

环肋加劲圆柱壳在静水压力作用下的初始后屈曲分析

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摘 要

本文用 Koiter 理论分析环肋加劲圆柱壳在静水压力作用下的后屈曲性能, 前屈曲状态采用与边界条件一致的非线性有矩方程, 本征值问题的解用伽辽金方法求出, 得到的临界载荷与经典线性解作了比较. 具体计算了三种不同环肋参数的外肋加劲圆柱壳, 结果表明, 肋的强弱不仅显著影响临界载荷值, 同时也改变了柱壳的缺陷敏感度.

一、引 言

长期以来, 薄壳稳定性的经典理论和实验结果之间的差异曾引起许多研究者的重视和努力. 早在30年代, Donnell^[1]就开始研究薄壳的后屈曲性能. 1941年, Karman 和钱学森^[2]在解释 Donnell 大挠度方程的基础上提出非线性跳跃理论, 指出在远低于线性临界压力情况下存在着一种后屈曲平衡位形. 1942年, 钱氏^[3]又提出等能量准则以说明这种非线性跳跃的条件. 后来的研究表明, 在后屈曲阶段可能存在一种对应于最小载荷的平衡位形, 但这个最小载荷却不是临界载荷, 因此这种理论仍不能充分解释理论和实验临界压力之间的分歧.

从60年代以后, Stein^[4]等人提出非线性前屈曲一致理论, 抛弃了经典稳定分析中前屈曲状态线性的假设, 认为前屈曲解必须与边界条件相一致. Stein 理论与近代的精确模型实验结果比较接近, 但是它的处理对象仍然停留在理想的完善壳体上.

为了研究初始缺陷对屈曲的影响, 早在1945年Koiter^{[5][6]}就提出了非完善结构稳定性的一般准则, 即缺陷敏感度理论, 或称之为初始后屈曲理论. 到60年代初期, Koiter 的论文被译成英文后, 得到了广泛的重视和研究. Budiansky 和 Amazigo^[7]曾用 Koiter 理论对圆柱壳受侧向外压和静水外压力情况进行了缺陷敏感度分析, 文中假定柱壳屈曲前为线性无矩状态.

本文试用Koiter理论分析承受静水外压的环肋加劲圆柱壳的初始后屈曲性能. 为了考察边界条件对此的影响, 在前屈曲状态采用了与边界条件相一致的非线性有矩方程. 首先推导

出后屈曲系数 a 和 b 的一般表达式, 然后建立环肋加劲圆柱壳的基本方程, 并用摄动法求解。

二、考虑非线性前屈曲态状的 Koiter 一般理论

假设结构的广义载荷为 q , 它的广义位移、广义应变和广义应力用 u , ε 和 σ 表示。设广义应变和广义位移之间的关系式是

$$\varepsilon = L_1(u) + L_2(u)/2 \quad (2.1)$$

它的变分形式可以写成

$$\delta\varepsilon = L_1(\delta u) + L_{11}(u, \delta u) \quad (2.2)$$

其中 L_1 为线性算子, L_2 为二次算子, 双线性算子 L_{11} 定义为

$$L_2(u+v) = L_2(u) + 2L_{11}(u, v) + L_2(v) \quad (2.3)$$

弹性应力应变关系为

$$\sigma = H(\varepsilon) \quad (2.4)$$

式中 H 为线性函数。虚功方程可以表示为

$$\sigma \cdot \delta\varepsilon = q \cdot \delta u \quad (2.5)$$

而互等定理是

$$H(\varepsilon^{(1)}) \cdot \varepsilon^{(2)} = H(\varepsilon^{(2)}) \cdot \varepsilon^{(1)} \quad (2.6)$$

在载荷 $q = \lambda q_0$ 时, 可以将位移、应变和应力展开成摄动表示式

$$\left. \begin{aligned} u &= u_0(\lambda) + \xi u_1 + \xi^2 u_2 + \xi^3 u_3 + \dots \\ \varepsilon &= \varepsilon_0(\lambda) + \xi \varepsilon_1 + \xi^2 \varepsilon_2 + \xi^3 \varepsilon_3 + \dots \\ \sigma &= \sigma_0(\lambda) + \xi \sigma_1 + \xi^2 \sigma_2 + \xi^3 \sigma_3 + \dots \end{aligned} \right\} \quad (2.7)$$

其中 u_0 , ε_0 , σ_0 是载荷参数 λ 的非线性函数, u_1 为法化了的临界屈曲波形, 摄动参数 ξ 是屈曲位移幅度的一个量度, 当 λ 趋近于临界载荷参数 λ_c 时, ξ 趋近于零。

为了研究在初始屈曲状态下 λ 和 ξ 的关系, 将 u_0 , ε_0 和 σ_0 在临界点 λ_c 附近展开成泰勒级数

$$\left. \begin{aligned} u_0 &= u_{0c} + (\lambda - \lambda_c) u_0^{(1)} + (\lambda - \lambda_c)^2 u_0^{(2)}/2 + \dots \\ \varepsilon_0 &= \varepsilon_{0c} + (\lambda - \lambda_c) \varepsilon_0^{(1)} + (\lambda - \lambda_c)^2 \varepsilon_0^{(2)}/2 + \dots \\ \sigma_0 &= \sigma_{0c} + (\lambda - \lambda_c) \sigma_0^{(1)} + (\lambda - \lambda_c)^2 \sigma_0^{(2)}/2 + \dots \end{aligned} \right\} \quad (2.8)$$

