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时间导数项含小参数的抛物方程的数值解法

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摘　要

本文讨论时间导数项含小参数的抛物方程．我们依Бахволов 构造非均匀网格的差分格式，并证明了格式的一阶一致收敛性．给出了数值结果．

关键词　差分格式　一致收敛　抛物型方程　非均匀网格

一、引　言

本文讨论时间变量上含小参数ε的抛物方程

$$Lu\equiv a(x,t)\partial^2u/\partial x^2+b(x,t)\partial u/\partial x-c(x,t)u-\varepsilon\partial u/\partial t$$

$$=f(x,t,\varepsilon),\qquad (x,t)\in D \tag{1.1}$$

$$u(x,0)=\varphi(x),\ u(0,t)=\psi_0(t),\ u(1,t)=\psi_1(t) \tag{1.2}$$

其中$\bar{D}=\{0<x<1,\ 0<t\leqslant T\}$，$a$，$b$，$c$，$f$，$\varphi$，$\psi_0$及$\psi_1$充分光滑，并且在$\bar{D}$上

$$a(x,t)\geqslant\tilde{a}>0,\ c(x,t)\geqslant\tilde{c}>0 \tag{1.3}$$

Титов[1]曾对(1.1)～(1.2)构造了指数拟合差分格式，但只对$t\geqslant M\delta(0<\delta<1)$得到了一致收敛性，Hsiao，Jordan[2]曾对

$$\varepsilon\frac{\partial u}{\partial t}-\sum_{i,j=1}^{n}\frac{\partial}{\partial x_i}\left[a_{ij}(x)\frac{\partial u}{\partial x_j}\right]+c(x)u=f(x,t)$$

构造了修正的Crank-Nicolson-Galerkin格式，但也只对$t\geqslant M\delta(0<\delta<1)$得到了一致收敛性，这实质上都忽视了边界层$t=0$．本文，我们用Бахволов[3]对时间采用非均匀步长构造差分格式，并证明对一切$t\in[0,T]$关于ε一致收敛．

二、解的导数估计

下面我们假设a和b不依赖于t，且$\psi_0(t)=\psi_1(t)=0$．

定理1　设$u(x,t)$是(1.1)～(1.3)的解，若$Lu\leqslant0$，在边界Γ上$u(x,t)\geqslant0$，则在$\bar{D}$上$u(x,t)\geqslant0$．

证明　由Ильин[4]易得证．

引理1 设$u(x,t)$为(1.1)～(1.3)的解，若在$\bar{D}$上$|f(x,t,\varepsilon)|\leqslant N$，在$\Gamma$上$|u(x,t)|\leqslant m$，则$|u(x,t)|\leqslant\max\{N/\bar{c},m\}$，其中$N$和$m$为不依赖于$\varepsilon$的任意正常数。

证明 设$w(x,t)=\max\{N/\bar{c},\ m\}\pm u(x,t)$，则

$$Lw=-c\cdot\max\{N/\bar{c},\ m\}\pm Lu(x,t)\leqslant -N\pm Lu(x,t)\leqslant 0$$

且在Γ上，$w(x,t)\geqslant m\pm u(x,t)\geqslant 0$，故由定理1，知$|u(x,t)|\leqslant\max\{N/\bar{c},\ m\}$。

定理2 设$u(x,t)$为(1.1)～(1.3)的解，则

$$\left|\frac{\partial u(x,t)}{\partial t}\right|\leqslant M\left\{1+\varepsilon^{-1}\exp\left[-\frac{\bar{c}t}{\varepsilon}\right]\right\},\qquad 若(x,t)\in\bar{D}$$

$$\left|\frac{\partial^2 u(x,t)}{\partial t^2}\right|\leqslant M\left\{1+\varepsilon^{-2}\exp\left[-\frac{\bar{c}t}{2\varepsilon}\right]\right\},\qquad 若(x,t)\in\bar{D}$$

$$\left|\frac{\partial^k u(x,t)}{\partial x^k}\right|\leqslant M,\qquad k=1,2,3,4,\qquad 若(x,t)\in\bar{D}$$

$$\left|\frac{\partial^{k+1} u(x,t)}{\partial x^k\partial t}\right|\leqslant \frac{M}{\varepsilon},\qquad k=1,2,\qquad 若(x,t)\in\bar{D}$$

