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一类向量四阶非线性微分方程边值问题的奇摄动

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摘　要

我们研究伴有边界摄动的向量边值问题：

$$\varepsilon^2 y^{(4)}=f(x,y,y'',\varepsilon,\mu)\qquad(\mu<x<1-\mu)$$

$$y(x,\varepsilon,\mu)|_{x=\mu}=A_1(\varepsilon,\mu),\quad y(x,\varepsilon,\mu)|_{x=1-\mu}=B_1(\varepsilon,\mu)$$

$$y''(x,\varepsilon,\mu)|_{x=\mu}=A_2(\varepsilon,\mu),\quad y''(x,\varepsilon,\mu)|_{x=1-\mu}=B_2(\varepsilon,\mu)$$

其中 y,f,A_j 和 B_j （$j=1,\ 2$）是 n 维向量函数和 ε,μ 是两个正的小参数．虽然纯量边值问题曾有人研究过，但这样的向量边值问题尚未被研究．在适当的假设下，利用微分不等式方法，我们找到向量边值问题的一个解和获得一致有效的渐近展开式．

一、引　　言

我们考虑伴有边界摄动的向量四阶非线性微分方程边值问题：

$$\varepsilon^2 y^{(4)}=f(x,y,y'',\varepsilon,\mu)\qquad(\mu<x<1-\mu)\tag{1.1}$$

$$y(x,\varepsilon,\mu)|_{x=\mu}=A_1(\varepsilon,\mu),\quad y(x,\varepsilon,\mu)|_{x=1-\mu}=B_1(\varepsilon,\mu)\tag{1.2}$$

$$y''(x,\varepsilon,\mu)|_{x=\mu}=A_2(\varepsilon,\mu),\quad y''(x,\varepsilon,\mu)|_{x=1-\mu}=B_2(\varepsilon,\mu)\tag{1.3}$$

其中 ε,μ 是两个正的小参数，对于 y,f,A_j 和 $B_j(j=1,2)$不依赖于 μ 且是它们变元的纯量函数的情形，这种边值问题 Howes[1]已讨论过，然而，对于实值 n 维向量函数 $y=(y^1,y^2,$ $y^n)$，$f=(f^1,f^2,\cdots,f^n)$，$A_j=(A_j^1,A_j^2,\cdots,A_j^n)$ 和 $B_j=(B_j^1,B_j^2,\cdots,B_j^n)$ （$j=1,2$)，且它们同时依赖于 ε，μ 的这种向量边值问题似乎还无人问津．在这篇文章里，我们使用微分不等式方法来讨论这个向量边值问题，我们通过引入两个具有边界层性质的函数，我们得到对于所求函数的每一个分量和它们在整个区间 $\mu\leqslant x\leqslant 1-\mu$ 上的二阶导数准确到任一精度的一致有效的渐近开式．

二、构造形式解

首先，我们定义区域：

$$D_i=\{0\leqslant x\leqslant 1,\ |y^i-Y_m^i|\leqslant d_1^i(t),$$

$$|y^{i''}-Y_m^{i''}|\leqslant d_2^i(t),\ 0\leqslant\varepsilon\leqslant\varepsilon_1,\ 0\leqslant\mu\leqslant\mu_1\}$$

其中 $\varepsilon_1>0$，$\mu_1>0$ 为常数，而 Y_m^i 稍后给出，每一个 $d_j^i(x)(j=1,2)$ 为一个光滑的正函数，使得在$[0,0+\delta/2]$中，$d_j^i(x)=|A_j^i|+\delta$；在$[1-\delta/2,1]$中，$d_j^i(x)=|B_j^i|+\delta$；在$[0+\delta,1-\delta]$中，$d_j^i(x)\equiv\delta\ (0<\delta<1)$。

现在我们构造原问题(1.1)～(1.3)的外解。假设它的各分量有下列形式展开式

$$y^i(x,\varepsilon,\mu)=\sum_{s=0}^{\infty}\sum_{k=0}^{s}\varepsilon^{s-k}\mu^k y_{s-k,k}^i \tag{2.1}$$

其中 $y_{s-k,k}^i(k=0,1\cdots,s;\ s=0,1,\cdots)$是待定函数。其次，我们假设退化问题

$$f(x,y,y'',0,0)=0 \qquad (0<x<1) \tag{2.2}$$

$$y(0,0,0)=A_1(0,0),\ y(1,0,0)=B_1(0,0) \tag{2.3}$$

有一个解 $y_{0,0}(x)=(y_{0,0}^1(x),\ y_{0,0}^2(x),\cdots,y_{0,0}^i(x),\cdots,y_{0,0}^n(x))\in C^{(4)}[0,1]$使得,对于$[0,1]$中的 x 和 $D_j(j\neq i)$中的所有y^j，有

$$f^i(x,y_{y_{0,0}^i},y''_{y_{0,0}^{i''}},0,0)=0,\quad y_{0,0}^i(0)=A_1^i(0,0),\quad y_{0,0}^i(1)=B_1^i(0,0).$$

其中 $y_{y_{0,0}^i}=(y^1,\cdots,y_{0,0}^i,\cdots,y^n)$，$y''_{y_{0,0}^{i''}}=(y^{1''},\cdots,y_{0,0}^{i''},\cdots,y^{n''})$。这个假设具有把向量二阶系统分解成 n 个二阶纯量方程的作用,使得纯量理论的方法能够应用于估计每一个分量(参看文[2])。最后，我们假设已知函数 $f(x,y,y'',\varepsilon,\mu)$，$A_j(\varepsilon,\mu)$，$B_j(\varepsilon,\mu)(j=1,2)$，全都有分别关于变量的连续偏导数直到$(m+1)$阶且存在 n 个正常数$m_i(i=1,\ 2,\ \cdots,\ n)$，使得

$$f_{y^{i''}}^i(x,y,y'',\varepsilon,\mu)\geqslant m_i \tag{2.4}$$

于是

$$f^i(x,y_{y^i},y''_{y^{i''}},\varepsilon,\mu)\equiv F^i(\varepsilon,\mu)=\sum_{s=0}^{m}\sum_{k=0}^{s}F_{s-k,k}^i\varepsilon^{s-k}\mu^k+r^i \tag{2.5}$$

$$A_j^i(\varepsilon,\mu)=\sum_{s=0}^{m}\sum_{k=0}^{s}A_{j,s-k,k}^i\varepsilon^{s-k}\mu^k+r_{j1}^i \quad (j=1,2) \tag{2.6}$$

$$B_j^i(\varepsilon,\mu)=\sum_{s=0}^{m}\sum_{k=0}^{s}B_{j,s-k,k}^i\varepsilon^{s-k}\mu^k+r_{j2}^i \quad (j=1,2) \tag{2.7}$$