式中记 $(\partial^n u_0 / \partial \lambda^n)_{\lambda=\lambda_c} = u_0^{(n)}$ ($n=0, 1, 2, \dots$), 对于 $\varepsilon_0^{(n)}$ 和 $\sigma_0^{(n)}$ 皆同。同时应用 Koiter 理论, 在初始后屈曲阶段, 载荷参数比为

$$\lambda/\lambda_c = 1 + a\xi + b\xi^2 + \dots \quad (2.9)$$

上式中 a 和 b 为后屈曲系数。

将(2.7), (2.8)式代入(2.1), (2.4), (2.5)式, 并运用公式(2.2)和(2.9), 然后分别比较 ξ 各次项的系数。对于 ξ 的零次项系数, 可得临界状态的前屈曲解

$$\left. \begin{aligned} \varepsilon_{0c} &= L_1(u_{0c}) + L_2(u_{0c})/2 \\ \sigma_{0c} &= H(\varepsilon_{0c}) \\ \sigma_{0c} \cdot [L_1(\delta u) + L_{11}(u_{0c}, \delta u)] &= \lambda_c q_0 \delta u \end{aligned} \right\} \quad (2.10)$$

比较 ξ 的一次项系数, 得临界屈曲解

$$\left. \begin{aligned} \varepsilon_1 &= L_1(u_1) + L_{11}(u_{0c}, u_1), \quad \sigma_1 = H(\varepsilon_1) \\ \sigma_1 [L_1(\delta u) + L_{11}(u_{0c}, \delta u)] + \sigma_{0c} L_{11}(u_1, \delta u) &= 0 \end{aligned} \right\} \quad (2.11)$$

比较 ξ 的二次项系数, 得

$$\left. \begin{aligned} \varepsilon_2 &= L_1(u_2) + L_2(u_1)/2 + a\lambda_c L_{11}(u_{0c}^{(1)}, u_1) + L_{11}(u_{0c}, u_2) \\ \sigma_2 &= H(\varepsilon_2) \\ \sigma_2 [L_1(\delta u) + L_{11}(u_{0c}, \delta u)] + \sigma_1 L_{11}(u_1, \delta u) + \sigma_{0c} L_{11}(u_2, \delta u) \\ &\quad + a\lambda_c [\sigma_{0c}^{(1)} L_{11}(u_1, \delta u) + \sigma_1 L_{11}(u_{0c}^{(1)}, \delta u)] = 0 \end{aligned} \right\} \quad (2.12)$$

取 $\delta u = u_1$, 并考虑互等关系式(2.6), 即由上式中的第三式得

$$a = - \frac{3/2 \cdot \sigma_1 L_2(u_1)}{\lambda_c [\sigma_{0c}^{(1)} L_2(u_1) + 2\sigma_1 L_{11}(u_{0c}^{(1)}, u_1)]} \quad (2.13)$$

又比较 ξ 的三次项系数, 得

$$\left. \begin{aligned} \varepsilon_3 &= L_1(u_3) + L_{11}(u_{0c}, u_3) + L_{11}(u_1, u_2) + a\lambda_c L_{11}(u_{0c}^{(1)}, u_2) \\ &\quad + \frac{a^2 \lambda_c^2}{2} L_{11}(u_1, u_{0c}^{(2)}) + b\lambda_c L_{11}(u_{0c}^{(1)}, u_1) \\ \sigma_3 &= H(\varepsilon_3) \\ \sigma_{0c} L_{11}(u_3, \delta u) + \sigma_1 L_{11}(u_2, \delta u) + \sigma_2 L_{11}(u_1, \delta u) + \sigma_3 [L_1(\delta u) \\ &\quad + L_{11}(u_{0c}, \delta u)] + a\lambda_c [\sigma_{0c}^{(1)} L_{11}(u_2, \delta u) + \sigma_2 L_{11}(u_{0c}^{(1)}, \delta u)] \\ &\quad + \frac{a^2 \lambda_c^2}{2} [\sigma_1 L_{11}(u_{0c}^{(2)}, \delta u) + \sigma_{0c}^{(2)} L_{11}(u_1, \delta u)] \\ &\quad + b\lambda_c [\sigma_{0c}^{(1)} L_{11}(u_1, \delta u) + \sigma_1 L_{11}(u_{0c}^{(1)}, \delta u)] = 0 \end{aligned} \right\} \quad (2.14)$$

同样, 取 $\delta u = u_1$, 且利用公式(2.6), 即由上式中的第三式得

$$\left. \begin{aligned} b &= - \{ 2\sigma_1 L_{11}(u_1, u_2) + \sigma_2 L_2(u_1) + a\lambda_c [\sigma_{0c}^{(1)} L_{11}(u_1, u_2) \\ &\quad + \sigma_1 L_{11}(u_{0c}^{(1)}, u_2) + \sigma_2 L_{11}(u_{0c}^{(1)}, u_1)] + \frac{a^2 \lambda_c^2}{2} [\sigma_{0c}^{(2)} L_2(u_1) \\ &\quad + 2\sigma_1 L_{11}(u_{0c}^{(2)}, u_1)] \} / \{ \lambda_c [2\sigma_1 L_{11}(u_{0c}^{(1)}, u_1) + \sigma_{0c}^{(1)} L_2(u_1)] \} \end{aligned} \right\} \quad (2.15)$$

至此, 已求得后屈曲系数 a 和 b 。当结构屈曲时具有和特征位移的方向无关的对称性质时, 由公式(2.9)可以看出 $a=0$, 且 b 具有较简单的形式

$$b = - \frac{2\sigma_1 L_{11}(u_1, u_2) + \sigma_2 L_2(u_1)}{\lambda_c [\sigma_{0c}^{(1)} L_2(u_1) + 2\sigma_1 L_{11}(u_{0c}^{(1)}, u_1)]} \quad (2.16)$$

结构的初始后屈曲性能取决于系数 b ; 若 b 为正值, 则结构屈曲后载荷可继续增加; 若 b 为负值, 则屈曲后载荷降低, 即不可能继续加载, 且结构是缺陷敏感的。