证明 首先，我们估计$\partial u/\partial t$。设$w_1(x,t)=M\{1+\varepsilon^{-1}\exp[-\bar{c}t/\varepsilon]\}\pm\partial u/\partial t$，在$t=0$上，由(1.1)～(1.2)知$|\partial u/\partial t|\leqslant M_1/\varepsilon$，则取$M$充分大，$w_1(x,t)>0$。在$x=0$和$x=1$上有$u\equiv 0$，则$\partial u/\partial t=0$，从而$w_1(x,t)>M>0$。而

$$Lw_1=M\left\{-c\cdot\left[1+\varepsilon^{-1}\exp\left[-\frac{\bar{c}t}{\varepsilon}\right]\right]-\varepsilon\cdot\varepsilon^{-1}\cdot\left(-\frac{\bar{c}}{\varepsilon}\right)\cdot\exp\left[-\frac{\bar{c}t}{\varepsilon}\right]\right\}\pm L\left(\frac{\partial u}{\partial t}\right)$$

$$=M\left\{-c-(c-\bar{c})\varepsilon^{-1}\exp\left[-\frac{\bar{c}t}{\varepsilon}\right]\right\}\pm\left[\frac{\partial f}{\partial t}+u\frac{\partial c}{\partial t}\right]$$

$$\leqslant -M\bar{c}\pm[\partial f/\partial t+u\partial c/\partial t]$$

则当M充分大时，$Lw_1\leqslant 0$，从而由定理1，知

$$|\partial u/\partial t|\leqslant M\{1+\varepsilon^{-1}\exp[-\bar{c}t/\varepsilon]\}$$

下面，我们来估计$\partial^2u/\partial t^2$。设

$$w_2(x,t)=M\{1+\varepsilon^{-2}\exp[-\bar{c}t/2\varepsilon]\}\pm\partial^2u/\partial t^2$$

在$t=0$，对(1.1)关于t微分并令$t=0$，得$|\partial^2u/\partial t^2|\leqslant M_2/\varepsilon^2$，则当$M$充分大有$w_2>0$。在$x=0$及$x=1$，由假设$u\equiv 0$，则$\partial^2u/\partial t^2=0$，从而$w_2>0$。而

$$Lw_2=M\left\{-c\left[1+\varepsilon^{-2}\exp\left[-\frac{\bar{c}t}{2\varepsilon}\right]\right]-\varepsilon\cdot\varepsilon^{-2}\cdot\left(-\frac{\bar{c}}{2\varepsilon}\right)\exp\left[-\frac{\bar{c}t}{2\varepsilon}\right]\right\}\pm L\left(\frac{\partial^2 u}{\partial t^2}\right)$$

$$=M\left[-c-\left(c-\frac{1}{2}\bar{c}\right)\varepsilon^{-2}\exp\left[-\frac{\bar{c}t}{2\varepsilon}\right]\right]\pm\left[\frac{\partial^2 f}{\partial t^2}+2\frac{\partial c}{\partial t}\frac{\partial u}{\partial t}-u\frac{\partial^2 c}{\partial t^2}\right]$$

$$\leqslant M\left[-\bar{c}-\frac{\bar{c}}{2}\varepsilon^{-1}\exp\left[-\frac{\bar{c}t}{\varepsilon}\right]\right]\pm\left[\frac{\partial^2 f}{\partial t^2}+2\frac{\partial c}{\partial t}\frac{\partial u}{\partial t}-u\frac{\partial^2 c}{\partial t^2}\right]$$