其中

$$F_{0,0}^i=F^i(0,0)=f^i(x,\ y_{y_{0,0}^i},y''_{y_{0,0}^{i''}},0,0) \tag{2.8}$$

$$\begin{aligned}F_{s-k,k}^i=\frac{1}{(s-k)!k!}\frac{\partial^s F^i}{\partial\varepsilon^{s-k}\partial\mu^k}\Big|_{\varepsilon=\mu=0}&=\frac{\partial f^i}{\partial y^{i''}}(x,y_{y_{0,0}^i},y''_{y_{0,0}^{i''}},0,0)\,y_{s-k,k}^{i''}\\&+\frac{\partial f^i}{\partial y^i}(x,y_{y_{0,0}^i},y''_{y_{0,0}^{i''}},0,0)y_{s-k,k}^i+C_{s-k,k}^i(x)\quad(i=1,2,\cdots,n)\end{aligned} \tag{2.9}$$

$$A_{j,s-k,k}^i=\frac{1}{(s-k)!k!}\frac{\partial^s A_j^i(\varepsilon,\mu)}{\partial\varepsilon^{s-k}\partial\mu^k}\Big|_{\varepsilon=\mu=0}$$

$$(j=1,2;\ k=0,1,\cdots,s;\ s=0,1,\cdots,m) \tag{2.10}$$

$$B_{j,s-k,k}^i=\frac{1}{(s-k)!k!}\frac{\partial^s B_j^i(\varepsilon,\mu)}{\partial\varepsilon^{s-k}\partial\mu^k}\Big|_{\varepsilon=\mu=0}$$

$$(j=1,2;\ k=0,1,\cdots,s;\ s=0,1,\cdots,m) \tag{2.11}$$

$r^i = r^i_{jl} = O\left(\sum_{s=0}^{m+1} \varepsilon^{m+1-s}\mu^s\right)$ $(j,l=1,2;\ 0<\varepsilon,\ \mu\ll 1)$；而 $C^i_{s-k,k}(x)$ 可以通过 x 和 $y^i_{l,k}$, $y^{i''}_{l,t}(0\leqslant l\leqslant s-k-1;\ 0\leqslant t\leqslant k-1)$ 逐次得到，我们略去详细过程。

同样地，上面的符号显而易见也适合于其它的展开式。

将(2.1)代入(1.1)的第 i 个分量，我们有

$$\varepsilon^2 \frac{d^4y^i}{dx^4} = f^i(x, y_{y^i}, y''_{y^{i''}}, \varepsilon, \mu) \tag{2.12}$$

从(2.5)，(2.8)和(2.9)，合并 ε,μ 的同次幂的项且令其系数等于零，我们有

$$f^i(x, y_{y^i_{0,0}}, y''_{y^{i''}_{0,0}}, 0, 0) = 0,$$

$$f^i_{y^{i''}}(x, y_{y^i_{0,0}}, y''_{y^{i''}_{0,0}}, 0, 0) y^{i''}_{s-k,k} + f^i_{y^i}(x, y_{y^i_{0,0}}, y''_{y^{i''}_{0,0}}, 0, 0) y^i_{s-k,k}$$

$$+ c^i_{s-k,k}(x) = y^{i(4)}_{s-k-2,k} \qquad (k=0,1,\cdots,s;\ s=1,2,\cdots,m) \tag{2.13}$$

前面和下面带有负下标的量都认为是零。为了从(2.13)得到 $y^i_{s-k,k}(x)$，我们需要适当的边界条件 $y^i_{s-k,k}(0)$ 和 $y^i_{s-k,k}(1)$，它们将在下面给出。将 $y^i_{s-k,k}(x)$ 代入(2.1)，我们得到问题(1.1)～(1.3)的外解的每一个分量的 m 阶近似式。显然，它关于 x 的二阶导数一般地不可能满足边界条件(1.3)。因此，我们将分别在 $x=\mu$ 和 $x=1-\mu$ 附近构造具有边界性质的函数。

首先，我们在 $x=\mu$ 附近构造具有边界层性质的函数 $U^i(t,\varepsilon,\mu)$，令

$$Y^i = y^i(x,\varepsilon,\mu) + U^i(t,\varepsilon,\mu) \quad (t=(x-\mu)/\varepsilon) \tag{2.14}$$

其中 t 是一个伸展变量。我们假设 $U^i(t,\varepsilon,\mu)$ 有下列的形式展开式：

$$U^i(t,\varepsilon,\mu) = \varepsilon^2 \sum_{s=0}^{\infty} \sum_{k=0}^{s} u^i_{s-k,k}\varepsilon^{s-k}\mu^k \tag{2.15}$$

将 Y^i 代入(1.1)的第 i 个分量，我们有

$$\varepsilon^2 \frac{d^4Y^i}{dx^4} = f^i(x, y_{Y^i}, y''_{Y^{i''}}, \varepsilon, \mu) \tag{2.16}$$

从(2.1)和(2.12)，(2.15)，我们得到

$$\varepsilon^{-2}\frac{d^4U^i}{dt^4} = f^i(\mu+\varepsilon t, y_{y^i+U^i}, y''_{y^{i''}+\varepsilon^{-2}U^{i''}_t}, \varepsilon, \mu)$$

$$- f^i(\mu+\varepsilon t, y_{y^i}, y''_{y^{i''}}, \varepsilon, \mu)$$

$$\equiv \bar F^i(\varepsilon,\mu) = \sum_{s=0}^{m} \sum_{k=0}^{s} \bar F^i_{s-k,k}\varepsilon^{s-k}\mu^k + \bar r^i \tag{2.17}$$

