应用数学和力学，第7卷第9期（1986年9月）
Applied Mathematics and Mechanics

应用数学和力学编委会编
重庆出版社出版

层状二维流动的基本方程式

蔡 树 棠

（中国科技大学，上海市应用数学和力学研究所）

吴 烽

（中国科技大学近代力学系）

（1985年12月8日收到）

摘 要

在很多海洋、大气等二维流动问题中所用的动力学方程往往沿用推广后的河流水力学方程

$$\frac{\partial}{\partial t}U_\alpha+U_\beta\frac{\partial U_\alpha}{\partial x_\beta}=-g\frac{\partial h}{\partial x_\alpha}+g\left(i_\alpha-\frac{|\mathbf{U}|U_\alpha}{c^2R}\right)+F_\alpha$$

或"纳维-斯托克斯方程"

$$\frac{\partial}{\partial t}U_\alpha+U_\beta\frac{\partial U_\alpha}{\partial x_\beta}=-g\frac{\partial h}{\partial x_\alpha}+gi_\alpha+F_\alpha+\frac{\partial}{\partial x_\beta}\left(\nu_T\frac{\partial U_\alpha}{\partial x_\beta}\right)$$

其中把湍流阻力项写成

$$-g\frac{|\mathbf{U}|U_\alpha}{c^2R}\quad 或\quad \frac{\partial}{\partial x_\beta}\left(\nu_T\frac{\partial U_\alpha}{\partial x_\beta}\right)$$

这样的方程式和湍流阻力项用到实际问题上去，无疑是存在着极大的局限性，而将导致矛盾百出．本文则从雷诺方程出发，把所有的物理量沿深度加以平均，求出平均以后的物理量所满足的运动方程、连续方程和扩散方程．

一、引 言

河口、海洋或大气的流动通常并不能作为一维问题来处理。由于深度比起宽度来小得多，很多情况又并不需要了解物理量沿深度的分布，而只要知道它整个深度的平均值。这样，我们就可以把这类问题看做层状的二维流动问题。关于描述这类运动的运动方程式，通常沿用水力学中的动力学方程式，并且把它推广到二维，也就是下列方程式

$$\frac{\partial U_\alpha}{\partial t}+U_\beta\frac{\partial U_\alpha}{\partial x_\beta}=-g\frac{\partial h}{\partial x_\alpha}+gi_\alpha-g\frac{|\mathbf{U}|U_\alpha}{c^2R}+F_\alpha \tag{1.1}$$

式中U_α为速度，$\alpha=1,2$，h为水深度，i_α为坡降，g为重力加速度，c为谢齐系数，R为水力半径。F_α为科氏力。有时也采用所谓的纳维-斯托克斯方程，即

$$\frac{\partial U_\alpha}{\partial t}+U_\beta\frac{\partial U_\alpha}{\partial x_\beta}=-g\frac{\partial h}{\partial x_\alpha}+gi_\alpha+F_\alpha+\frac{\partial}{\partial x_\beta}\left(\nu_T\frac{\partial U_\alpha}{\partial x_\beta}\right) \tag{1.2}$$

式中ν_T为湍流运动粘性系数。阻力项写成

$$-g\frac{|\tilde{\mathbf{U}}|\tilde{U}_\alpha}{c^2R} \quad 或 \quad \frac{\partial}{\partial x_\beta}\left(\nu_T\frac{\partial\tilde{U}_\alpha}{\partial x_\beta}\right)$$

一般说来是并不正确的。为了正确地得到层状流动的基本方程组，本文从雷诺方程出发，通过对沿深度平均着得到正确的方程式，从而讨论上述两种表达式的适用范围。

二、沿深度平均值和它们的偏导数

设某一个物理量A，它沿深度的平均值为

$$\tilde{A}=\frac{1}{h}\int_b^{b+h}Adz \tag{2.1}$$

令

$$A=\tilde{A}+\Delta A \tag{2.2}$$

那么

$$\widetilde{AB}=\tilde{A}\tilde{B}+\widetilde{\Delta A\Delta B} \tag{2.3}$$

现在取(2.1)式对t的偏导数

$$\frac{\partial\tilde{A}}{\partial t}=-\frac{1}{h^2}\frac{\partial h}{\partial t}\int_b^{b+h}Adz+\frac{1}{h}\left[A^{b+h}\left(\frac{\partial b}{\partial t}+\frac{\partial h}{\partial t}\right)-A^b\frac{\partial b}{\partial t}\right]$$
$$+\frac{1}{h}\int_b^{b+h}\frac{\partial A}{\partial t}dz$$