由假设、引理1及$\partial u/\partial t$的估计，有$Lw_2\leqslant 0$，因而

$$|\partial^2u(x,t)/\partial t^2|\leqslant M\{1+\varepsilon^{-2}\exp[-\bar{c}t/2\varepsilon]\}$$

下面我们来估计$\partial u/\partial x$和$\partial^2u/\partial x^2$。由(1.1)得

$$\frac{\partial u(x,t)}{\partial x}=\exp\left[-\int\frac{b}{a}dx\right]\cdot\left\{c_1+\int\frac{f+cu+\varepsilon\partial u/\partial t}{a}\exp\left[\int\frac{b}{a}dx\right]dx\right\}$$

由已得的结论，我们有$|\partial u/\partial x|\leqslant M$，因此由(1.1)，$|\partial^2 u/\partial x^2|\leqslant M$。

现在我们估计$\partial^2 u/\partial x\partial t$和$\partial^3 u/\partial x^2\partial t$。对(1.1)关于$t$微分得

$$a\frac{\partial^3 u}{\partial x^2\partial t}+b\frac{\partial^2 u}{\partial x\partial t}=u\frac{\partial c}{\partial t}+c\frac{\partial u}{\partial t}+\varepsilon\frac{\partial^2 u}{\partial t^2}+\frac{\partial f}{\partial t}$$

则

$$\frac{\partial^2 u(x,t)}{\partial x\partial t}=\exp\left[-\int\frac{b}{a}dx\right]\left\{c_2+\int\exp\left[\int\frac{b}{a}dx\right]\left(u\frac{\partial c}{\partial t}+c\frac{\partial u}{\partial t}+\varepsilon\frac{\partial^2 u}{\partial t^2}+\frac{\partial f}{\partial t}\right)\Big/a\right\}$$

由$\partial u/\partial t$及$\partial^2 u/\partial t^2$的估计，有$|\partial^2 u/\partial x\partial t|\leqslant M/\varepsilon$，则$|\partial^3 u/\partial x^2\partial t|\leqslant M/\varepsilon$。

最后我们估计$\partial^3 u/\partial x^3$和$\partial^4 u/\partial x^4$。对(1.1)关于$x$微分两次，得到

$$a\frac{\partial^4 u}{\partial x^4}+\left(b+2\frac{\partial a}{\partial x}\right)\frac{\partial^3 u}{\partial x^3}=u\frac{\partial^2 c}{\partial x^2}+2\frac{\partial c}{\partial x}\cdot\frac{\partial u}{\partial x}+c\frac{\partial^2 u}{\partial x^2}+\varepsilon\frac{\partial^3 u}{\partial x^2\partial t}$$

$$+\frac{\partial^2 f}{\partial x^2}-\frac{\partial^2 a}{\partial x^2}\frac{\partial^2 u}{\partial x^2}-\frac{\partial^2 b}{\partial x^2}\cdot\frac{\partial u}{\partial x}-2\frac{\partial b}{\partial x}\cdot\frac{\partial u}{\partial x}\equiv F(x,t,\varepsilon)$$

则

$$\frac{\partial^3 u}{\partial x^3}=\exp\left[-\int\frac{2\partial a/\partial x+b}{a}dx\right]\left\{c_3+\int\frac{F}{a}\exp\left[\int\frac{2\partial a/\partial x+b}{a}dx\right]dx\right\}$$

由所得结果，有$|F(x,t,\varepsilon)|\leqslant M_3$，因而取$M$充分大，使

$$|\partial^3 u/\partial x^3|\leqslant M,\quad |\partial^4 u/\partial x^4|\leqslant M$$

从而定理得证。

注 定理2中$a=a(x)$，$b=b(x)$的假设只是为了导出$u(x,t)$的估计．如果$a=a(x,t)$，$b=b(x,t)$，定理2中的估计成立，则下列结果仍成立．