如果结构具有微小的初始缺陷 \bar{u} , 其形状和临界屈曲波形相一致, 即 $\bar{u} = \bar{\xi} u_1$, $\bar{\xi}$ 为缺陷参数。于是, 载荷 λq_0 引起的位移、应变和应力为

$$\left. \begin{aligned} u &= u_0 + \xi u_1 + \xi^2 u_2 + \xi^3 u_3 + \dots \\ \varepsilon &= L_1(u) + L_2(u)/2 + L_{11}(u, \bar{u}) \\ &= \varepsilon_0 + \xi \varepsilon_1 + \xi^2 \varepsilon_2 + \dots + \bar{\xi} L_{11}(u_0, u_1) + \xi \bar{\xi} L_{11}(u_1, u_1) + \dots \\ \sigma &= \sigma_0 + \xi \sigma_1 + \xi^2 \sigma_2 + \dots + \bar{\xi} H[L_{11}(u_0, u_1)] + \xi \bar{\xi} H[L_{11}(u_1, u_1)] + \dots \end{aligned} \right\} \quad (2.17)$$

它的应变变分为

$$\begin{aligned} \delta\varepsilon = & L_1(\delta u) + L_{11}(u_0, \delta u) + \xi L_{11}(u_1, \delta u) \\ & + \xi^2 L_{11}(u_2, \delta u) + \dots + \bar{\xi} L_{11}(u_1, \delta u) \end{aligned} \quad (2.18)$$

因为当 $\bar{\xi} = 0$ 时, 载荷参数比由公式

$$\lambda/\lambda_c = 1 + a\xi + b\xi^2 + \dots$$

表示, 现设

$$\lambda/\lambda_c = 1 + a\xi + b\xi^2 + \dots + \bar{\xi} f(\xi, \bar{\xi}) \quad (2.19)$$

将(2.17), (2.18), (2.19)式代入虚功方程式(2.5), 略去 $\xi\bar{\xi}$ 和 $\bar{\xi}^2$ 以上的高阶项, 可以得到

$$\lambda_c \xi \bar{\xi} f(\xi, \bar{\xi}) = \frac{\sigma_1 L_1(u_1) - (\lambda - \lambda_c) \sigma_0^{(1)} L_{11}(u_1, u_1)}{\sigma_0^{(1)} L_{11}(u_1, u_1) + \sigma_1 L_{11}(u_0^{(1)}, u_1)} \bar{\xi}$$

于是有

$$\begin{aligned} (\lambda - \lambda_c) \xi = & a \lambda_c \xi^2 + b \lambda_c \xi^3 + \dots \\ & + \bar{\xi} \frac{\sigma_1 L_1(u_1) - (\lambda - \lambda_c) \sigma_0^{(1)} L_{11}(u_1, u_1)}{\sigma_0^{(1)} L_{11}(u_1, u_1) + \sigma_1 L_{11}(u_0^{(1)}, u_1)} \end{aligned} \quad (2.20)$$

公式(2.20)就是在 $\bar{\xi}$ 比较小的情况下, λ 和 ξ , $\bar{\xi}$ 之间的近似关系式。

将公式(2.20)对 $\bar{\xi}$ 进行导微, 且当 $d\lambda/d\bar{\xi} = 0$ 时 $\lambda = \lambda_m$ 和 $\bar{\xi} = \bar{\xi}_m$, 于是

$$(\lambda_m - \lambda_c) = 2a\lambda_c \bar{\xi}_m + 3b\lambda_c \bar{\xi}_m^2 \quad (2.21a)$$

由(2.20), (2.21a)式就可以确定载荷参数的极值 λ_m , 它与缺陷参数 $\bar{\xi}$ 以及初始后屈曲系数 a 和 b 有关。

在特殊情况 $a = 0$ 时, 由(2.21a)式得

$$\bar{\xi}_m = \left(\frac{\lambda_m - \lambda_c}{3b\lambda_c} \right)^{1/2} \quad (2.21b)$$

代入(2.20)式, 并略去高阶微量, 有

$$1 - \frac{\lambda_m}{\lambda_c} = 3 \left(\frac{-b}{4} \right)^{1/3} \left[\bar{\xi} \frac{\sigma_1 L_1(u_1)}{\lambda_c [\sigma_0^{(1)} L_{11}(u_1, u_1) + \sigma_1 L_{11}(u_0^{(1)}, u_1)]} \right]^{2/3} \quad (2.22)$$

可见, 当 $a = 0$, $b < 0$ 时, 由上式得出 $\lambda_m < \lambda_c$ 。也就是说, 由于缺陷的存在致使结构的屈曲强度降低, 所以结构是缺陷敏感的。而且当 b 的绝对值愈大时, 屈曲强度降低愈多, 因此 b 值的大小反映了结构对缺陷敏感的程度。

上面推导了考虑非线性前屈曲状态的 Koiter 一般理论。如假定结构的前屈曲状态为线性, 亦即

$$\sigma_0^{(1)} = \frac{\sigma_{0c}}{\lambda_c}, \quad u_0^{(1)} = \frac{u_{0c}}{\lambda_c}$$

且

$$L_{11}(u_{0c}, u_1) = 0$$

那么, 后屈曲系数 a 和 b 的表达式(2.13), (2.15), (2.16)以及 λ 和 ξ , $\bar{\xi}$ 之间的近似关系式(2.20)完全回复到文献[7]中的结果。

三、环肋加劲圆柱壳的基本方程

用上面推导的 Koiter 理论分析在静水压力 p 作用下的环肋加劲圆柱薄壳。设壳长为 L , 半径为 R , 壳厚为 t , 环肋截面积 A , 惯性矩 I , 偏心距 e (环肋中性轴到壳中面的距离, 环肋