其中

$$\bar F^i_{0,0} = \bar F^i(0,0) = f^i(0, y_{y^i_{0,0}}, y''_{y^{i''}_{0,0}+u^{i''}_{0,0,t}}, 0, 0) - f^i(0, y_{y^i_{0,0}}, y''_{y^{i''}_{0,0}}, 0, 0)$$

$$= f^i_{y^{i''}}(0, y_{y^i_{0,0}}, y''_{y^{i''}_{0,0}+\theta_1 u^{i''}_{0,0,t}}, 0, 0)\frac{d^2u^i_{0,0}}{dt^2} \qquad (0<\theta_1<1)$$

$$\bar{F}^i_{s-k,k}=\frac{1}{(s-k)!k!}\frac{\partial^s \bar{F}^i}{\partial\varepsilon^{s-k}\partial\mu^k}\Big|_{\varepsilon=\mu=0}$$

$$=f^i_{y^{i''}}(0,y_{y^i_{0,0}},y''_{y^{i''}_{0,0}}+u^{i''}_{0,0,t},0,0)\frac{d^2u^i_{s-k,k}}{dt^2}+\bar{c}^i_{s-k,k}(t)$$

而 $\bar{c}^i_{s-k,k}(t)$是由t和$y^i_{l,\tau}$ $(0\leqslant l\leqslant s-k;\ \ 0\leqslant\tau\leqslant k)$，$u^i_{p,q}$ $(0\leqslant p\leqslant s-k-1,\ 0\leqslant q\leqslant k)$的多项式逐次得到。令$\varepsilon$，$\mu$的同次幂系数相等，将产生

$$\frac{d^4u^i_{0,0}}{dt^4}=f^i_{y^{i''}}(0,y_{y^i_{0,0}},y''_{y^{i''}_{0,0}}+\theta_1u^{i''}_{0,0,t},0,0)\frac{d^2u^i_{0,0}}{dt^2}\qquad(0<\theta_1<1)\tag{2.18}$$

$$\frac{d^4u^i_{s-k,k}}{dt^4}=f^i_{y^{i''}}(0,y_{y^i_{0,0}},y''_{y^{i''}_{0,0}}+u^{i''}_{0,0,t},0,0)\frac{d^2u^i_{s-k,k}}{dt^2}+\bar{c}^i_{s-k,k}(t),$$

$$(k=0,1\cdots,s;\ \ s=1,2,\cdots,m)\tag{2.19}$$

为了从(2.18)，(2.19)逐次解出$u^i_{s-k,k}(t)$ $(k=0,1,\cdots,s;\ s=0,1,\cdots,m)$，我们将在下面给出$u^i_{s-k,k}$的适当的初始条件。

其次，我们在$x=1-\mu$附近构造具有边界层性质的函数$V^i(\tau,\varepsilon,\mu)$，设

$$\bar{Y}^i=Y^i+V^i=Y^i(x,\varepsilon,\mu)+U^i(t,\varepsilon,\mu)+V^i(\tau,\varepsilon,\mu)$$

$$\tau=(1-\mu-x)/\varepsilon\tag{2.20}$$

其中τ也是一个伸展变量。我们假设$V^i(\tau,\varepsilon,\mu)$有下列的形式展开式：

$$V^i(\tau,\varepsilon,\mu)=\varepsilon^2\sum_{s=0}^{\infty}\sum_{k=0}^{s}v^i_{s-k,k}(\tau)\varepsilon^{s-k}\mu^k\tag{2.21}$$

将$\bar{Y}^j$代入方程(1.1)的第i个分量，我们有

$$\varepsilon^2\frac{d^4\bar{Y}^i}{dx^4}=f^i(x,y_{\bar{Y}^i},y''_{\bar{Y}^{i''}},\varepsilon,\mu)\tag{2.22}$$

从(2.1)，(2.15)，(2.16)和(2.21)，我们得到

$$\varepsilon^{-2}\frac{d^4V^j}{d\tau^4}=f^i(1-\mu-\varepsilon\tau,\ y_{y^i+U^i+V^i},\ y''_{y^{i''}}+\varepsilon^{-2}(U^{i''}_t+V^{i''}_\tau),\varepsilon,\mu)$$

$$-f^i(1-\mu-\varepsilon\tau,\ y_{y^i+U^i},y''_{y^{i''}}+\varepsilon^{-2}U^{i''}_t,\varepsilon,\mu)$$

$$\equiv\tilde{F}^i(\varepsilon,\mu)=\sum_{s=0}^{m}\sum_{k=0}^{s}\tilde{F}^i_{s-k,k}\varepsilon^{s-k}\mu^k+\tilde{\gamma}^i_{s-k,k}\tag{2.23}$$

其中

$$\tilde{F}^i_{0,0}\equiv\tilde{F}^i(0,0)=f^i(1,\ y_{y^i_{0,0}},y''_{y^{i''}_{0,0}}+u^{i''}_{0,0,t}+v^{i''}_{0,0,\tau},0,0)$$

$$-f^i(1,y_{y^i_{0,0}},y''_{y^{i''}_{0,0}}+u^{i''}_{0,0,t},0,0)$$

$$=f^i_{y^{i''}}(1,y_{y^i_{0,0}},y''_{y^{i''}_{0,0}}+u^{i''}_{0,0,t}+\theta_2v^{i''}_{0,0,\tau},0,0)\frac{d^2v^i_{0,0}}{d\tau^2}\qquad(0<\theta_2<1);$$

$$\tilde{F}^i_{s-k,k}=\frac{1}{(s-k)!k!}\frac{\partial^s\tilde{F}^j}{\partial\varepsilon^{s-k}\partial\mu^k}\Big|_{\varepsilon=\mu=0}$$

$$=f^i_{y^{i''}}(1,y_{y^i_{0,0}},y''_{y^{i''}_{0,0}}+u^{i''}_{0,0,t}+v^{i''}_{0,0,\tau},0,0)\frac{d^2v^i_{s-k,k}}{d\tau^2}+\tilde{c}^i_{s-k,k}(\tau),$$

$$(k=0,1,\cdots,s;\ \ s=1,2,\cdots,m)$$

而$\tilde{c}^i_{s-k,k}(\tau)$ 是由 $\tau, y^i_{l,k}, u^i_{l,k}$ $(0\leqslant l, k\leqslant s)$和 $v^i_{l,k}$ $(0\leqslant l, k\leqslant s-1)$的多项式逐次确定的。

在这里我们假设了 $v^i_{l,k}$ 是具有边界层性质的函数。令 ε，μ 的同次幂相等，我们有

$$\frac{d^4v^i_{0,0}}{d\tau^4}=f^i_{y'''}(1, y_{y^i_{0,0}}, y''_{y^{i''}_{0,0}}+u^{i''}_{0,0,t}+\theta_2 u^{i''}_{0,0,\tau}, 0, 0)\frac{d^2v^i_{0,0}}{d\tau^2} \tag{2.24}$$

$$\frac{d^4v^i_{s-k,k}}{d\tau^4}=f^i_{y''}(1, y_{y^i_{0,0}}, y''_{y^{i''}_{0,0}}+u^{i''}_{0,0,t}+v^{i''}_{0,0,\tau}, 0, 0)\frac{d^2v^i_{s-k,k}}{d\tau^2}$$

$$+\tilde{c}^i_{s-k,k}(\tau) \qquad (k=0,1,\cdots,s;\ s=1,2,\cdots,m) \tag{2.25}$$

