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压电材料中两平行不相等界面裂纹的动态特性研究*

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(我刊编委王彪来稿)

摘要: 利用 Schmidt 方法, 研究了压电材料中两个平行不相等的可导通界面裂纹对简谐反平面剪切波的散射问题• 利用 Fourier 变换, 使问题的求解转换为对两对以裂纹面张开位移为未知变量的对偶积分方程的求解• 数值计算结果表明, 动态应力强度因子及电位移强度因子受裂纹的几何参数、入射波频率的影响• 在特殊情况下, 与已有结果进行了比较分析• 同时, 电位移强度因子远小于不可导通电边界条件下相应问题的结果•

关 键 词: 界面裂纹; 弹性波; 强度因子; 压电材料; Schmidt 方法

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引 言

因为固有的电机耦合效应, 压电材料在电子器件中有广泛的应用• 然而, 大多数的压电材料是非常脆的, 在制备压电材料及极化过程中, 很容易产生缺陷• 因此, 研究压电材料的电弹作用及断裂性质具有非常重要的意义•

在过去的很长时间内, 压电材料的断裂问题引起了很多学者的注意, 并取得了很大的成就• Pak[1], Chen 和 Yu[2] 分别研究了单裂纹的静态和动态问题• Meguid 和 Wang[3] 研究了反平面剪切波下压电体中两个裂纹的相互作用• Soh, Fang 和 Lee[4], Li 和 Tang[5] 对弹性体与压电材料间界面裂纹的静态问题进行了研究• Narita, Shindo 和 Watanabe[6] 分析了压电材料与弹性材料间的界面裂纹在反平面剪切波作用下的动态问题• 文献[7] 分析了一种压电材料中两个平行对称裂纹的静态问题, 文献[8] 对两种压电材料间的对称界面裂纹的静态问题进行了研究• 到目前为止, 位于两种压电材料间平行不等长界面裂纹的动态问题还没有研究过•

本文研究了两个平行不等长可导通界面裂纹, 在反平面简谐剪切波作用下的散射问题• 通过 Fourier 变换, 得到了两对对偶积分方程, 利用 Schmidt 方法求解这些方程(见 Morse[9], Yan

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[10])• 数值计算结果表明, 动态应力强度因子受到几何参数、入射波频率的影响•

1 问题的提出

假定有一个厚度为 $2h$ 的压电层位于另外一种压电材料的两个半平面中• 沿着界面有两个不同长度$(2a, 2b)$的平行界面裂纹• 指定笛卡儿坐标系(x, y, z)的原点位于两个裂纹的中间• 坐标的 z_轴跟压电材料的极化方向一致, x_y 是各向同性平面, $x = 0$ 是平面的几何对称中心• 为了描述方便, 变量上标 $k(k = 1, 2, 3, 4)$ 分别表示上半平面 _1, 压电层 _2, 压电层 _3 和下半平面 _4, 这里假设上半平面材料 _1 与下半平面材料 _4 相同, 压电层 _2 的材料与压电层 _3 的材料相同, 如图 1 所示• 假定反平面简谐剪切波垂直于裂纹入射, 弹性波的频率为 ω, 入射波的形式为 $\tau_{yz}(x, y, t) = \tau_0 e^{-i\omega t}$• 时间因子 $e^{-i\omega t}$ 略去不考虑, 同时假定 τ_0 为正值•

压电材料的动态反平面控制方程为:

$$c_{44}^{(k)} \nabla^2 w^{(k)} + e_{15}^{(k)} \nabla^2 \phi^{(k)} = \rho^{(k)} \frac{\partial^2 w^{(k)}}{\partial t^2}, \tag{1}$$

$$e_{15}^{(k)} \nabla^2 w^{(k)} - \varepsilon_{11}^{(k)} \nabla^2 \phi^{(k)} = 0, \tag{2}$$