三、差 分 格 式

将[0,1]分成L个子区间。设$h=1/L$为空间的步长，$x_i=ih$，$i=0,\cdots,L$。将[0,T]按非均匀网格划分。对问题(1.1)～(1.2)，构造差分格式

$$\left.\begin{aligned}&L_{ij}u_{ij}\equiv a(x_i,t_j)\frac{u_{i+1,j}-2u_{ij}+u_{i-1,j}}{h^2}+b(x_i,t_j)\frac{u_{i+1,j}-u_{i-1,j}}{2h}\\&\qquad-c(x_i,t_j)u_{ij}-\varepsilon\frac{u_{ij}-u_{i,j-1}}{t_j-t_{j-1}}=f(x_i,t_j)\\&u_{i,0}=\varphi(x_i)\\&u_{0,j}=\psi_0(t_j),\quad u_{L,j}=\psi_1(t_j)\end{aligned}\right\}\qquad(3.1)$$

则有

定理3 若在Γ_h上$u_{ij}\geqslant 0$，在$\bar D_h$上$L_{ij}u_{ij}\leqslant 0$，则对$h\leqslant h_0$，有$u_{ij}\geqslant 0$在$\bar D_h$，其中

$$h_0=\inf_{i,j}\{2a(x_i,t_j)/|b(x_i,t_j)|\}$$

若$b(x_i,t_j)=0$，则h任意。

证明 改写$L_{ij}u_{ij}\leqslant 0$成$A\bar u_{j-1}\leqslant B\bar u_j$，其中$\bar u_j=(u_{1j},\cdots,u_{N-1,j})^T$；对角矩阵$A=(a_{ij})$，$a_{ij}=0$ $(i\neq j)$，$a_{ii}=\varepsilon/(t_j-t_{j-1})$；三对角矩阵$B=(b_{ij})$，$b_{ij}=0$ $(|i-j|>1)$，

$$b_{ii}=c(x_i,t_j)+\frac{\varepsilon}{t_j-t_{j-1}}+\frac{2a(x_i,t_j)}{h^2},$$

$$b_{i,i+1}=b_{i+1,i}=-\frac{a(x_i,t_j)}{h^2}-\frac{b(x_i,t_j)}{2h}$$

$$b_{i-1,i}=b_{i,i-1}=\frac{b(x_i,t_j)}{2h}-\frac{a(x_i,t_j)}{h^2}$$

由假设，有

$$l_1=\frac{b(x_i,t_j)}{2h}-\frac{a(x_i,t_j)}{h^2}<0,\ l_3=-\frac{a(x_i,t_j)}{h^2}-\frac{b(x_i,t_j)}{2h}<0$$

$$l_2=c(x_i,t_j)+\frac{\varepsilon}{t_j-t_{j-1}}+\frac{2a(x_i,t_j)}{h^2}>0$$

$$l_1+l_2+l_3>0$$

故$l_2>|l_1|+|l_3|$。从而B为不可约M-矩阵，因此$B^{-1}>0$([5])。又由

$$\vec{u}_0\geqslant 0,\ 及\ \vec{u}_j\geqslant B^{-1}A\vec{u}_{j-1}$$

得$\vec{u}_j\geqslant 0$，定理得证。

现在，我们考虑收敛性。设$R_{ij}=u(x_i,t_j)-u_{ij}$，则

$$L_{ij}R_{ij}=L_{ij}(u(x_i,t_j))-L_{ij}u_{ij}=L_{ij}(u(x_i,t_j))-Lu(x_i,t_j)$$

从而

$$L_{ij}R_{ij}=r_{ij},\ R_{ij}|_{\Gamma_h}=0$$

定理4 $|R_{ij}|\leqslant\tilde{c}^{-1}\max\limits_{i,j}|r_{ij}|$。

证明 设$w_{ij}=\tilde{c}^{-1}\max\limits_{i,j}|r_{ij}|\pm R_{ij}$，则

$$w_{ij}|_{\Gamma_h}=\tilde{c}^{-1}\max_{i,j}|r_{ij}|\geqslant 0$$

$$L_{ij}w_{ij}=-c(x_i,t_j)\cdot\tilde{c}^{-1}\max_{i,j}|r_{ij}|\pm L_{ij}R_{ij}\leqslant 0$$