为了从(2.24)和(2.25)逐次解出 $v^i_{s-k,k}(\tau)(k=0,1,\cdots,s;\ s=0,1,\cdots,m)$，我们也将在下面给出 $v^i_{s-k,k}$的适当的初始条件。

现在我们给出 $y^i_{s-k,k}$ 的边界条件和 $u^i_{s-k,k}$和$v^i_{s-k,k}$的初始条件如下：

我们先确定 $u^i_{0,0}(t)$和 $v^i_{0,0}(\tau)$的初始条件，

$$\frac{d^2u^i_{0,0}(0)}{dt}=A^i_{2,0,0}-y^{i''}_{0,0}(0) \tag{2.26}$$

$$\frac{d^2v^i_{0,0}(0)}{d\tau^2}=B^i_{2,0,0}-y^{i''}_{0,0}(0) \tag{2.27}$$

在(2.26)和(2.27)中，我们略去了高阶小量。

我们注意到 $u^i_{0,0}$ 和$v^i_{0,0}$分别满足方程(2.18)和(2.24)。不难看出，存在一对函数$u^i_{0,0}(t)$，$v^i_{0,0}(\tau)\in C^{(4)}$，它们具有边界层性质[3]：

$$\frac{d^ju^i_{0,0}(t)}{dt^j}=O(\exp[-m_i(1-k_0)t]) \qquad (t\gg 1, j=0,1,2);$$

$$\frac{d^jv^i_{0,0}(\tau)}{d\tau^j}=O(\exp[-m_i(1-k_0)\tau])$$

其中 k_0 是一个任意小的正常数。

其次，我们确定 $y^i_{s-k,k}(x)$，$u^i_{s-k,k}(t)$和 $v^i_{s-k,k}(\tau)$ $(k=0, 1, \cdots, s;\ s=1, 2, \cdots, m)$ 的条件，考虑到边界摄动（参看[4]），我们有

$$y^i_{s-k,k}(0)=A^i_{1,s-k,k}-u^i_{s-k-2,k}(0)-\sum_{j=1}^{k}y^{i(j)}_{s-k,k-j}(0)/j! \tag{2.28a}$$

$$y^i_{s-k,k}(1)=B^i_{1,s-k,k}-v^i_{s-k-2,k}(0)-\sum_{j=1}^{k}y^{i(j)}_{s-k,k-j}(1)/j! \tag{2.28b}$$

$$\frac{d^2u^i_{s-k,k}(0)}{dt^2}=A^i_{2,s-k,k}-\sum_{j=0}^{k}y^i_{s-k,k-j}(0)/j! \tag{2.29}$$

$$\frac{d^2v^i_{s-k,k}(0)}{d\tau^2}=B^i_{2,s-k,k}-\sum_{j=0}^{k}y^i_{s-k,k-j}(1)/j! \tag{2.30}$$

其中若出现负的下标的字母一律取为零且略去高阶小量。从方程(2.13)和边界条件(2.28a)(2.28b)（在适当的假设下，其解的存在性的证明可仿照文[5]进行）和方程(2.19)、(2.25)

以及初始条件(2.29),(2.30)可逐次得到 $y^i_{s-k,k}(x)$, $u^i_{s-k,k}(t)$和$v^i_{s-k,k}(\tau)$, 其中$u^i_{s-k,k}(t)$和$v^i_{s-k,k}(\tau)$也满足条件[3]:

$$\frac{d^j u^i_{s-k,k}(t)}{dt^j}=O(\exp[-m_i(1-\delta_{s-k,k})t])$$

$$\frac{d^j v^i_{s-k,k}(\tau)}{d\tau^j}=O(\exp[-m_i(1-\sigma_{s-k,k})\tau])$$

$$(t,\tau\gg 1, k=0,1,\cdots,s;\ s=1,2,\cdots,m;\ j=0,1,2)$$

其中$\delta^i_{s-k,k}$和$\sigma^i_{s-k,k}$是任意小的正常量.

将上面确定的$y^i_{s-k,k}$, $u^i_{s-k,k}$和$v^i_{s-k,k}$ ($k=0,1,\cdots,s$; $s=0,1,\cdots,m$)逐次代入(2.1), (2.15), (2.21)和(2.20)且用伸展变量代替x, 我们得到形式渐近展开式的前m项的的和$\tilde{Y}^i_m$, 它是问题(1.1)~(1.3)的解$y(x,\varepsilon,\mu)$的第i个分量:

$$\tilde{Y}^i_m=\sum_{s=0}^{m}\sum_{k=0}^{s}\left\{y^i_{s-k,k}(x)+\varepsilon^2\left[u^i_{s-k,k}\left(\frac{x-\mu}{\varepsilon}\right)+v_{s-k,k}\left(\frac{1-\mu-x}{\varepsilon}\right)\right]\right\}\varepsilon^{s-k}\mu^k$$

三、余 项 估 计

现在我们证明在适当的条件下原问题(1.1)~(1.3)有一个解$y(x,\varepsilon,\mu)\in C^{(4)}$, 它可用下面的一致有效展开式来表示:

$$y^j(x,\varepsilon,\mu)=\tilde{Y}^i_m+R^i_1=\sum_{s=0}^{m}\sum_{k=0}^{s}\left\{y^i_{s-k,k}(x)+\varepsilon^2\left[u^i_{s-k,k}\left(\frac{x-\mu}{\varepsilon}\right)+v^i_{s-k,k}\left(\frac{1-\mu-x}{\varepsilon}\right)\right]\right\}\varepsilon^{s-k}\mu^k+R^i_1\qquad(\mu\leqslant x\leqslant 1-\mu)\tag{3.1}$$

因此,

$$y^{i\prime\prime}(x,\varepsilon,\mu)=\tilde{Y}^{i\prime\prime}_m+R^i_2=\sum_{s=0}^{m}\sum_{k=0}^{s}\left[y^{i\prime\prime}_{s-k,k}(x)+\frac{d^2u^i_{s-k,k}\left(\frac{x-\mu}{\varepsilon}\right)}{dt^2}+\frac{d^2u^i_{s-k,k}\left(\frac{1-\mu-x}{\varepsilon}\right)}{d\tau^2}\right]\varepsilon^{s-k}\mu^k+R^i_2\qquad(\mu\leqslant x\leqslant 1-\mu)\tag{3.2}$$

这里R^i_1和R^i_2是余项, 满足

$$R^i_j=O\left(\sum_{s=0}^{m+1}\varepsilon^{m+1-s}\mu^s\right)\qquad(\mu\leqslant x\leqslant 1-\mu,\ 0\leqslant\varepsilon,\ \mu\ll 1,\ j=1,\ 2)$$

下面的引理是文[6]中定理5的一种常见的特殊情况(也可看[7]).

