}$$
$$-X_v v_x u_{xy}-X_v u_x v_{xy}-2Y_u u_y u_{xy}-Y_v v_y u_{xy}$$
$$-Y_u u_x u_{yy}-Y_v v_x u_{yy}-Y_v u_y v_{xy} \tag{3.14}$$

同理得

$$u_{y^1y^1}=u_{yy}+\varepsilon[U_{yy}]+O(\varepsilon^2) \tag{3.15}$$
$$v_{x^1x^1}=v_{xx}+\varepsilon[V_{xx}]+O(\varepsilon^2) \tag{3.16}$$
$$v_{y^1y^1}=v_{yy}+\varepsilon[V_{yy}]+O(\varepsilon^2) \tag{3.17}$$
$$v_{x^1y^1}=v_{xy}+\varepsilon[V_{xy}]+O(\varepsilon^2) \tag{3.18}$$

式中 $[U_{yy}]$，$[V_{xx}]$，$[V_{yy}]$和$[V_{xy}]$分别由$[U_{xx}]$和$[U_{xy}]$作相应的置换而得．置换方法与一阶偏导数中的置换方法相同．

若因变量只有一个时，只要在上面给出的关系式中令v为零即可．

四、相似变量和相似性解中新的因变量

设在（x，y，u）空间存在一曲面S．若物理问题的原方程在变换群（2.1）下保持不变，如图1所示．即

$$u(x+\varepsilon X+O(\varepsilon^2),\ y+\varepsilon Y+O(\varepsilon^2))$$
$$=u+\varepsilon U+O(\varepsilon^2)$$

将上式左边展开，并使两边$O(\varepsilon)$的各项相等，得

$$X\frac{\partial u}{\partial x}+Y\frac{\partial u}{\partial y}=U \tag{4.1}$$

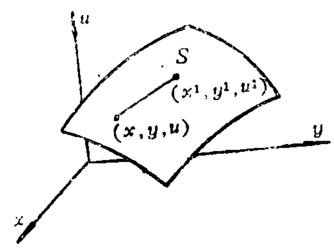

图 1

这就是“不变曲面的条件”，它是一阶拟线性偏微分方程．它的解可由其特征方程

$$\frac{dx}{X(x,y,u)}=\frac{dy}{Y(x,y,u)}=\frac{du}{U(x,y,u)} \tag{4.2}$$

求得[14]，若为两个因变量，则特征方程为

$$\frac{dx}{X(x,y,u,v)}=\frac{dy}{Y(x,y,u,v)}$$
$$=\frac{du}{U(x,y,u,v)}=\frac{dv}{V(x,y,u,v)} \tag{4.3}$$

将其改写为下列三个方程

$$\frac{dx}{dy}=\frac{X(x,y,u,v)}{Y(x,y,u,v)} \tag{4.4}$$

$$\frac{du}{dx}=\frac{U(x,y,u,v)}{X(x,y,u,v)} \tag{4.5}$$

$$\frac{dv}{dx}=\frac{V(x,y,u,v)}{X(x,y,u,v)} \tag{4.6}$$

积分之，则包含着三个积分常数，它们分别为自变量和因变量的绝对不变量ζ，$F(\zeta)$和$G(\zeta)$，ζ称为相似变量，$F(\zeta)$和$G(\zeta)$起因变量的作用，即为相似性解中新的因变量.

五、应　用

1. 平板在非牛顿流体中的突然运动

剪应力τ_{yx}为速度梯度$\partial u/\partial y$的任意函数时，其控制方程组为(τ_{xy}记为τ)

$$\rho\frac{\partial u}{\partial t}=\frac{\partial \tau}{\partial y} \tag{5.1}$$

$$f\left(\tau,\frac{\partial u}{\partial y}\right)=0 \tag{5.2}$$

引入变换群

$$\left.\begin{aligned}t^1&=t+\varepsilon T(t,y,u,\tau)+O(\varepsilon^2)\\y^1&=y+\varepsilon Y(t,y,u,\tau)+O(\varepsilon^2)\\u^1&=u+\varepsilon U(t,y,u,\tau)+O(\varepsilon^2)\\\tau^1&=\tau+\varepsilon V(t,y,u,\tau)+O(\varepsilon^2)\end{aligned}\right\} \tag{5.3}$$

将式(3.5)，(3.8)和(5.3)，(3.6)的形式代入方程(5.1)和(5.2)，得

$$\begin{aligned}\rho u^1_{t^1}-\tau^1_{y^1}=&(\rho u_t-\tau_y)+\varepsilon[(\rho U_t-V_y)+(U_u-T_t-V_\tau+Y_y)\rho u_t\\&-(\rho Y_t+V_u)u_y+(\rho U_\tau+T_y)\tau_t-(T_u-\rho Y_\tau)\rho u_t^2\\&-(\rho Y_\tau-T_u)u_y\tau_t]+O(\varepsilon^2)\end{aligned} \tag{5.4}$$

$$\begin{aligned}f(\tau^1,u^1_{y^1})=&f\{\tau+\varepsilon V,u_y+\varepsilon[U_y+(U_u-Y_y)u_y-T_yu_t\\&+U_\tau\tau_y-Y_uu_y^2-Y_\tau u_y\tau_y-T_uu_tu_y-T_\tau u_t\tau_y]+O(\varepsilon^2)\}\end{aligned} \tag{5.5}$$