位于壳外面时定为正), 环肋间距为 d , 如图 1 所示。

假定环肋刚度均匀分布在壳面上, 肋只承受其自身轴线方向的拉伸与弯曲, 它的弯曲率与壳相同, 且法向应变在肋和壳上作线性分布。肋的侧向弯曲及扭转略去不计。

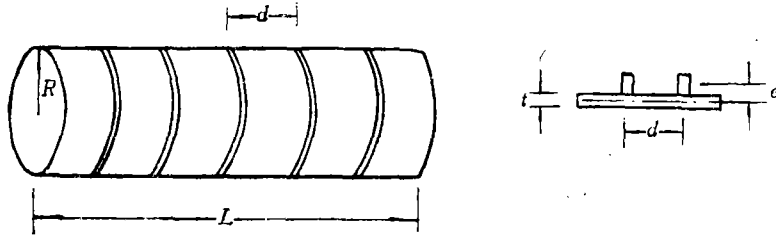


图 1 环肋加劲圆柱壳

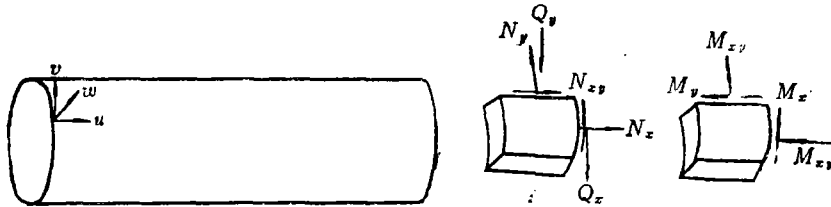


图 2 圆柱壳的内力和位移

设加劲壳的内力为 $N_x, N_y, N_{xy}, Q_x, Q_y, M_x, M_y, M_{xy}$; 其中仅与壳有关的量加注足标 s , 与环肋有关的量加注足标 r 。壳中面应变为 $\epsilon_x, \epsilon_y, \gamma_{xy}$; 弯曲应变为 $\kappa_x, \kappa_y, \kappa_{xy}$ 。环肋中性轴处的轴向应变为 ϵ_r , 轴向应力为 σ_r , 弯曲应变为 κ_r , 壳的中面挠度为 w 。

根据假定, 环肋弯曲应变和壳相同, 即 $\kappa_r = -\partial^2 w / \partial y^2$, 并认为法向应变在环肋和壳上作线性分布, 因此环肋中性轴处的应变为 $\epsilon_r = \epsilon_y - e \partial^2 w / \partial y^2$, 且环肋的轴力为 $\sigma_r A$, 它对壳中面处的弯矩是 $(-EI \partial^2 w / \partial y^2 + \sigma_r A e)$ 。由于假设肋的作用均匀分布在壳面上, 故有

$$N_{yr} = \frac{\sigma_r A}{d}, \quad M_{yr} = -\frac{EI d^2 w / dy^2}{d} + \frac{\sigma_r A e}{d}$$

综合壳中内力, 加劲壳的内力和应变关系为

$$\left. \begin{aligned} M_x &= -D \left(\frac{\partial^2 w}{\partial x^2} + \nu \frac{\partial^2 w}{\partial y^2} \right) \\ M_y &= -D \left(\frac{\partial^2 w}{\partial y^2} + \nu \frac{\partial^2 w}{\partial x^2} \right) - \frac{EI}{d} \frac{\partial^2 w}{\partial y^2} + \frac{\sigma_r A e}{d} \\ M_{xy} &= -(1-\nu) D \frac{\partial^2 w}{\partial x \partial y} \end{aligned} \right\} \quad (3.1)$$

而
$$\sigma_r = \frac{d}{A + dt} \left(N_y - \nu N_x - E t e \frac{\partial^2 w}{\partial y^2} \right) \quad (3.2)$$

式中 E 为杨氏模量, ν 为泊桑比; 弯曲刚度 $D = Et^3 / 12(1 - \nu^2)$ 。

限于篇幅, 环肋加劲圆柱薄壳的基本方程的推导, 这里不再赘述, 推导过程和非线性圆柱薄壳相同。仅将结果列之于下。

引入应力函数

$$N_x = \frac{\partial^2 F}{\partial y^2}, \quad N_y = \frac{\partial^2 F}{\partial x^2}, \quad N_{xy} = -\frac{\partial^2 F}{\partial x \partial y} \quad (3.3)$$

并记 $\alpha = \frac{A}{dt+A}$, $D' = D + \frac{EI}{d} + Et\alpha e^2$

又引入线性微分算子

$$L_1 = D \frac{\partial^4}{\partial x^4} + 2D \frac{\partial^4}{\partial x^2 \partial y^2} + D' \frac{\partial^4}{\partial y^4}$$

$$L_{\mathbf{I}} = -e\alpha \frac{\partial^4}{\partial x^2 \partial y^2} + \nu e\alpha \frac{\partial^4}{\partial y^4} + \frac{1}{R} \frac{\partial^2}{\partial x^2}$$

$$L_{\mathbf{II}} = \frac{1-\alpha}{Et} \frac{\partial^4}{\partial x^4} + \frac{2(1+\nu\alpha)}{Et} \frac{\partial^4}{\partial x^2 \partial y^2} + \frac{(1-\nu^2\alpha)}{Et} \frac{\partial^4}{\partial y^4}$$

于是, 环肋加劲圆柱薄壳的基本方程为

$$\left. \begin{aligned} L_1(w) + L_{\mathbf{I}}(F) &= \frac{\partial^2 F}{\partial x^2} \frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 F}{\partial y^2} \frac{\partial^2 w}{\partial x^2} - 2 \frac{\partial^2 F}{\partial x \partial y} \frac{\partial^2 w}{\partial x \partial y} - p \\ L_{\mathbf{I}}(F) - L_{\mathbf{I}}(w) &= \left(\frac{\partial^2 w}{\partial x \partial y} \right)^2 - \frac{\partial^2 w}{\partial x^2} \frac{\partial^2 w}{\partial y^2} \end{aligned} \right\} \quad (3.4)$$