移项

$$\left(\widetilde{\frac{\partial A}{\partial t}}\right)=\frac{\partial\tilde{A}}{\partial t}+\frac{1}{h}\frac{\partial h}{\partial t}\tilde{A}-\frac{1}{h}\left[A^{b+h}\left(\frac{\partial b}{\partial t}+\frac{\partial h}{\partial t}\right)-A^b\frac{\partial b}{\partial t}\right] \tag{2.4}$$

或

$$h\left(\widetilde{\frac{\partial A}{\partial t}}\right)=\frac{\partial}{\partial t}(h\tilde{A})-\left[A^{b+h}\left(\frac{\partial b}{\partial t}+\frac{\partial h}{\partial t}\right)-A^b\frac{\partial b}{\partial t}\right] \tag{2.4$'$}$$

式中A^b代表物理量A在高程b处取值。

把(2.1)对x_α微分，就得到

$$\frac{\partial\tilde{A}}{\partial x_\alpha}=-\frac{1}{h^2}\frac{\partial h}{\partial x_\alpha}\int_b^{b+h}Adz+\frac{1}{h}\left[A^{b+h}\left(\frac{\partial b}{\partial x_\alpha}+\frac{\partial h}{\partial x_\alpha}\right)\right.$$
$$\left.-A^b\frac{\partial b}{\partial x_\alpha}\right]+\frac{1}{h}\int_b^{b+h}\frac{\partial A}{\partial x_\alpha}dz$$

移项

$$\left(\widetilde{\frac{\partial A}{\partial x_\alpha}}\right)=\frac{\partial\tilde{A}}{\partial x_\alpha}+\frac{1}{h}\frac{\partial h}{\partial x_\alpha}\tilde{A}-\frac{1}{h}\left[A^{b+h}\left(\frac{\partial b}{\partial x_\alpha}+\frac{\partial h}{\partial x_\alpha}\right)-A^b\frac{\partial b}{\partial x_\alpha}\right] \quad (\alpha=1,2) \tag{2.5}$$

或

$$h\left(\widetilde{\frac{\partial A}{\partial x_\alpha}}\right)=\frac{\partial}{\partial x_\alpha}(h\tilde{A})-\left[A^{b+h}\left(\frac{\partial b}{\partial x_\alpha}+\frac{\partial h}{\partial x_\alpha}\right)-A^b\frac{\partial b}{\partial x_\alpha}\right] \quad (\alpha=1,2) \tag{2.5$'$}$$

另外还有下列公式

$$\left[\widetilde{\frac{\partial(AB)}{\partial x_\alpha}}\right]=\frac{\partial}{\partial x_\alpha}(\widetilde{AB})+\frac{1}{h}\frac{\partial h}{\partial x_\alpha}(\widetilde{AB})-\frac{1}{h}\left[(AB)^{b+h}\left(\frac{\partial b}{\partial x_\alpha}+\frac{\partial h}{\partial x_\alpha}\right)-(AB)^b\frac{\partial b}{\partial x_\alpha}\right]$$

$$= \frac{\partial}{\partial x_\alpha}(\tilde{A}\tilde{B}) + \frac{1}{h}\frac{\partial h}{\partial x_\alpha}(\tilde{A}\tilde{B}) + \frac{\partial}{\partial x_\alpha}(\widetilde{\Delta A \Delta B}) + \frac{1}{h}\frac{\partial h}{\partial x_\alpha}\widetilde{\Delta A \Delta B}$$

$$-\frac{1}{h}\left[(AB)^{b+h}\left(\frac{\partial b}{\partial x_\alpha} + \frac{\partial h}{\partial x_\alpha}\right) - (AB)^b \frac{\partial b}{\partial x_\alpha}\right] \qquad (2.6)$$

$$h\left[\widetilde{\frac{\partial(AB)}{\partial x_\alpha}}\right] = \frac{\partial}{\partial x_\alpha}(h\tilde{A}\tilde{B}) + \frac{\partial}{\partial x_\alpha}(h\widetilde{\Delta A \Delta B}) - \left[(AB)^{b+h}\left(\frac{\partial b}{\partial x_\alpha} + \frac{\partial h}{\partial x_\alpha}\right)\right.$$