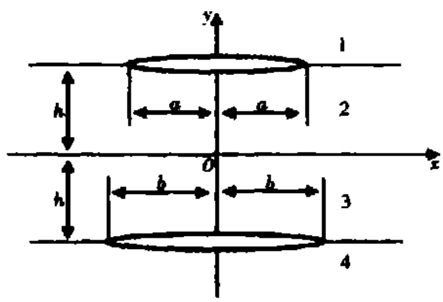

图 1 压电材料中两个不相等的界面裂纹

其中 $\nabla^2 = \partial^2/\partial x^2 + \partial^2/\partial y^2$ 是二维 Laplace 算子, $w^{(k)}$ 和 $\phi^{(k)}$ 是位移和电势, $\rho^{(k)}$、$c_{44}^{(k)}$、$e_{15}^{(k)}$ 和 $\varepsilon_{11}^{(k)}$ 分别是压电材料的密度、剪切模量、压电常数和介电常数•

压电材料的本构方程为

$$\tau_{jz}^{(k)} = c_{44}^{(k)} w_{,j}^{(k)} + e_{15}^{(k)} \phi_{,j}^{(k)}, \tag{3}$$

$$D_j^{(k)} = e_{15}^{(k)} w_{,j}^{(k)} - \varepsilon_{11}^{(k)} \phi_{,j}^{(k)}, \tag{4}$$

这儿 $\tau_{jz}^{(k)}$、$w_{,j}^{(k)}$、$\phi_{,j}^{(k)}$ 和 $D_j^{(k)}(j = x, y)$ 分别是应力、应变、电场强度和电位移张量• 波速可表示为 $c^{(k)} = \sqrt{\mu^{(k)}/\rho^{(k)}}$, 其中 $\mu^{(k)} = c_{44}^{(k)} + (e_{15}^{(k)})^2/\varepsilon_{11}^{(k)}$• 因为几何和载荷的对称性, 只需考虑 x_y 平面的右半部分, 即 $x \geqslant 0$, $|y| < \infty$•

本问题的边界条件为

$$\begin{cases} \tau_{yz}^{(1)}(x, h, t) = \tau_{yz}^{(2)}(x, h, t), \quad \phi^{(1)}(x, h, t) = \phi^{(2)}(x, h, t), \\ D_y^{(1)}(x, h, t) = D_y^{(2)}(x, h, t), \end{cases} \qquad x > 0; \tag{5}$$

$$\begin{cases} w^{(2)}(x, 0, t) = w^{(3)}(x, 0, t), \quad \tau_{yz}^{(2)}(x, 0, t) = \tau_{yz}^{(3)}(x, 0, t), \\ \phi^{(2)}(x, 0, t) = \phi^{(3)}(x, 0, t), \quad D_y^{(2)}(x, 0, t) = D_y^{(3)}(x, 0, t), \end{cases} \qquad x > 0; \tag{6}$$

$$\begin{cases} \tau_{yz}^{(3)}(x, -h, t) = \tau_{yz}^{(4)}(x, -h, t), \\ \phi^{(3)}(x, -h, t) = \phi^{(4)}(x, -h, t), \qquad x > 0; \\ D_y^{(3)}(x, -h, t) = D_y^{(4)}(x, -h, t), \end{cases} \tag{7}$$

$$\tau_{yz}^{(1)}(x, h, t) = \tau_{yz}^{(2)}(x, h, t) = -\tau_0, \qquad 0 \leqslant x \leqslant a; \tag{8}$$

$$\tau_{yz}^{(3)}(x, -h, t) = \tau_{yz}^{(4)}(x, -h, t) = -\tau_0 e^{-i\omega 2h/c_2}, \qquad 0 \leqslant x \leqslant b; \tag{9}$$