则由定理3，知$w_{ij}\leqslant 0$，因此

$$|R_{ij}|\leqslant\tilde{c}^{-1}\max_{i,j}|r_{ij}|$$

四、网格和截断误差

根据[3]，将时间变量非均匀剖分，选取常数d，q满足$d\tilde{c}\geqslant 1$，$0<q<0.5$。过点(0.5, 1)作$\psi(y)=d\varepsilon\ln(q/(q-y))$的切线，设切点为$(\alpha,\ \beta)$，$\sigma=\psi'(\alpha)$。当$\varepsilon\geqslant\mu_1=2q/d$时，$\psi(y)$位于直线$2y-1$之上，即由$\psi(y)$得到的网格比等距网格要大，此时$\varepsilon$不是小参数。设

$$\lambda(y)=\begin{cases}\psi(y), & 0\leqslant y\leqslant\alpha\\ \beta+\sigma(y-\alpha), & \alpha\leqslant y\leqslant 1\end{cases}$$

定理5 若$t_j=\lambda(j/N)$，则

$$(u|x_i,t_j)-u_{ij}|\leqslant M(1/N+h^2)$$

其中$u(x_i,t_j)$是(1.1)～(1.3)的解，u_{ij}是(3.1)的解。

证明 首先，考虑$j<(N-1)/2$情形。设

$$\psi'(\alpha_1)=1/(0.5-q)\equiv q_3$$

则$a_1=q-d\varepsilon/q_3$。设$\psi'(a_2)=2$，$q_2=d/2$，则$a_2=q-q_2\varepsilon$。由[3]，易得到：

$$t_{j+1}-t_j\leqslant q_3/N, \quad 若0\leqslant j\leqslant (N-1)/2 \tag{4.1}$$

$$t_{j+1}-t_j\leqslant 2d\varepsilon/(q-y_j)N, \quad 若y_j\leqslant q-2/N \tag{4.2}$$

$$\exp[-\bar{c}t_{j-1}/\varepsilon]=((q-y_j)/q)^{d\bar{c}}, \quad 若y_j\leqslant a \tag{4.3}$$

$$\exp[-\bar{c}t_j/\varepsilon]\leqslant \exp[-\bar{c}\lambda(a_2)/\varepsilon]=(q_2\varepsilon/q)^{d\bar{c}}, \quad 若a_2\leqslant y_i\leqslant a \tag{4.4}$$

$$\exp[-\bar{c}t_j/\varepsilon]\leqslant \exp[-\bar{c}\lambda(q-3/N)/\varepsilon]=(3/qN)^{d\bar{c}}, \quad 若q-3/N\leqslant y_i \tag{4.5}$$

运用Taylor展式

$$u(x_{i\pm1},t_j)=u(x_i,t_j)\pm h\frac{\partial u(x_i,t_j)}{\partial x}+\frac{h^2}{2}\ \frac{\partial^2u(x_i,t_j)}{\partial x^2}\pm\frac{h^3}{3!}\ \frac{\partial^3u(x_i,t_j)}{\partial x^3}$$

$$+\frac{h^4}{4!}\ \frac{\partial^4u(\xi,t_j)}{\partial x^4}, \qquad (i-1)h\leqslant\xi\leqslant(i+1)h$$

$$u(x_i,t_{j-1})=u(x_i,t_j)+(t_{j-1}-t_j)\frac{\partial u(x_i,t_j)}{\partial t}+\frac{1}{2}(t_{j-1}-t_j)^2\frac{\partial^2u(x_i,\eta)}{\partial t^2}$$

$$t_{j-1}\leqslant\eta\leqslant t_j$$

则有

$$|r_{ij}|\leqslant Mh^2\sup_{x_{i-1}\leqslant x\leqslant x_{i+1}}\left|\frac{\partial^4u(x,t_j)}{\partial x^4}\right|+Mh^2\sup_{x_{i-1}\leqslant x\leqslant x_{i+1}}\left|\frac{\partial^3u(x,t_j)}{\partial x^3}\right|$$