$$\left. - (AB)^b \frac{\partial b}{\partial x_\alpha}\right) \qquad (2.6)'$$

$$\left(\widetilde{\frac{\partial^2 A}{\partial x_\alpha \partial x_\alpha}}\right) = \frac{1}{h}\frac{\partial^2}{\partial x_\alpha \partial x_\alpha}(h\tilde{A}) - \frac{2}{h}\left[\left(\frac{\partial A}{\partial x_\alpha}\right)^{b+h}\frac{\partial}{\partial x_\alpha}(b+h) - \left(\frac{\partial A}{\partial x_\alpha}\right)^b \frac{\partial b}{\partial x_\alpha}\right]$$

$$-\frac{1}{h}\left[\left(\frac{\partial A}{\partial z}\right)^{b+h}\left(\frac{\partial b}{\partial x_\alpha} + \frac{\partial h}{\partial x_\alpha}\right)^2 - \left(\frac{\partial A}{\partial z}\right)^b \left(\frac{\partial b}{\partial x_\alpha}\right)^2\right]$$

$$-\frac{1}{h}\left[A^{b+h}\frac{\partial^2(b+h)}{\partial x_\alpha \partial x_\alpha} - A^b \frac{\partial^2 b}{\partial x_\alpha \partial x_\alpha}\right] \qquad (2.7)$$

$$\left(\widetilde{\frac{\partial A}{\partial z}}\right) = \frac{1}{h}[A^{b+h} - A^b] \qquad (2.8)$$

或

$$h\left(\widetilde{\frac{\partial A}{\partial z}}\right) = A^{b+h} - A^b \qquad (2.8)'$$

三、表面和底部的条件

如果某一点$P(x,y,z)$在曲面$F(x,y,z,t)=0$上面。经过时间δt移动到$P'(x+\delta x, y+\delta y, z+\delta z)$，这时仍旧在曲面$F=0$上面。于是我们有

$$\delta F = \frac{\partial F}{\partial x_\alpha}\delta x_\alpha + \frac{\partial F}{\partial z}\delta z + \frac{\partial F}{\partial t}\delta t = 0$$

也就有

$$U_\alpha \frac{\partial F}{\partial x_\alpha} + U_z \frac{\partial F}{\partial z} + \frac{\partial F}{\partial t} = 0 \qquad (3.1)$$

现在设流场上表面的方程式为

$$z - f(x,y,t) = 0 \qquad (3.2)$$

式中

$$f(x,y,t) = b + h \qquad (3.3)$$

b就是底部高程，h为水深。于是在表面上有

$$U_z^f - U_\alpha^f \frac{\partial f}{\partial x_\alpha} - \frac{\partial f}{\partial t} = 0 \qquad (3.4)$$

或

$$U_z^{b+h} - U_\alpha^{b+h}\frac{\partial}{\partial x_\alpha}(b+h) - \frac{\partial}{\partial t}(b+h) = 0 \qquad (3.4)'$$

同样在底部为

$$z - b(x,y,t) = 0 \qquad (3.5)$$

这时满足的条件为

$$U_z^b - U_\alpha^b \frac{\partial b}{\partial x_\alpha} - \frac{\partial b}{\partial t} = 0 \qquad (3.6)$$

四、层状二维流动的基本方程式

(A) 连续方程

在没有平均以前的连续方程式为

$$\frac{\partial U_j}{\partial x_j}=\frac{\partial U_\alpha}{\partial x_\alpha}+\frac{\partial U_z}{\partial x_z}=0 \qquad (j=1,2,3;\alpha=1,2) \tag{4.1}$$

速度分量U_z就是U_3。把(4.1)式取平均，利用上面第二节推导出来的公式，就得到

$$\frac{\partial}{\partial x_\alpha}(h\tilde{U}_\alpha)-\left[U_\alpha^{b+h}\left(\frac{\partial b}{\partial x_\alpha}+\frac{\partial h}{\partial x_\alpha}\right)-U_\alpha^b\frac{\partial b}{\partial x_\alpha}\right]+U_z^{b+h}-U_z^b=0$$

再利用表面和底部条件，把$U_z^{b+h}-U_z^b$代入，就可得到

$$\frac{\partial}{\partial x_\alpha}(h\tilde{U}_\alpha)+\frac{\partial h}{\partial t}=0 \tag{4.2}$$