为了使在变换群(5.3)下方程组保形不变，则必须使

$$\left.\begin{aligned}&\rho U_t-V_y=0,\ U_u-T_t-V_\tau+Y_y=0,\ \rho Y_t+V_u=0\\&\rho U_\tau+T_y=0,\ \rho Y_\tau-T_u=0,\ V=0,\ U_y=0\\&U_u-Y_y=0,\ T_y=0,\ U_\tau=0,\ Y_u=0\\&Y_\tau=0,\ T_u=0,\ T_\tau=0\end{aligned}\right\} \tag{5.6}$$

所以

$$T=2at+b,\ Y=ay+c,\ U=au+d,\ V=0 \tag{5.7}$$

其中　a，b，c，d为任意常数. 代入特征方程(4.3)，得

$$\frac{dt}{2at+b}=\frac{dy}{ay+c}=\frac{du}{au+d}=\frac{d\tau}{0}$$

解之，得相似变量，新的因变量为

$$\zeta=\frac{ay+c}{(2at+b)^{1/2}} \tag{5.8}$$

$$F(\zeta)=\frac{au+d}{(2at+b)^{1/2}} \tag{5.9}$$

$$G(\zeta)=\tau(\zeta) \tag{5.10}$$

将式(5.8)，(5.9)，(5.10)代入方程(5.1)，(5.2)得

$$\rho(f-\zeta F')-G'=0 \tag{5.11}$$

$$f(G,F'')=0 \tag{5.12}$$

若速度的边界条件为

$$y=0,\ u=U_0t^{1/2};\ y\to\infty:\ u=0 \tag{5.13}$$

不失一般性，令 $a=2/U_0^2$，$b=c=d=0$，则

$$\zeta=\frac{1}{U_0}\frac{y}{t^{1/2}} \tag{5.14}$$

$$F(\zeta)=\frac{1}{U_0}\frac{u}{t^{1/2}} \tag{5.15}$$

$$G(\zeta)=\tau(\zeta) \tag{5.16}$$

此时方程组仍为(5.11)，(5.12)，而边界条件为

$$F(0)=1,\qquad F(\infty)=0 \tag{5.17}$$

2. 半无限杆件的冲击问题

对于非线性Maxwell体的半无限杆件的冲击问题，在杆中的一维运动方程为

$$\frac{\partial\sigma}{\partial x}=\rho\frac{\partial v}{\partial t} \tag{5.18}$$

非线性Maxwell体的本构方程为

$$\frac{\partial v}{\partial x}=\frac{\partial}{\partial t}\left[\left(\frac{\sigma}{\mu}\right)\right]^q+\left(\frac{\sigma}{\lambda}\right)^n \tag{5.19}$$

x为杆轴线方向的坐标，t为时间，σ为沾杆轴线方向的法向应力，v为轴线方向的速度分量，ρ为杆件材料的质量密度，q，n，λ，μ为材料常数。

引入变换群

$$\left.\begin{aligned}
x^1&=x+\varepsilon X(x,t,\sigma,v)+O(\varepsilon^2)\\
t^1&=t+\varepsilon T(x,t,\sigma,v)+O(\varepsilon^2)\\
\sigma^1&=\sigma+\varepsilon U(x,t,\sigma,v)+O(\varepsilon^2)\\
v^1&=v+\varepsilon V(x,t,\sigma,v)+O(\varepsilon^2)
\end{aligned}\right\} \tag{5.20}$$

则

$$\sigma^1_{x^1}=\sigma_x+\varepsilon[U_x]+O(\varepsilon^2),\quad \sigma^1_{t^1}=\sigma_t+\varepsilon[U_t]+O(\varepsilon^2)$$

$$v^1_{x^1}=v_x+\varepsilon[V_x]+O(\varepsilon^2),\quad v^1_{t^1}=v_t+\varepsilon[V_t]+O(\varepsilon^2)$$