即适用于偏心环肋加劲圆柱壳的广义 Von Karman-Donell 型基本微分方程式。

四、摄动解法

将载荷 p 、挠度 w 和应力函数 F 展开成摄动级数的形式

$$\left. \begin{aligned} \frac{p}{p_c} &= 1 + a \left(\frac{\delta}{t} \right) + b \left(\frac{\delta}{t} \right)^2 + \dots \\ \left. \begin{aligned} \left\{ \begin{matrix} w \\ F \end{matrix} \right\} &= \left\{ \begin{matrix} w_0 \\ F_0 \end{matrix} \right\} + \left(\frac{\delta}{t} \right) \left\{ \begin{matrix} w_1 \\ F_1 \end{matrix} \right\} + \left(\frac{\delta}{t} \right)^2 \left\{ \begin{matrix} w_2 \\ F_2 \end{matrix} \right\} + \dots \end{aligned} \right\} \quad (4.1)$$

其中 p_c 为临界载荷, w_0, F_0 是前屈曲解, w_1, F_1 是临界屈曲解, δ 为 w_1 的波幅值。

考虑简支边界条件, 当 $x=0, L$ 时有

$$w=0, \quad v=0 \quad (4.2)$$

$$\frac{\partial^2 F}{\partial y^2} = N_s = -\frac{pR}{2}, \quad M_s = 0 \quad (4.3)$$

式中 v 为薄壳的环向位移。从上式 $v=w=0$ 可知壳体边界中面环向应变 $\epsilon_r=0$, 即

$$\frac{\partial^2 F}{\partial x^2} - \frac{\sigma_r A}{d} - \nu \frac{\partial^2 w}{\partial y^2} = 0$$

再利用(3.2), (3.3)式和(3.1)式, 可以将边界条件(4.2), (4.3)式写成当 $x=0, L$ 时有

$$w_0 = \frac{\partial^2 w_0}{\partial x^2} = 0, \quad \frac{\partial^2 F_0}{\partial y^2} = -\frac{pR}{2}, \quad \frac{\partial^2 F_0}{\partial x^2} = -\frac{\nu pR}{2} \quad (4.4)$$

$$\text{以及 } w_j = \frac{\partial^2 w_j}{\partial x^2} = \frac{\partial^2 F_j}{\partial x^2} = \frac{\partial^2 F_j}{\partial y^2} = 0 \quad (j=1, 2, 3, \dots) \quad (4.5)$$

1. 前屈曲状态

壳体在前屈曲状态, 其挠度 w_0 和应力函数 F_0 必须满足(3.4)式。由于结构和载荷均为轴对称, 故前屈曲变形和应力是轴对称的。可以设

$$w_0 = w_0(x, p), \quad F_0 = -\frac{pR}{4}y^2 + f_0(x, p) \quad (4.6)$$

将上式代入(3.4)式, 得到前屈曲方程

$$\left. \begin{aligned} D \frac{d^4 w_0}{dx^4} + \frac{pR}{2} \frac{d^2 w_0}{dx^2} + \frac{Et}{R^2(1-\alpha)} w_0 &= \frac{p}{2}(v-2) \\ \frac{d^2 f_0}{dx^2} &= \frac{Et}{R(1-\alpha)} w_0 - \frac{v p R}{2} \end{aligned} \right\} \quad (4.7)$$

2. 临界屈曲解

为了研究结构在临界点邻近的后屈曲性能, 将 $\begin{Bmatrix} w_0 \\ F_0 \end{Bmatrix}$ 在临界载荷邻近展开成泰勒级数

$$\left. \begin{aligned} w_0 &= w_{0c} + (p-p_c) \left. \frac{\partial w_0}{\partial p} \right|_{p=p_c} + \frac{1}{2} (p-p_c)^2 \left. \frac{\partial^2 w_0}{\partial p^2} \right|_{p=p_c} + \dots \\ F_0 &= F_{0c} + (p-p_c) \left. \frac{\partial F_0}{\partial p} \right|_{p=p_c} + \frac{1}{2} (p-p_c)^2 \left. \frac{\partial^2 F_0}{\partial p^2} \right|_{p=p_c} + \dots \end{aligned} \right\} \quad (4.8)$$

式中 $p-p_c = a\left(\frac{\delta}{t}\right) + b p_c \left(\frac{\delta}{t}\right)^2 + \dots$

把摄动展开式(4.1)和(4.8)代入基本方程式(3.4), 然后比较 (δ/t) 的一次项系数, 得到关于临界屈曲解的微分方程组

$$\left. \begin{aligned} L_1(w_1) + L_1(F_1) - \left(\frac{\partial^2 f_0}{\partial x^2} \right)_c \frac{\partial^2 w_1}{\partial y^2} - \frac{\partial^2 F}{\partial y^2} \left(\frac{\partial^2 w_0}{\partial x^2} \right)_c + \frac{p_c R}{2} \frac{\partial^2 w_1}{\partial x^2} &= 0 \\ L_1(F_1) - L_1(w_1) + \left(\frac{\partial^2 w_0}{\partial x^2} \right)_c \frac{\partial^2 w_1}{\partial y^2} &= 0 \end{aligned} \right\} \quad (4.9)$$