(B) 运动方程式

在没有平均以前，x和y方向的方程式为

$$\frac{\partial U_\alpha}{\partial t}+\frac{\partial}{\partial x_\beta}(U_\alpha U_\beta)+\frac{\partial}{\partial z}(U_\alpha U_z)=-\frac{1}{\rho}\frac{\partial p}{\partial x_\alpha}-\frac{\partial}{\partial x_\beta}\overline{u_\alpha u_\beta}$$
$$-\frac{\partial}{\partial z}\overline{u_\alpha u_z}+\nu\left(\frac{\partial^2 U_\alpha}{\partial x_\beta\partial x_\beta}+\frac{\partial^2 U_\alpha}{\partial z^2}\right)+F_\alpha-g_\alpha \tag{4.3}$$

其中$\alpha=1,2$，$\beta=1,2$。

对(4.3)式取平均后，得到方程式

$$h\left(\frac{\partial\tilde{U}_\alpha}{\partial t}+\tilde{U}_\beta\frac{\partial\tilde{U}_\alpha}{\partial x_\beta}\right)=-\frac{\partial}{\partial x_\beta}(h\widetilde{\Delta U_\alpha\Delta U_\beta})-\frac{1}{\rho}\frac{\partial}{\partial x_\alpha}(h\tilde{P})$$
$$+\frac{1}{\rho}\left[P^{b+h}\frac{\partial}{\partial x_\alpha}(b+h)-P^b\frac{\partial b}{\partial x_\alpha}\right]$$
$$-\frac{\partial}{\partial x_\beta}(h\widetilde{u_\alpha u_\beta})+\left[\overline{u_\alpha u_\beta}^{b+h}\frac{\partial}{\partial x_\beta}(b+h)-\overline{u_\alpha u_\beta}^b\frac{\partial b}{\partial x_\beta}\right]$$
$$-(\overline{u_\alpha u_z}^{b+h}-\overline{u_\alpha u_z}^b)+\nu\frac{\partial^2}{\partial x_\beta\partial x_\beta}(h\tilde{U}_\alpha)+\nu\left[\left(\frac{\partial U_\alpha}{\partial z}\right)^{b+h}-\left(\frac{\partial U_\alpha}{\partial z}\right)^b\right]$$
$$+h\tilde{F}_\alpha-hg_\alpha-2\nu\left[\left(\frac{\partial U_\alpha}{\partial x_\beta}\right)^{b+h}\frac{\partial}{\partial x_\beta}(b+h)-\left(\frac{\partial U_\alpha}{\partial x_\beta}\right)^b\frac{\partial b}{\partial x_\beta}\right]$$
$$-\nu\left[\left(\frac{\partial U_\alpha}{\partial z}\right)^{b+h}\left(\frac{\partial(b+h)}{\partial x_\beta}\right)^2-\left(\frac{\partial U_\alpha}{\partial z}\right)^b\left(\frac{\partial b}{\partial x_\beta}\right)^2\right]$$
$$-\nu\left[U_\alpha^{b+h}\frac{\partial^2(b+h)}{\partial x_\beta\partial x_\beta}-U_\alpha^b\frac{\partial^2 b}{\partial x_\beta\partial x_\beta}\right] \tag{4.4}$$

没有平均以前z方向的运动方程式为

$$\frac{\partial U_z}{\partial t}+\frac{\partial}{\partial x_\beta}(U_zU_\beta)+\frac{\partial}{\partial z}(U_z^2)=-\frac{1}{\rho}\frac{\partial P}{\partial z}-\frac{\partial}{\partial x_\beta}(\overline{u_zu_\beta})$$
$$-\frac{\partial}{\partial z}\overline{u_z^2}+\nu\left(\frac{\partial^2 U_z}{\partial x_\beta\partial x_\beta}+\frac{\partial^2 U_z}{\partial z^2}\right)-g_z+F_z \tag{4.5}$$