$$+\varepsilon M\cdot\sup_{t_{j-1}\leqslant t\leqslant t_j}\left|\frac{\partial^2u(x_i,t)}{\partial t^2}\right|\cdot|t_j-t_{j-1}|$$

$$\leqslant Mh^2+M\varepsilon\cdot|t_j-t_{j-1}|\cdot[1+\varepsilon^{-2}\cdot\exp[-\bar{c}t_{j-1}/\varepsilon]] \tag{4.6}$$

若$y_{j-1}\geqslant a_2$，由(4.1)和(4.4)，知

$$M\varepsilon^{-1}|t_j-t_{j-1}|\exp\left[-\frac{\bar{c}t_{j-1}}{\varepsilon}\right]\leqslant M\varepsilon^{-1}\frac{q_3}{N}\left(\frac{q_2\varepsilon}{q}\right)^{d\bar{c}}\leqslant\frac{M}{N}$$

若$y_{j-1}\leqslant\min(a,q-3/N)$，由(4.2)和(4.3)，得

$$M\varepsilon^{-1}|t_j-t_{j-1}|\exp\left[-\frac{\bar{c}t_{j-1}}{\varepsilon}\right]\leqslant M\varepsilon^{-1}\ \frac{2d\varepsilon}{(q-y_{j-1})N}\left(\frac{q-y_{j-1}}{q}\right)^{d\bar{c}}\leqslant\frac{M}{N}$$

若$q-3/N<y_{j-1}<a_2$，由$q-3/N<q-d\varepsilon/2$，得$\varepsilon\leqslant 6/dN$。运用一阶Taylor展式，得到

$$|r_{ij}|\leqslant Mh^2+\varepsilon\left|\frac{\partial u(x_i,\eta)}{\partial t}-\frac{\partial u(x_i,t_j)}{\partial t}\right|, \qquad t_{j-1}\leqslant\eta\leqslant t_j$$

$$\leqslant Mh^2+2\varepsilon\{1+\varepsilon^{-1}\exp[-\bar{c}t_{j-1}/\varepsilon]\}$$

再由(4.5)，得

$$|r_{ij}|\leqslant Mh^2+2\varepsilon+2\left(\frac{3}{dN}\right)^{d\bar{c}}\leqslant Mh^2+2\frac{6}{dN}+2\ \frac{3}{dN}\leqslant M\left(\frac{1}{N}+h^2\right)$$

若$j\geqslant(N-1)/2$，则$\varepsilon^{-2}\exp[-\bar{c}t_{j-1}/\varepsilon]\leqslant M$，由(4.6)，

$$|r_{ij}|\leqslant M(1/N+h^2)$$

则由定理 4 就得到本定理的证明。

五、数值结果

考虑问题

$$\begin{cases}\partial^2u/\partial x^2-u-\varepsilon\partial u/\partial t=-\pi^2\cdot\exp[-t/\varepsilon]\sin\pi x\\ u(0,t)=u(1,t)=0,\ u(x,0)=\sin\pi x\end{cases}$$

精确解为$u(x,t)=\exp[-t/\varepsilon]\sin\pi x$，我们用(3.1)求其数值解．我们可以由下列迭代过程求得$\lambda(t)$中的α和β．假设已知近似值$\bar{\alpha}_n$，$0<\bar{\alpha}_n<q$，设$\bar{\beta}_n=\psi(\bar{\alpha}_n)$，过点$(\bar{\alpha}_n,\ \bar{\beta}_n)$和$(0.5,1)$作直线，由等式$\psi'(\alpha_{n+1})=(1-\bar{\beta}_n)/(0.5-\bar{\alpha}_n)$，求得$\bar{\alpha}_{n+1}$，并取$\alpha_1=q-d\varepsilon(0.5-q)$作为$\bar{\alpha}_0$，数值结果如下．