我们以足标 c 表示临界状态值。

设临界屈曲波形 w_1 和应力函数 F_1 为

$$w_1 = w_{11}(x) \cos(ny/R), \quad F_1 = f_1(x) \cos(ny/R)$$

代入(4.9)式得

$$\left. \begin{aligned} D \frac{d^4 w_{11}}{dx^4} + \left[\frac{p_c R}{2} - 2D \left(\frac{n}{R} \right)^2 \right] \frac{d^2 w_{11}}{dx^2} + D' \left(\frac{n}{R} \right)^4 w_{11} \\ + \left[ea \left(\frac{n}{R} \right)^2 + \frac{1}{R} \right] \frac{d^2 f_1}{dx^2} + v ea \left(\frac{n}{R} \right)^4 f_1 + \left(\frac{n}{R} \right)^2 \left(\frac{\partial^2 f_0}{\partial x^2} \right)_c w_{11} \\ + \left(\frac{n}{R} \right)^2 \left(\frac{\partial^2 w_0}{\partial x^2} \right)_c f_1 = 0 \\ 1-\alpha \frac{d^4 f_1}{Et dx^4} - \frac{2(1+\nu\alpha)}{Et} \left(\frac{n}{R} \right)^2 \frac{d^2 f_1}{dx^2} + \frac{(1-\nu^2\alpha)}{Et} \left(\frac{n}{R} \right)^4 f_1 \\ - \left[ea \left(\frac{n}{R} \right)^2 + \frac{1}{R} \right] \frac{d^2 w_{11}}{dx^2} - v ae \left(\frac{n}{R} \right)^4 w_{11} \\ - \left(\frac{n}{R} \right)^2 \left(\frac{d^2 w_0}{dx^2} \right)_c w_{11} = 0 \end{aligned} \right\} \quad (4.10)$$

由此, 可以确定临界载荷 p_c , 并求出后屈曲系数 a . 由于本问题具有和特征位移方向无关的对称性质, 故可预期到 $a=0$.

3. 关于 w_2 和 F_2

把 (4.1) 和 (4.8) 式代入基本微分方程 (3.4) 式后, 比较 (δ/t) 二次项的系数, 得到关于 w_2 和 F_2 的微分方程组. 并设

$$w_2 = w_{21}(x) + w_{22} \cos \frac{2ny}{R}, \quad F_2 = f_{21}(x) + f_{22} \cos \frac{2ny}{R}$$

得到两组常微分方程组 (已用了 $a=0$ 这一结论)

$$\left. \begin{aligned} D \frac{d^4 w_{21}}{dx^4} + \frac{p_c R}{2} \frac{d^2 w_{21}}{dx^2} + \frac{1}{R} \frac{d^2 f_{21}}{dx^2} &= -\frac{1}{2} \left(\frac{n}{R} \right)^2 \\ &\cdot \left(\frac{d^2 f_1}{dx^2} w_{11} + 2 \frac{df_1}{dx} \frac{dw_{11}}{dx} + f_1 \frac{d^2 w_{11}}{dx^2} \right) \\ 1 - \alpha \frac{d^4 f_{21}}{Et dx^4} - \frac{1}{R} \frac{d^2 w_{21}}{dx^2} &= \frac{1}{2} \left(\frac{n}{R} \right)^2 \left[\left(\frac{dw_{11}}{dx} \right)^2 + w_{11} \frac{d^2 w_{11}}{dx^2} \right] \end{aligned} \right\} \quad (4.11)$$

和

$$\left. \begin{aligned} D \frac{d^4 w_{22}}{dx^4} + \left[\frac{p_c R}{2} - 2D \left(\frac{2n}{R} \right)^2 \right] \frac{d^2 w_{22}}{dx^2} + \left[D' \left(\frac{2n}{R} \right)^4 \right. \\ \left. + \left(\frac{\partial^2 f_0}{\partial x^2} \right)_c \left(\frac{2n}{R} \right)^2 \right] w_{22} + \left[e\alpha \left(\frac{2n}{R} \right)^2 + \frac{1}{R} \right] \frac{d^2 f_{22}}{dx^2} \\ \left. + \left[\nu e\alpha \left(\frac{2n}{R} \right)^4 + \left(\frac{\partial^2 w_0}{\partial x^2} \right)_c \left(\frac{2n}{R} \right)^2 \right] f_{22} \right. \\ = -\frac{1}{2} \left(\frac{n}{R} \right)^2 \left(\frac{d^2 f_1}{dx^2} w_{11} - 2 \frac{df_1}{dx} \frac{dw_{11}}{dx} + f_1 \frac{d^2 w_{11}}{dx^2} \right) \\ 1 - \alpha \frac{d^4 f_{22}}{Et dx^4} - \frac{2(1+\nu\alpha)}{Et} \left(\frac{2n}{R} \right)^2 \frac{d^2 f_{22}}{dx^2} + \frac{1-\nu^2\alpha}{Et} \left(\frac{2n}{R} \right)^4 f_{22} \\ \left. - \left[e\alpha \left(\frac{2n}{R} \right)^2 + \frac{1}{R} \right] \frac{d^2 w_{22}}{dx^2} - \left[\nu e\alpha \left(\frac{2n}{R} \right)^4 + \left(\frac{\partial^2 w_0}{\partial x^2} \right)_c \left(\frac{2n}{R} \right)^2 \right] w_{22} \right. \\ = \frac{1}{2} \left(\frac{n}{R} \right)^2 \left[\frac{d^2 w_{11}}{dx^2} w_{11} - \left(\frac{dw_{11}}{dx} \right)^2 \right] \end{aligned} \right\} \quad (4.12)$$