平均以后的方程式为

$$h\left(\frac{\partial\tilde{U}_z}{\partial t}+\tilde{U}_\beta\frac{\partial\tilde{U}_z}{\partial x_\beta}\right)=-\frac{1}{\rho}(P^{b+h}-P^b)-\frac{\partial}{\partial x_\beta}(\widetilde{\Delta U_z\Delta U_\beta h})$$

$$-\frac{\partial}{\partial x_\beta}(\widetilde{u_z u_\beta h})+\left[\overline{u_z u_\beta}^{b+h}\frac{\partial}{\partial x_\beta}(b+h)-\overline{u_z u_\beta}^b\frac{\partial b}{\partial x_\beta}\right]$$

$$-(\overline{u_z^2}^{b+h}-\overline{u_z^2}^b)+\nu\frac{\partial^2(h\tilde{U}_z)}{\partial x_\beta\partial x_\beta}-2\nu\left[\left(\frac{\partial U_z}{\partial x_\beta}\right)^{b+h}\frac{\partial}{\partial x_\beta}(b+h)\right.$$

$$\left.-\left(\frac{\partial U_z}{\partial x_\beta}\right)^b\frac{\partial b}{\partial x_\beta}\right]-\nu\left[U_z^{b+h}\frac{\partial^2(b+h)}{\partial x_\beta\partial x_\beta}-U_z^b\frac{\partial^2 b}{\partial x_\beta\partial x_\beta}\right]$$

$$-\nu\left[\left(\frac{\partial U_z}{\partial z}\right)^{b+h}\left(\frac{\partial b}{\partial x_\beta}+\frac{\partial h}{\partial x_\beta}\right)^2-\left(\frac{\partial U_z}{\partial z}\right)^b\left(\frac{\partial b}{\partial x_\beta}\right)^2\right]$$

$$+\nu\left[\left(\frac{\partial U_z}{\partial z}\right)^{b+h}-\left(\frac{\partial U_z}{\partial z}\right)^b\right]-g_z h+h\tilde{F}_a \tag{4.6}$$

上两式中

$$\tilde{F}_x=-2(\omega_y\tilde{U}_z-\omega_z\tilde{U}_y),\quad \tilde{F}_y=-2(\omega_z\tilde{U}_x-\omega_x\tilde{U}_z),\quad \tilde{F}_z=-2(\omega_x\tilde{U}_y-\omega_y\tilde{U}_x)$$

(C） 扩散方程

如果某一物质的浓度为C。它在没有平均以前的方程式为

$$\frac{\partial C}{\partial t}+\frac{\partial}{\partial x_a}(U_a C)+\frac{\partial}{\partial z}(U_z C)=D\left(\frac{\partial^2 C}{\partial x_a\partial x_a}+\frac{\partial^2 C}{\partial z^2}\right)$$

$$-\frac{\partial}{\partial x_a}(\overline{u_a C'})-\frac{\partial}{\partial z}(\overline{u_z C'})+f_C \tag{4.7}$$

式中D为扩散系数，f_C为源项。

将(4.7)式对深度求平均后得到

$$h\left(\frac{\partial\tilde{C}}{\partial t}+\tilde{U}_a\frac{\partial\tilde{C}}{\partial x_a}\right)=-\frac{\partial}{\partial x_a}(h\widetilde{\Delta C\Delta U_a})+D\frac{\partial^2(h\tilde{C})}{\partial x_a\partial x_a}$$

$$-2D\left[\left(\frac{\partial C}{\partial x_\beta}\right)^{b+h}\frac{\partial}{\partial x_\beta}(b+h)-\left(\frac{\partial C}{\partial x_\beta}\right)^b\frac{\partial b}{\partial x_\beta}\right]$$

$$-D\left[\left(\frac{\partial C}{\partial z}\right)^{b+h}\left(\frac{\partial b}{\partial x_\beta}+\frac{\partial h}{\partial x_\beta}\right)^2-\left(\frac{\partial C}{\partial z}\right)^b\left(\frac{\partial b}{\partial x_\beta}\right)^2\right]$$

$$-D\left[C^{b+h}\frac{\partial^2(b+h)}{\partial x_\beta\partial x_\beta}-C^b\frac{\partial^2 b}{\partial x_\beta\partial x_\beta}\right]+D\left[\left(\frac{\partial C}{\partial z}\right)^{b+h}-\left(\frac{\partial C}{\partial z}\right)^b\right]$$

$$-\frac{\partial}{\partial x_a}(\widetilde{u_a C' h})+\left[\overline{u_a C'}^{b+h}\frac{\partial}{\partial x_a}(b+h)-\overline{u_a C'}^b\frac{\partial b}{\partial x_a}\right]$$