$$w^{(1)}(x, h, t) = w^{(2)}(x, h, t), \qquad x > a; \tag{10}$$

$$w^{(3)}(x, -h, t) = w^{(4)}(x, -h, t), \qquad x > b• \tag{11}$$

假定方程(1)和(2)的解为

$$\begin{cases} w^{(1)}(x,y,t) = \dfrac{2}{\pi}\displaystyle\int_0^\infty A_1(s)\mathrm{e}^{-\gamma^{(1)}y}\cos(sx)\mathrm{d}s, \\ \phi^{(1)}(x,y,t) = \dfrac{e_{15}^{(1)}}{\varepsilon_{11}^{(1)}}w^{(1)}(x,y,t) + \dfrac{2}{\pi}\displaystyle\int_0^\infty C_1(s)\mathrm{e}^{-sy}\cos(sx)\mathrm{d}s; \end{cases} \tag{12}$$

$$\begin{cases} w^{(2)}(x,y,t) = \dfrac{2}{\pi}\displaystyle\int_0^\infty \left[A_2(s)\mathrm{e}^{-\gamma^{(2)}y} + B_2(s)\mathrm{e}^{\gamma^{(2)}y}\right]\cos(sx)\mathrm{d}s, \\ \phi^{(2)}(x,y,t) = \dfrac{e_{15}^{(2)}}{\varepsilon_{11}^{(2)}}w^{(2)}(x,y,t) + \dfrac{2}{\pi}\displaystyle\int_0^\infty \left[C_2(s)\mathrm{e}^{-sy} + D_2(s)\mathrm{e}^{sy}\right]\cos(sx)\mathrm{d}s; \end{cases} \tag{13}$$

$$\begin{cases} w^{(3)}(x,y,t) = \dfrac{2}{\pi}\displaystyle\int_0^\infty \left[A_3(s)\mathrm{e}^{\gamma^{(2)}y} + B_3(s)\mathrm{e}^{-\gamma^{(2)}y}\right]\cos(sx)\mathrm{d}s, \\ \phi^{(3)}(x,y,t) = \dfrac{e_{15}^{(2)}}{\varepsilon_{11}^{(2)}}w^{(3)}(x,y,t) + \dfrac{2}{\pi}\displaystyle\int_0^\infty \left[C_3(s)\mathrm{e}^{sy} + D_3(s)\mathrm{e}^{-sy}\right]\cos(sx)\mathrm{d}s; \end{cases} \tag{14}$$

$$\begin{cases} w^{(4)}(x,y,t) = \dfrac{2}{\pi}\displaystyle\int_0^\infty A_4(s)\mathrm{e}^{\gamma^{(1)}y}\cos(sx)\mathrm{d}s, \\ \phi^{(4)}(x,y,t) = \dfrac{e_{15}^{(1)}}{\varepsilon_{11}^{(1)}}w^{(4)}(x,y,t) + \dfrac{2}{\pi}\displaystyle\int_0^\infty C_4(s)\mathrm{e}^{sy}\cos(sx)\mathrm{d}s; \end{cases} \tag{15}$$

其中 $(\gamma^{(k)})^2 = s^2 - \omega^2/c_k^2$, $c_k^2 = \mu^{(k)}/\rho^{(k)}$, $\mu^{(k)} = c_{44}^{(k)} + (e_{15}^{(k)})^2/\varepsilon_{11}^{(k)}$• $A_k(s)$, $B_k(s)$, $C_k(s)$, $D_k(s)$ 是未知函数•

将方程(12)~(15)代入方程(3)和(4),可以得到

$$\begin{cases} \tau_{yz}^{(1)} = -\dfrac{2}{\pi}\displaystyle\int_0^\infty \left\{\mu^{(1)}\gamma^{(1)}A_1(s)\mathrm{e}^{-\gamma^{(1)}y} + e_{15}^{(1)}sC_1(s)\mathrm{e}^{-sy}\right\}\cos(sx)\mathrm{d}s, \\ D_y^{(1)} = \dfrac{2}{\pi}\displaystyle\int_0^\infty \varepsilon_{11}^{(1)}sC_1(s)\mathrm{e}^{-sy}\cos(sx)\mathrm{d}s; \end{cases} \tag{16}$$