引理1 我们考虑一个向量非线性微分方程边值问题:

$$y_1''=f_1(x,y_1,y_2),\qquad y_1(0)=A_1,\qquad y_1(1)=B$$

$$(0<x<1),$$

$$y_2''=f_2(x,y_1,y_2),\qquad y_2(0)=A_2,\qquad y_2(1)=B_2,$$

如果有满足下列条件的函数$\alpha^j_i(x)$, $\beta^j_i(x)\in C^{(2)}[0,1]$ ($j=1,2$; $i=1,2,\cdots,m$)

$$\alpha_j^i(0)\leqslant A_j^i\leqslant\beta_j^i(0),\ \alpha_j^i(1)\leqslant B_j^i\leqslant\beta_j^i(1)\qquad(j=1,2;\ i=1,2,\cdots,n)$$

$$\left.\begin{array}{l}\alpha_1^{i\prime\prime}(x)\geqslant f_1^i(x,y_1^1,\cdots,\alpha_1^i(x),\cdots,y_1^n,y_2^1,\cdots,y_2^n)\\ \beta_1^{i\prime\prime}(x)\leqslant f_1^i(x,y_1^1,\cdots,\beta_1^i(x),\cdots,y_1^n,y_1^1,\cdots,y_2^n)\end{array}\right.\quad\left(\begin{array}{l}0<x<1\\ \alpha_2^i(x)\leqslant y_2^i\leqslant\beta_2^i(x)\end{array}\right)$$

$$\left.\begin{array}{l}\alpha_2^{i\prime\prime}(x)\geqslant f_2^i(x,y_1^1,\cdots,y_1^n,y_2^1,\cdots,\alpha_2^i(x),\cdots,y_2^n)\\ \beta_2^{i\prime\prime}(x)\leqslant f_2^i(x,y_1^1,\cdots,y_1^n,y_2^1,\cdots,\beta_2^i(x),\cdots,y_2^n)\end{array}\right.\quad\left(\begin{array}{l}0<x<1\\ \alpha_1^i(x)\leqslant y_1^i\leqslant\beta_1^i(x)\end{array}\right)$$

和$f_j^i(x,y_1,y_2)\in C^{(1)}(D)$，其中

$$D=\{0\leqslant x\leqslant 1,\alpha_j^i(x)\leqslant y_j^i\leqslant\beta_j^i(x),j=1,2\}$$

则原来的问题可通过满足下列条件

$$\alpha_j^i(x)\leqslant y_j^i(x)\leqslant\beta_j^i(x)\qquad(0\leqslant x\leqslant 1,\ j=1,2)$$

的一对函数$y_1(x)$，$y_2(x)\in C^{(2)}[0,1]$解得。

利用上述引理1，我们能够证明下列定理：

定理1 假设

1. 退化问题(2.2)～(2.3)有一个解$y_{0,0}(x)=(y_{0,0}^1(x),\cdots,y_{0,0}^i(x),\cdots,y_{0,0}^n(x))\in C^{(4)}[0,1]$使得$f^i(x,y_{y_{0,0}^i},y''_{y_{0,0}^{i\prime\prime}},0,0)=0,y_{0,0}^i(0)=A_1^i(0,0),\ y_{0,0}^i(1)=B_1^i(0,0)$；

2. $f^i(,y,y'',\varepsilon,\mu)\in C^{(m+1)}(D_i)$；

3. $f_{y^{i\prime\prime}}^i\geqslant m_i$，$(x,y,y'',\varepsilon,\mu)\in D_i$，其中$m_i(i=1,\cdots,n)$是$n$个正常数；

4. $A_j(\varepsilon,\mu)$，$B_j(\varepsilon,\mu)\in C^{(m+1)}([0,\varepsilon,]\times[0,\mu_1])$ $(j=1,2)$，则边值问题(1.1)～(1.3)有一个解$y^i(x,\varepsilon,\mu)\in C^{(4)}([\mu,1-\mu]\times[0,\varepsilon_0]\times[0,\mu_0])$，它可用一致有效渐近展开式(3.1)来表示和它在$\mu\leqslant x\leqslant 1-\mu$上的二阶导数可用(3.2)表示，其中$\varepsilon_0$，$\mu_0$是两个正的小常数。

证明 设$y^{i\prime\prime}=z^i$，于是原来的问题(1.1)～(1.3)的第i个分量成为下列的边问题值：

$$y^{i\prime\prime}=z^i\tag{3.3}$$

$$y^i(\mu,\varepsilon,\mu)=A_1^i(\varepsilon,\mu),\ y^i(1-\mu,\varepsilon,\mu)=B_1^i(\varepsilon,\mu)\tag{3.4}$$

$$\varepsilon^2z^{i\prime\prime}=f^i(x,y^1,\cdots,y^i,\cdots,y^n,z^1,\cdots,z^i,\cdots,z^n,\varepsilon,\mu)\tag{3.5}$$

$$z^i(\mu,\varepsilon,\mu)=A_2^i(\varepsilon,\mu),z^i(1-\mu,\varepsilon,\mu)=B_2^i(\varepsilon,\mu)\tag{3.6}$$