$$-[\overline{u_z C'}^{b+h}-\overline{u_z C'}^b]+\tilde{f}_C \tag{4.8}$$

五、结论和讨论

如果在方程(4.6)中略去所有的速度项，并且认为P^{b+h}为常数大气压P_0，(4.6)式就变成

$$P^b=P_0+\rho g h \tag{5.1}$$

这样(4.4)式中的

$$-\frac{1}{\rho}\frac{\partial}{\partial x_\alpha}(h\widetilde{P})+\frac{1}{\rho}\left[P^{b+h}\frac{\partial}{\partial x_\alpha}(b+h)-P^{b}\frac{\partial b}{\partial x_\alpha}\right]$$

$$=-\frac{1}{\rho}\frac{\partial}{\partial x_\alpha}\left(P_0h+\frac{1}{2}\rho gh^2\right)+\frac{1}{\rho}\left[P_0\frac{\partial}{\partial x_\alpha}(b+h)-(P_0+\rho gh)\frac{\partial b}{\partial x_\alpha}\right]$$

$$=-gh\frac{\partial h}{\partial x_\alpha}-gh\frac{\partial b}{\partial x_\alpha}$$

$$=h\left(-g\frac{\partial h}{\partial x_\alpha}-g\frac{\partial b}{\partial x_\alpha}\right) \tag{5.2}$$

这就是一般水力学运动方程式中的“压力项”和坡降项。

(4.4)式中

$$-\frac{\partial}{\partial x_\beta}(\widetilde{hu_\alpha u_\beta})=-h\left[\frac{\partial}{\partial x_\beta}(\widetilde{u_\alpha u_\beta})+\frac{1}{h}\frac{\partial h}{\partial x_\beta}(\widetilde{u_\alpha u_\beta})\right] \tag{5.3}$$

(5.3)括弧中的两项相应于阻力项$-\frac{g|\tilde{\mathbf{U}}|\tilde{U}_\alpha}{c^2R}$或$\frac{\partial}{\partial x_\beta}\left(\nu_T\frac{\partial \tilde{U}_\alpha}{\partial x_\beta}\right)$。我们没有任何理由可以说明括弧中的阻力项必须和$\tilde{U}_\alpha$的方向一致或和$\frac{\partial}{\partial x_\beta}\left(\nu_T\frac{\partial \tilde{U}_\alpha}{\partial x_\beta}\right)$的方向一致。而且(4.4)式等式右边各项中除带有粘性系数ν的各项可能因粘性系数很小而较小以外，其它各项未必很小。特别存在滩地等复杂地形的情形，$\partial b/\partial x_\alpha$不仅不小，甚至可以是$\delta$函数。这时把有些项略去将会造成很大的误差。

The Fundamental Equations of Two-Dimensional Layer Flows

Tsai Shu-tang

(University of Science and Technology of China, Hefei; Shanghai Institute of Applied Mathematics and Mechanics, Shanghai)

Wu Feng

(University of Science and Technology of China, Hefei)

Abstract

In many studies on two-dimensional flows in field of atmosphere and ocean the equations which are extension of river-hydraulic equations

$$\frac{\partial}{\partial t}U_\alpha+U_\beta\frac{\partial U_\alpha}{\partial x_\beta}=-g\frac{\partial h}{\partial x_\alpha}+g\left(i_\alpha-\frac{|\mathbf{U}|U_\alpha}{c^2R}\right)+F_\alpha$$

or Navier-Stokes equations

$$\frac{\partial}{\partial t}U_\alpha+U_\beta\frac{\partial U_\alpha}{\partial x_\beta}=-g\frac{\partial h}{\partial x_\alpha}+gi_\alpha+F_\alpha+\frac{\partial}{\partial x_\beta}\left(\nu_T\frac{\partial U_\alpha}{\partial x_\beta}\right)$$

are usually used, In these equations $-\frac{|\mathbf{U}|U_\alpha}{c^2R}$ or $\frac{\partial}{\partial x_\beta}\left(\nu_T\frac{\partial U_\alpha}{\partial x_\beta}\right)$ stands for turbulent resistance. Obviously the use of these equations in practice may lead to contradiction. In this paper the average of Reynolds equations over depth is taken. The motion equations, continuity equation and diffusion equation are obtained for the average physical variables.