至此, 已能求出初始后屈曲系数 a 和 b . 如上所述, 由于结构具有对称性, 故 $a=0$ 而 b 值可由 (2.16) 式求得. 在本问题中, 它的有关项分别为

$$\sigma_1 = \begin{Bmatrix} \frac{\partial^2 F_1}{\partial y^2} \\ \frac{\partial^2 F_1}{\partial x^2} \\ -\frac{\partial^2 F_1}{\partial x \partial y} \\ M_{x1} \\ M_{y1} \\ M_{xy1} \end{Bmatrix}, \quad \sigma_2 = \begin{Bmatrix} \frac{\partial^2 F_2}{\partial y^2} \\ \frac{\partial^2 F_2}{\partial x^2} \\ -\frac{\partial^2 F_2}{\partial x \partial y} \\ M_{x2} \\ M_{y2} \\ M_{xy2} \end{Bmatrix}, \quad \sigma_0^{(1)} = \begin{Bmatrix} \frac{\partial^2 F_0^{(1)}}{\partial y^2} \\ \frac{\partial^2 F_0^{(1)}}{\partial x^2} \\ -\frac{\partial^2 F_0^{(1)}}{\partial x \partial y} \\ 0 \\ 0 \\ 0 \end{Bmatrix},$$

$$L_2(u_1) = \begin{pmatrix} \left(\frac{\partial w_1}{\partial x}\right)^2 \\ \left(\frac{\partial w_1}{\partial y}\right)^2 \\ 2 \frac{\partial w_1}{\partial x} \frac{\partial w_1}{\partial y} \\ 0 \\ 0 \\ 0 \end{pmatrix}, \quad L_{11}(u_1, u_2) = \begin{pmatrix} \frac{\partial w_1}{\partial x} \frac{\partial w_2}{\partial x} \\ \frac{\partial w_1}{\partial y} \frac{\partial w_2}{\partial y} \\ \frac{\partial w_1}{\partial y} \frac{\partial w_2}{\partial x} + \frac{\partial w_2}{\partial y} \frac{\partial w_1}{\partial x} \\ 0 \\ 0 \\ 0 \end{pmatrix},$$

$$L_{11}(u_{0c}, u_1) = \begin{pmatrix} \frac{\partial w_1}{\partial x} \frac{\partial w_{0c}^{(1)}}{\partial x} \\ \frac{\partial w_1}{\partial y} \frac{\partial w_{0c}^{(1)}}{\partial y} \\ \frac{\partial w_1}{\partial x} \frac{\partial w_{0c}^{(1)}}{\partial y} + \frac{\partial w_1}{\partial y} \frac{\partial w_{0c}^{(1)}}{\partial x} \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

式中 $w_{0c}^{(1)} = \frac{\partial w_0}{\partial p} \Big|_{p=p_c}$, $F_{0c}^{(1)} = \frac{\partial F_0}{\partial p} \Big|_{p=p_c}$

将以上诸式代入(2.16)式, 即得

$$b = b_1/b_2 \tag{4.13}$$

其中

$$b_1 = - \frac{1}{p_c} \left\{ 2 \iint \left[\frac{\partial^2 F_1}{\partial y^2} \frac{\partial w_1}{\partial x} \frac{\partial w_2}{\partial x} + \frac{\partial^2 F_1}{\partial x^2} \frac{\partial w_1}{\partial y} \frac{\partial w_2}{\partial y} - \frac{\partial^2 F_1}{\partial x \partial y} \left(\frac{\partial w_1}{\partial x} \frac{\partial w_2}{\partial y} + \frac{\partial w_2}{\partial x} \frac{\partial w_1}{\partial y} \right) \right] dx dy \right. \\ \left. + \iint \left[\frac{\partial^2 F_2}{\partial y^2} \left(\frac{\partial w_1}{\partial x} \right)^2 + \frac{\partial^2 F_2}{\partial x^2} \left(\frac{\partial w_1}{\partial y} \right)^2 - 2 \frac{\partial^2 F_2}{\partial x \partial y} \frac{\partial w_1}{\partial x} \frac{\partial w_1}{\partial y} \right] dx dy \right\} \tag{4.14}$$

$$b_2 = \iint \left[\frac{\partial^2 F_{0c}^{(1)}}{\partial y^2} \left(\frac{\partial w_1}{\partial x} \right)^2 + \frac{\partial^2 F_{0c}^{(1)}}{\partial x^2} \left(\frac{\partial w_1}{\partial y} \right)^2 - 2 \frac{\partial^2 F_{0c}^{(1)}}{\partial x \partial y} \frac{\partial w_1}{\partial x} \frac{\partial w_1}{\partial y} \right] dx dy + 2 \iint \left[\frac{\partial^2 F_1}{\partial y^2} \frac{\partial w_{0c}^{(1)}}{\partial x} \frac{\partial w_1}{\partial x} + \frac{\partial^2 F_1}{\partial x^2} \frac{\partial w_{0c}^{(1)}}{\partial y} \frac{\partial w_1}{\partial y} - \frac{\partial^2 F_1}{\partial x \partial y} \left(\frac{\partial w_{0c}^{(1)}}{\partial x} \frac{\partial w_1}{\partial y} + \frac{\partial w_{0c}^{(1)}}{\partial y} \frac{\partial w_1}{\partial x} \right) \right] dx dy$$

五、实例计算

以外肋加劲圆柱壳在静水压力作用下的稳定性分析为例, 讨论肋及其强弱对临界载荷和壳体后屈曲性能的影响。为此选取三组不同的环肋参数

	A dt	J dt^2	c t
I 无 肋	0	0	0
II 弱 肋	0.2	1	2
III 强 肋	0.5	5	5

计算时 $\nu=0.3$ 。

为计算结果简洁起见, 对于各变量无量纲化。设

$$\bar{p} = \frac{pL^2R}{\pi^2D}, \quad \bar{w} = \frac{w}{t}, \quad \bar{f} = \frac{f}{D}, \quad \bar{w}_{0c}^{(1)} = \frac{Et}{R^2} w_{0c}^{(1)}, \quad \xi = \frac{x}{L}$$