现在我们构造$\alpha_j^i(x,\varepsilon,\mu)$和$\beta_j^i(x,\varepsilon,\mu)$如下：

$$\alpha_1^i(x,\varepsilon,\mu)=\widetilde{Y}_m^i(x,\varepsilon,\mu)-r_i\left(\sum_{s=0}^{m+1}\varepsilon^{m+1-s}\mu^s\right)\{\sigma_1^i\cos[(l_i/m_i)^{1/2}(x-\mu)]-1+\sigma_2^i\cos[(l_i/m_i)^{1/2}(1-\mu-x)]-1\}\tag{3.7}$$

$$\beta_1^i(x,\varepsilon,\mu)=\widetilde{Y}_m^i(x,\varepsilon,\mu)+r_i\left(\sum_{s=0}^{m+1}\varepsilon^{m+1-s}\mu^s\right)\{\sigma_1^i\cos[l_i/m_i)^{1/2}(x-\mu)]-1+\sigma_2^i\cos[(l_i/m_i)^{1/2}(1-\mu-x)]-1\}\tag{3.8}$$

$$\alpha_2^i(x,\varepsilon,\mu)=\widetilde{Y}_m^{i\prime\prime}(x,\varepsilon,\tau)-(l_ir_i/m_i)\left(\sum_{s=0}^{m+1}\varepsilon^{m+1-s}\mu^s\right)\{\sigma_1^i\cos[(l_i/m_i)^{1/2}(x-\mu)]+\sigma_2^i\cos[(l_i/m_i)^{1/2}(1-\mu-x)]\}\tag{3.9}$$

$$\beta_2^i(x,\varepsilon,\mu)=\widetilde{Y}_m^{i\prime\prime}(x,\varepsilon,\mu)+(l_ir_i/m_i)\left(\sum_{s=0}^{m+1}\varepsilon^{m+1-s}\mu^s\right)\{\sigma_1^i\cos[(l_i/m_i)^{1/2}(x-\mu)]$$

$$+\sigma_2^i\cos[(l_i/m_i)^{1/2}(1-\mu-x)]\}\tag{3.10}$$

其中 r_i 是一个足够大的正常数，将在稍后选定；σ_1^i，σ_2^i 是正的常数，使得

$$\sigma_1^i\cos[(l_i/m_i)^{1/2}(x-\mu)]\geqslant 1$$

和

$$\sigma_2^i\cos[(l_i/m_i)^{1/2}(1-\mu-x)]\geqslant 1\quad(\mu\leqslant x\leqslant 1-\mu)$$

和 $|f_{y'}^i|\leqslant l_i,(x,y,y'',\varepsilon,\mu)\in D_i$，为了不使下面表达式含糊起见，我们先限制正常数 l_i，m_i 使得 $(l_i/m_i)^{1/2}<\pi/2$，然后我们再指出对于一般的 l_i,m_i 的证明。

从(3.7)～(3.8)，(2.24)～(2.30)，易见

$$\alpha_j^i(x,\varepsilon,\mu)\in C^{(2)}[\mu,1-\mu]\subset C^{(2)}[0,1]\qquad(j=1,2)\tag{3.11}$$

$$\beta_j^i(x,\varepsilon,\mu)\in C^{(2)}[\mu,1-\mu]\subset C^{(2)}[0,1]\qquad(j=1,2)\tag{3.12}$$

且从 α_j^i，β_j^i 和 $\widetilde{Y}_m^i(x,\varepsilon,\mu)$ 的结构，对于充分小的正数 ε_1，μ_1 和足够大的 $r_0^i>0$，当 $0<\varepsilon<\varepsilon_1$，$0<\mu<\mu_1$，$r_i>r_0^i$ 时，我们能够得到不等式

$$\alpha_j^i(\mu,\varepsilon,\mu)\leqslant A_j^i(\varepsilon,\mu)\leqslant\beta_j^i(\mu,\varepsilon,\mu)\quad(j=1,2)\tag{3.13}$$

$$\alpha_j^i(1-\mu,\varepsilon,\mu)\leqslant B_j^i(\varepsilon,\mu)\leqslant\beta_j^i(1-\mu,\varepsilon,\mu)\quad(j=1,2)\tag{3.14}$$

$$\alpha_1^{i\prime\prime}\geqslant z^i\qquad(\alpha_2^i(x,\varepsilon,\mu)\leqslant z^i\leqslant\beta_2^i(x,\varepsilon,\mu))\tag{3.15}$$

$$\beta_1^{i\prime\prime}\leqslant z^i\qquad(\alpha_2^i(x,\varepsilon,\mu)\leqslant z^i\leqslant\beta_2^i(x,\varepsilon,\mu))\tag{3.16}$$

因为 $y_{s-k,k}^i(x)$，$u_{s-k,k}^i(t)$ 和 $v_{s-k,k}^i(\tau)$ 满足(2.13)，(2.18)，(2.19)，(2.24)和(2.25)，并考虑到 $y_{m-1}^i(x)$ 和 $y_m^i(x)$ 是有界的，我们容易得到

$$\begin{aligned}
&f^i(x,y_{\widetilde{Y}_m^i},y''_{\widetilde{Y}_m^{i\prime\prime}},\varepsilon,\mu)=f(x,y_{y_{0,0}^i},y''_{y_{0,0}^{i\prime\prime}},0,0)\\
&\quad+\sum_{s=1}^{m}\sum_{k=0}^{s}\Big[f_{y''}^i(x,y_{y_{0,0}^i},y''_{y_{0,0}^{i\prime\prime}},0,0)y_{s-k,k}^{i\prime\prime}+f_{y'}^i(x,y_{y_{0,0}^i},y''_{y_{0,0}^{i\prime\prime}},0,0)y_{s-k,k}^i\\
&\quad+c_{s-k,k}^i(x)-y_{s-k-2,k}^{i(4)}+y_{s-k-2,k}^{i(4)}\Big]\varepsilon^{s-k}\mu^k\\
&\quad+\Big[f_{y''}^i(0,y_{y_{0,0}^i},y''_{y_{0,0}^{i\prime\prime}}+\theta_1u_{0,0,t}^{i\prime\prime},0,0)\frac{d^2u_{0,0}^i}{dt^2}-\frac{d^4u_{0,0}^i}{dt^4}+\frac{d^4u_{0,0}^i}{dt^4}\Big]\\
&\quad+\sum_{s=1}^{m}\sum_{k=0}^{s}\Big[f_{y''}^i(0,y_{y_{0,0}^i},y''_{y_{0,0}^{i\prime\prime}}+u_{0,0,t}^{i\prime\prime},0,0)\frac{d^2u_{s-k,k}^i}{dt^2}+\bar{c}_{s-k,k}^i(t)-\frac{d^4u_{s-k,k}^i}{dt^4}\\
&\quad+\frac{d^4u_{s-k,k}^i}{dt^4}\Big]\varepsilon^{s-k}\mu^k+\Big[f_{y''}^i(1,y_{y_{0,0}^i},y''_{y_{0,0}^i}+u_{0,0,t}^{i\prime\prime}+\theta_2u_{0,0,\tau}^{i\prime\prime},0,0)\frac{d^2v_{0,0}^i}{d\tau^2}\\
&\quad-\frac{d^4v_{0,0}^i}{d\tau^4}+\frac{d^4v_{0,0}^i}{d\tau^4}\Big]+\sum_{s=1}^{m}\sum_{k=0}^{s}\Big[f_{y_{0,0}^{i\prime\prime}}^i(1,y_{y_{0,0}^i},y''_{y_{0,0}^{i\prime\prime}}+u_{0,0,t}^{i\prime\prime}+\theta_2v_{0,0,\tau}^{i\prime\prime},0,0)\frac{d^2v_{s-k,k}^i}{d\tau^2}\\
&\quad+\tilde{c}_{s-k,k}^i(\tau)-\frac{d^4v_{s-k,k}^i}{d\tau^4}+\frac{d^4v_{s-k,k}^i}{d\tau^4}\Big]\varepsilon^{s-k}\mu^k+O\Big(\sum_{s=0}^{m+1}\varepsilon^{m+1-s}\mu^s\Big)\\
&=\varepsilon^2\widetilde{Y}_m^{i(4)}(x,\varepsilon,\mu)+O\Big(\sum_{s=0}^{m+1}\varepsilon^{m+1-s}\mu^s\Big)\qquad(0<\varepsilon,\ \mu\ll 1)
\end{aligned}$$