那么支配方程（5.18）和（5.19）

$$\begin{aligned}
\sigma^1_{x^1}-\rho v^1_{t^1}=(\sigma_x-\rho v_t)+\varepsilon[&(U_x-\rho v_t)+\rho(U_\sigma-X_x-V_v+T_t)v_t\\
&-(T_x+\rho V_\sigma)\sigma_t+(U_v-\rho X_t)v_x-\rho(\rho X_\sigma-T_v)v_t^2\\
&-(T_v-\rho X_\sigma)\sigma_t v_x]+O(\varepsilon^2)
\end{aligned} \tag{5.21}$$

$$\begin{aligned}
v^1_{x^1}-q\left(\frac{1}{\mu}\right)^q\sigma^{1(q-1)}\sigma^1_{t^1}-\left(\frac{\sigma^1}{\lambda}\right)^n=&\left[v_x-q\left(\frac{1}{\mu}\right)^q\sigma^{(q-1)}\sigma_t-\left(\frac{\sigma}{\lambda}\right)^n\right]\\
&+\varepsilon\left\{\left[V_x+(V_v-X_x)\left(\frac{\sigma}{\lambda}\right)^n-x_v\left(\frac{\sigma}{\lambda}\right)^{2n}-\frac{q}{\mu^q}U_t\sigma^{q-1}-\frac{n}{\lambda^n}U\sigma^{n-1}\right]\right.\\
&+\left[V_\sigma-X_\sigma\left(\frac{\sigma}{\lambda}\right)^n+\frac{q}{\mu^q}X_t\sigma^{q-1}\right]\sigma_x+\left[\frac{q}{\mu^q}(V_v-X_x)\sigma^{q-1}\right.\\
&\left.-2\frac{q}{\mu^q}X_v\sigma^{q-1}\left(\frac{\sigma}{\lambda}\right)^n-\frac{q}{\mu^q}\left(\frac{q-1}{\sigma}U+U_\sigma-T_t\right)\sigma^{q-1}\right]\sigma_t
\end{aligned}$$

$$-\left[T_x+T_v\left(\frac{\sigma}{\lambda}\right)^n+\frac{q}{\mu^q}U_v\sigma^{q-1}\right]v_t-\left[T_\sigma-\frac{q}{\mu^q}X_v\sigma^{q-1}\right]\sigma_x v_t$$

$$+\frac{q}{\mu^q}\sigma^{q-1}\left(T_\sigma-\frac{q}{\mu^q}X_v\sigma^{q-1}\right)\sigma_t^2\Big\}+O(\varepsilon^2) \tag{5.22}$$

为了使在变换群（5.20）下，方程组保形不变，则必须使

$$\left.\begin{aligned}
&U_x-\rho V_t=0,\ U_\sigma-X_x-V_v+T_t=0,\ T_x+\rho V_\sigma=0\\
&U_v+\rho X_t=0,\ T_v-\rho X_\sigma=0\\
&V_x+(V_v-X_x)\left(\frac{\sigma}{\lambda}\right)^n-X_v\left(\frac{\sigma}{\lambda}\right)^{2n}-\frac{q}{\mu^q}U_t\sigma^{q-1}-\frac{n}{\lambda^n}U\sigma^{n-1}=0\\
&V_\sigma-X_\sigma\left(\frac{\sigma}{\lambda}\right)^n+\frac{q}{\mu^q}X_t\sigma^{q-1}=0\\
&V_v-X_x-2X_v\left(\frac{\sigma}{\lambda}\right)^n-\left(\frac{q-1}{\sigma}U+U_\sigma-T_t\right)=0\\
&T_x+T_v\left(\frac{\sigma}{\lambda}\right)^n+\frac{q}{\mu^q}U_v\sigma^{q-1}=0,\ T_\sigma-\frac{q}{\mu^q}X_v\sigma^{q-1}=0
\end{aligned}\right\} \tag{5.23}$$

所以

$$\left.\begin{aligned}
&X=ax,\ T=\frac{2(q-n)}{q-2n+1}at\\
&U=\frac{2}{q-2n+1}a\sigma,\ V=\frac{q+1}{q-2n+1}av
\end{aligned}\right\} \tag{5.24}$$

其中 a为任意常数。将式（5.24）代入特征方程(4.3)，得

$$\frac{dx}{ax}=\frac{dt}{\frac{2(q-n)}{q-2n+1}at}=\frac{d\sigma}{\frac{2}{q-2n+1}a\sigma}=\frac{dv}{\frac{q+1}{q-2n+1}av} \tag{5.25}$$

解之，得相似变量，新的因变量为

$$\zeta=\frac{x}{t^{\frac{q-2n+1}{2(q-n)}}} \tag{5.26}$$

$$F(\zeta)=t^{\frac{1}{n-q}}\sigma \tag{5.27}$$

$$G(\zeta)=t^{\frac{q+1}{2(n-q)}}v \tag{5.28}$$

将(5.26)，(5.27)，(5.28)式代入方程(5.18)和(5.19)，得

$$F'=\frac{\rho}{2(q-n)}[(q+1)G-(q-2n+1)\zeta G'] \tag{5.29}$$

$$G'=\frac{q}{\mu^q}\frac{1}{q-n}[F^q-(q-2n+1)\zeta F^{q-1}F']+\frac{1}{\lambda^n}F^n \tag{5.30}$$

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Application of One-Parameter Groups of Transformation in Mechanics

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Abstract

In this paper, including some partial differential equations with a number of independent variables, which can be reduced by the infinitesimal form of the group, we obtain the theory of similarity transformation and its application of the second order nonlinear partial differential equations which have two independent variables and two dependent variables in mechanics.