注　由例子看出，实际结果要比理论结果好．

表1　　$\varepsilon=0.1,\ a=4,\ q=0.4,\ N=1000,\ L=100$

(x,t)	(0.01,0.0001)	(0.21,0.0001)	(0.61,0.0001)	(0.81,0.0001)
精确解	0.0310978	0.6068009	0.9315073	0.5564837
数值解	0.0310998	0.6068385	0.9315469	0.5565181
(x,t)	(0.01,0.01)	(0.21,0.01)	(0.61,0.01)	(0.81,0.01)
精确解	0.0283855	0.5538766	0.8506273	0.5131778
数值解	0.0283977	0.5541142	0.8502626	0.5133836
(x,t)	(0.01,0.2)	(0.21,0.2)	(0.61,0.2)	(0.81,0.2)
精确解	0.0019632	0.0383067	0.0588050	0.0351302
数值解	0.0019655	0.0383519	0.0588744	0.0351716

表2　　$\varepsilon=0.0001,\ a=2,\ q=0.4,\ N=1000,\ L=100$

(x,t)	(0.01,0.0001)	(0.21,0.0001)	(0.41,0.0001)	(0.81,0.0001)
精确解	0.0312539	0.6098463	0.9554982	0.5592765
数值解	0.0312544	0.6098559	0.9555133	0.5592863
(x,t)	(0.01,0.01)	(0.21,0.01)	(0.41,0.01)	(0.81,0.01)
精确解	0.0298599	0.5826448	0.9128792	0.5343306
数值解	0.0298637	0.5827208	0.9129983	0.5344003
(x,t)	(0.01,0.2)	(0.21,0.2)	(0.61,0.2)	(0.81,0.2)
精确解	0.0078527	0.1532268	0.2400734	0.1405209
数值解	0.0078571	0.1533133	0.2402090	0.1406002

表3　　$\varepsilon=0.000001,\ a=2,\ q=0.2,\ N=1000,\ L=100$

(x,t)	(0.01,0.0001)	(0.21,0.0001)	(0.61,0.0001)	(0.81,0.0001)
精确解	0.0310974	0.6067933	0.9314955	0.5564767
数值解	0.0310994	0.6068310	0.9315534	0.5565112
(x,t)	(0.01,0.01)	(0.21,0.01)	(0.61,0.01)	(0.81,0.01)
精确解	0.0283482	0.5531486	0.8491450	0.5072803
数值解	0.0283606	0.5533903	0.8495159	0.5075019
(x,t)	(0.01,0.2)	(0.21,0.2)	(0.61,0.2)	(0.81,0.2)
精确解	0	0	0	0
数值解	0	0.0000001	0.0000001	0.0000001

表4 $\varepsilon=0.01,\ a=2,\ q=0.3,\ N=1000,\ L=100$

(x,t)	(0.01,0.0001)	(0.21,0.0001)	(0.61,0.0001)	(0.81,0.0001)
精确解	0.0312017	0.6088278	0.9346188	0.5583425
数值解	0.0312027	0.6088474	0.9346489	0.5583605
(x,t)	(0.01,0.01)	(0.21,0.01)	(0.61,0.01)	(0.81,0.01)
精确解	0.0293516	0.5727276	0.8792009	0.5252358
数值解	0.0293590	0.5728719	0.8794225	0.5253681
(x,t)	(0.01,0.2)	(0.21,0.2)	(0.61,0.2)	(0.81,0.2)
精确解	0.0034901	0.0681008	0.1045423	0.0624537
数值解	0.0034941	0.0681793	0.1046628	0.0625257

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Numerical Methods for Parabolic Equation with a Small Parameter in Time Variable

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Abstract

In this paper we discuss the parabolic equation with a small parameter on derivative in time variable. We construct difference scheme on the non-uniform mesh according to Bakhavlov, and prove the one-order uniform convergence of this scheme. Numerical results are presented.

Key words difference scheme, uniform convergence, parabolic type equation, non-uniform mesh