因此，存在 $\delta_i>0$，$\varepsilon_2>0$，$\mu_2>0$，使得不等式

$$|f^i(x,y_{\tilde{Y}^i_m},y_{\tilde{Y}^{i''}_m},\varepsilon,\mu)-\varepsilon^2\tilde{Y}^{i(4)}_m(x,\varepsilon,\mu)|\leqslant\delta_i\left[\sum_{s=0}^{m+1}\varepsilon^{m+1-s}\mu^s\right] \tag{3.17}$$

对于 $0<\varepsilon\leqslant\varepsilon_2$，$0<\mu\leqslant\mu_2$ 成立。

从中值定理，我们有

$$\begin{aligned}f^i(x,y,y''_{\alpha^i_2},\varepsilon,\mu)&=f^i(x,y_{\tilde{Y}^i_m},y''_{\tilde{Y}^{i''}_m},\varepsilon,\mu)\\&+f^i_{y^i}(x,y_{\tilde{Y}^i_m+\theta_3(y^i-\tilde{Y}^i_m)},y''_{\tilde{Y}^i_m},\varepsilon,\mu)(y^i-\tilde{Y}^i_m)\\&+f^i_{y^{i''}}(x,y,y''_{\tilde{Y}^i_m+\theta_4(\alpha^i_2-\tilde{Y}^{i''}_m)},\varepsilon,\mu)(\alpha^i_2-\tilde{Y}^{i''}_m)\qquad(0<\theta_3,\theta_4<1)\end{aligned} \tag{3.18}$$

$$\begin{aligned}f^i(x,y,y_{\beta^i_2},\varepsilon,\mu)&=f^i(x,y_{\tilde{Y}^i_m},y''_{\tilde{Y}^{i''}_m},\varepsilon,\mu)\\&+f^i_{y^i}(x,y_{\tilde{Y}^i_m+\tilde{\theta}_3(y^i-\tilde{Y}^i_m)},y''_{\tilde{Y}^{i''}_m},\varepsilon,\mu)(y-\tilde{Y}^i_m)\\&+f^i_{y^{i''}}(x,y,y''_{\tilde{Y}^i_m+\tilde{\theta}_4(\beta_2-\tilde{Y}^{i''}_m)},\varepsilon,\mu)(\beta^i_2-\tilde{Y}^{i''})\qquad(0<\tilde{\theta}_3,\tilde{\theta}_4<1)\end{aligned} \tag{3.19}$$

当 $\alpha^i_1(x,\varepsilon,\mu)\leqslant y^i\leqslant\beta^i_1(x,\varepsilon,\mu)$时，从(3.7)，(3.8) 和 $|f^i_{y^i}|\leqslant l_i$，我们能够得到

$$\begin{aligned}|f^i_{y^i}(\cdots)(y^i-\tilde{Y}^i_m)|\leqslant r_il_i\Big(\sum_{s=0}^{m+1}\varepsilon^{m+1-s}\mu^s\Big)\{\sigma^i_1\cos[l_i/m_i)^{1/2}(x-\mu)]-1\\+\sigma^i_2\cos[(l_i/m_i)^{1/2}(1-\mu-x)]-1\}\end{aligned}$$

和从(3.17)～(3.19)，我们有

$$f^i(x,y,y''_{\alpha^i_2},\varepsilon,\mu)\leqslant\varepsilon^2\tilde{Y}^{i(4)}_m+\delta_i\Big(\sum_{s=0}^{m+1}\varepsilon^{m+1-s}\mu^s\Big)-2r_il_i\Big(\sum_{s=0}^{m+1}\varepsilon^{m+1-s}\mu^s\Big)$$

$$f^i(x,y,y''_{\beta^i_2},\varepsilon,\mu)\geqslant\varepsilon^2\tilde{Y}^{i(4)}_m-\delta_i\Big(\sum_{s=0}^{m+1}\varepsilon^{m+1-s}\mu^s\Big)+2r_il_i\Big(\sum_{s=0}^{m+1}\varepsilon^{m+1-s}\mu^s\Big)$$

于是我们得到

$$\varepsilon^2\alpha^{i''}_2-f^i(x,y,y''_{\alpha^i_2},\varepsilon,\mu)\geqslant(2r_il_i-\delta_i)\Big(\sum_{s=0}^{m+1}\varepsilon^{m+1-s}\mu^s\Big),$$

$$\varepsilon^2\beta^{i''}_2-f^i(x,y,y''_{\beta^i_2},\varepsilon,\mu)\leqslant(\delta_i-2r_il_i)\Big(\sum_{s=0}^{m+1}\varepsilon^{m+1-s}\mu^s\Big).$$