并引入无量纲柱壳参数

$$Z = \frac{L^2}{Rt} \sqrt{1-\nu^2}$$

计算步骤: 从(4.7)式可求得前屈曲解 \bar{w}_0, \bar{f}_0 , 然后利用(4.10)式确定临界载荷 \bar{p}_c , 即求方程组的最小本征值问题。由于该方程组为变系数微分方程组, 本文采用伽辽金法求解。

考虑到一般总体失稳波形在轴向总是以一个半波为主 (轴对称失稳除外), 故 \bar{w}_{11}, \bar{f}_1 取下列函数形式

$$\bar{w}_{11} = g_1 \sin \pi \xi, \quad \bar{f}_1 = g_2 \sin \pi \xi \quad (5.1)$$

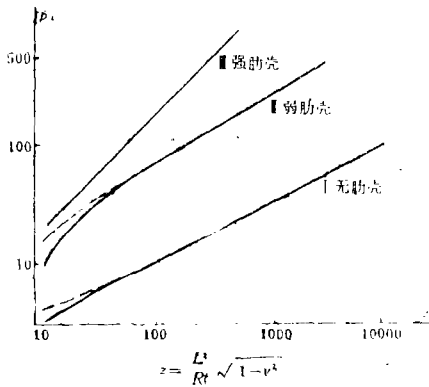
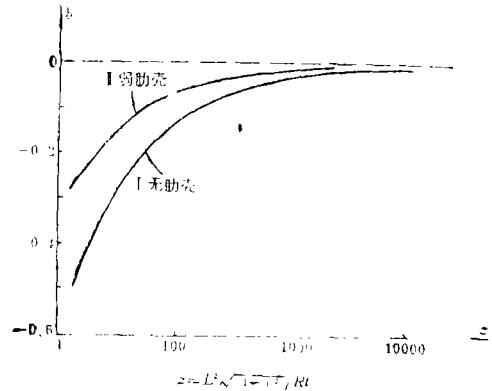
显然上式满足边界条件。一般地说, 上式并不满足(4.10)式, 这里用 $S_1(\bar{w}_{11}, \bar{f}_1)$ 和 $S_2(\bar{w}_{11}, \bar{f}_1)$ 分别表示方程(4.10)中第一式和第二式中的左边部分, 运用伽辽金法可得

$$\left. \begin{aligned} \int_0^1 S_1(\bar{w}_{11}, \bar{f}_1) \sin \pi \xi \delta g_1 d\xi &= 0 \\ \int_0^1 S_2(\bar{w}_{11}, \bar{f}_1) \sin \pi \xi \delta g_2 d\xi &= 0 \end{aligned} \right\} \quad (5.2)$$

为使 \bar{w}_{11}, \bar{f}_1 有非零解, 方程式(5.2)中 g_1 和 g_2 的系数行列式必须为零, 由此可以确定临界载荷 \bar{p}_c , 并进而求出 \bar{w}_{11} 和 \bar{f}_1 。对于(4.11)和(4.12)式的计算, 我们用差分法求解, 得到 $\bar{w}_{21}, \bar{f}_{21}$ 与 $\bar{w}_{21}, \bar{f}_{22}$ 。最后, 从(4.13), (4.14)式籍数值积分算出 b 。

用上述三组不同的环肋参数算得的 \bar{p}_c 绘于图3。图中虚线表示经典性解, 实线为考虑了非线性前屈曲状态后得来的临界载荷。从图中可见, 对于无肋和弱肋加劲的圆柱壳, 当柱壳较短时, 实线和虚线相差较大。也就是说, 由于考虑了边界条件, 使临界载荷值大为降低, 从而更接近于实验结果。随着 Z 的增大至某一定值之后, 屈曲就不再是轴对称了。设从轴对称失稳向非轴对称失稳过渡时的 Z 为 Z_c , 可以发现, 用边界条件一致的非线性前屈曲理论求得的 Z_c 要大于经典线性理论的相应结果。这是因为边界条件限制使周向应力分量起作用的实际长度减少的缘故。如肋 III 这种环肋加劲壳, 若用线性理论计算 $Z=400$ 时就已发生非轴对称屈曲, 而用前屈曲非线性理论, Z 约等于500。

图4是表征缺陷敏感度的系数 b 与 Z 的关系曲线。可以看出, 肋条强度越大, 缺陷敏感度越低。由(4.14)式显见, 强肋加劲壳Ⅲ在发生轴对称失稳时 b 恒为零。此时缺陷敏感度将由(4.1)式中 (δ/t) 的更高次项系数来反映。当 $Z \sim 500$ 时, 柱壳Ⅲ非轴对称失稳, 算出的 b 值几近于零。

图3. $\bar{p} \sim Z$ 关系曲线图4. $b \sim Z$ 关系曲线

六、结 论

上面讨论了外部用环肋加劲的圆柱壳在静水压力作用下的屈曲问题。如前屈曲状态采用与边界条件相符的非线性有矩方程, 求得的临界载荷值将低于经典线性解, 更接近于实验结果。

柱壳用环肋加劲后, 不但临界载荷值显著提高, 同时也降低了缺陷敏感度。因此, 对于承受静水压力的圆柱壳, 外部用环肋加劲能有效地增强壳体抵抗失稳的能力。

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The Initial Post-Buckling Behavior of Ring Stiffened Shells under Hydrostatic Pressure

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Abstract

In this article the initial post-buckling behavior of ring stiffened cylindrical shells under hydrostatic pressure is analyzed by Koiter's theory. The nonlinear bending equations consistent with boundary conditions have been used in prebuckling state. The eigenvalue problem is solved by Galerkin's method. The obtained buckling loads are compared with the results which are based on classical stability theory. As calculating examples, three typical outside-stiffened cylinders with different ring stiffener parameters are chosen. The results show that the strength of stiffener not only influences buckling load obviously, but also changes the imperfection-sensitivity of cylindrical shells.