取 $\varepsilon_0=\min\{\varepsilon_1,\varepsilon_2,\varepsilon_3\}$，$\mu_0=\min\{\mu_1,\mu_2,\mu_3\}$和 $r_i=\max\{r^i_0,\delta_i/2l_i\}$，我们有

$$\varepsilon^2\alpha^{i''}_2\geqslant f^i(x,y,y''_{\alpha^i_2},\varepsilon,\mu) \tag{3.20}$$

$$\varepsilon^2\beta^{i''}_2\leqslant f^i(x,y,y''_{\beta^i_2},\varepsilon,\mu) \tag{3.21}$$

$$(\alpha^i_1(x,\varepsilon,\mu)\leqslant y^i\leqslant\beta^i_1(x,\varepsilon,\mu),\mu<x<1-\mu,0<\varepsilon\leqslant\varepsilon_0,0<\mu\leqslant\mu_0).$$

从关系式(3.11)～(3.16)，(3.20)和(3.21)，利用引理1，对于边值问题(3.3)～(3.6)，我们得到一对函数 $y^i(x,\varepsilon,\mu)$，$z^i(x,\varepsilon,\mu)\in C^{(2)}(\mu\leqslant x\leqslant1-\mu)$，它满足不等式

$$\alpha_1^i(x,\varepsilon,\mu)\leqslant y^i(x,\varepsilon,\mu)\leqslant\beta_1^i(x,\varepsilon,\mu)\quad(\mu\leqslant x\leqslant 1-\mu,\ 0<\varepsilon\leqslant\varepsilon_0,\ 0<\mu\leqslant\mu_0)$$

$$\alpha_2^i(x,\varepsilon,\mu)\leqslant z^i(x,\varepsilon,\mu)\leqslant\beta_2^i(x,\varepsilon,\mu)\quad(\mu\leqslant x\leqslant 1-\mu,0<\varepsilon\leqslant\varepsilon_0,0<\mu\leqslant\mu_0)$$

从(3.7)～(3.10)，我们得到

$$y^i(x,\varepsilon,\mu)=\sum_{s=0}^{m}\sum_{k=0}^{s}\Big[y_{s-k,k}^i(x)+\varepsilon^2 u_{s-k,k}^i\left(\frac{x-\mu}{\varepsilon}\right)$$

$$+\varepsilon^2 v_{s-k,k}^i\left(\frac{1-\mu-x}{\varepsilon}\right)\Big]\varepsilon^{s-k}\mu^k+O\Big(\sum_{s=0}^{m+1}\varepsilon^{m+1-s}\mu^s\Big),$$

$$z^i(x,\varepsilon,\mu)=\sum_{s=0}^{m}\sum_{k=0}^{s}\left[y_{s-k,k}^{i''}(x)+\frac{d^2u_{s-k,k}^i\left(\frac{x-\mu}{\varepsilon}\right)}{dt^2}+\frac{d^2v_{s-k,k}^i\left(\frac{1-\mu-x}{\varepsilon}\right)}{d\tau^2}\right]\varepsilon^{s-k}\mu^k$$

$$+O\Big(\sum_{s=0}^{m+1}\varepsilon^{m+1-s}\mu^s\Big)\qquad(\mu\leqslant x\leqslant 1-\mu,0<\varepsilon\ll 1,0<\mu\ll 1)$$

因为 $z^i(x,\varepsilon,\mu)=y^{i''}(x,\varepsilon,\mu)$，于是我们有 $y^i\in C^{(4)}([\mu,1-\mu]\times[0,\varepsilon_0]\times[0,\mu_0])$ 和关系式(3.1)，(3.2)在 $\mu\leqslant x\leqslant 1-\mu$ 中一致成立。最后，我们指出，在 $(l_i/m_i)^{1/2}\geqslant\pi/2$ 的情形，实质上可用同样的方法继续做下去，只不过这时是用 $\sin[(l_i/m_i)^{1/2}(x-\mu)]$，$\cos[(l_i\text{-}m_i)^{1/2}(x-\mu)]$ 和 $\sin[(l_i/m_i)^{1/2}(1-\mu-x)]$，$\cos[l_i/m_i)^{1/2}(1-\mu-x)]$ 的线性组合分别替代 $\sigma_1^i\cos[(l_i/m_i)^{1/2}(x-\mu)]$ 和 $\sigma_2^i\cos[(l_i/m_i)^{1/2}(1-\mu-x)]$ 而已，其计算是直接的，但是乏味的，我们略去(可参看文[1])。

参 考 文 献

[1] Howes, F. A., Differential inequalities and applications to nonlinear singular perturbation problem, *J. Diff. Eqs.*, **20** (1976), 133—149.

[2] Chang, K. W. and F. A. Howes, *Nonlinear Singular Perturbation Phenomena: Theory and Applications*, Springer-Verlag, New York, Berlin, Heidelberg, Tokyo (1984).

[3] O'Malley, R. E., *Introduction to Singular Perturbations*, Academic Press (1974).

[4] 林宗池、郑永树，高阶常微分方程边值问题的奇摄动(Ⅰ)，福建师大学报(自然科学版)，2 (1980), 13—28.

[5] 刘光旭，关于奇摄动拟线性系统，应用数学和力学，8, 11 (1987), 967—976.

[6] Kelley, W. G., A geometric method of studying two point boundary value problems for second order systems, *Rocky Moun. J. of Math.*, 7 (1977), 251—263.

[7] Kelley, W. G., Boundary value problems for pairs of second order equations containing a small parameter, *Rocky Moun. J. Math.* 12, 4 (1982), 655—667.

Singular Perturbation of Boundary Value Problem for a Vector Fourth Order Nonlinear Differential Equation

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Abstract

We study the vector boundary value problem with boundary perturbations:

$$\varepsilon^2 y^{(4)}=f(x,y,y'',\varepsilon,\mu) \qquad (\mu<x<1-\mu)$$

$$y(x,\varepsilon,\mu)|_{x=\mu}=A_1(\varepsilon,\mu),\quad y(x,\varepsilon,\mu)|_{x=1-\mu}=B_1(\varepsilon,\mu);$$

$$y''(x,\varepsilon,\mu)|_{x=\mu}=A_2(\varepsilon,\mu),\quad y''(x,\varepsilon,\mu)|_{x=1-\mu}=B_2(\varepsilon,\mu)$$

where y, f, A_j and $B_j(j=1,2)$ are n-dimensional vector functions and ε,μ are two small positive parameters. This vector boundary value problem does not appear to have been studied, although the scalar boundary value problem has been treated. Under appropriate assumptions, using the method of differential inequalities we find a solution of the vector boundary value problem and obtain the uniformly valid asymptotic expansions.