$$\begin{cases} \tau_{yz}^{(2)} = -\dfrac{2}{\pi}\displaystyle\int_0^\infty \Big\{\mu^{(2)}\gamma^{(2)}\left[A_2(s)\mathrm{e}^{-\gamma^{(2)}y} - B_2(s)\mathrm{e}^{\gamma^{(2)}y}\right] + \\ \qquad e_{15}^{(2)}s\left[C_2(s)\mathrm{e}^{-sy} - D_2(s)e^{sy}\right]\Big\}\cos(sx)\mathrm{d}s, \\ D_y^{(2)} = \dfrac{2}{\pi}\displaystyle\int_0^\infty \varepsilon_{11}^{(2)}s\left[C_2(s)\mathrm{e}^{-sy} - D_2(s)\mathrm{e}^{sy}\right]\cos(sx)\mathrm{d}s; \end{cases} \tag{17}$$

$$\begin{cases} \tau_{yz}^{(3)} = \dfrac{2}{\pi}\displaystyle\int_0^\infty \Big\{\mu^{(2)}\gamma^{(2)}\left[A_3(s)\mathrm{e}^{\gamma^{(2)}y} - B_3(s)\mathrm{e}^{-\gamma^{(2)}y}\right] + \\ \qquad e_{15}^{(2)}s\left[C_3(s)\mathrm{e}^{sy} - D_3(s)e^{-sy}\right]\Big\}\cos(sx)\mathrm{d}s, \\ D_y^{(3)} = -\dfrac{2}{\pi}\displaystyle\int_0^\infty \varepsilon_{11}^{(2)}s\left[C_3(s)\mathrm{e}^{sy} - D_3(s)\mathrm{e}^{-sy}\right]\cos(sx)\mathrm{d}s; \end{cases} \tag{18}$$

$$\begin{cases} \tau_{yz}^{(4)} = \dfrac{2}{\pi}\displaystyle\int_0^\infty \left\{\mu^{(1)}\gamma^{(1)}A_4(s)\mathrm{e}^{\gamma^{(1)}y} + e_{15}^{(1)}sC_4(s)\mathrm{e}^{-sy}\right\}\cos(sx)\mathrm{d}s, \\ D_y^{(4)} = -\dfrac{2}{\pi}\displaystyle\int_0^\infty \varepsilon_{11}^{(1)}sC_4(s)\mathrm{e}^{sy}\cos(sx)\mathrm{d}s• \end{cases} \tag{19}$$

为了求解以上方程,取两个裂纹上下表面的位移差分别为

$$f_1(x) = w^{(1)}(x,h,t) - w^{(2)}(x,h,t), \tag{20}$$

$$f_2(x) = w^{(3)}(x,-h,t) - w^{(4)}(x,-h,t)• \tag{21}$$

将方程(12)~(15)代入方程(20)~(21)并进行 Fourier 余弦变换(本文中上划线代表 Fourier 余弦变换),利用边界条件(5)、(7)、(10)和(11)可以得到

$$\overline{f}_1(s) = A_1(s)\mathrm{e}^{-\gamma^{(1)}h} - A_2(s)\mathrm{e}^{-\gamma^{(2)}h} - B_2(s)\mathrm{e}^{\gamma^{(2)}h}, \tag{22}$$

$$f_2(s) = A_3(s)e^{-\gamma^{(2)}h} + B_3(s)e^{\gamma^{(2)}h} - A_4(s)e^{-\gamma^{(1)}h}, \tag{23}$$

$$\frac{e_{15}^{(1)}}{\varepsilon_{11}^{(1)}}A_1(s)e^{-\gamma^{(1)}h} + C_1(s)e^{-sh} - \frac{e_{15}^{(2)}}{\varepsilon_{11}^{(2)}}\left[A_2(s)e^{-\gamma^{(2)}h} + B_2(s)e^{\gamma^{(2)}h}\right] - \left[C_2(s)e^{-sh} + D_2(s)e^{sh}\right] = 0, \tag{24}$$

$$\frac{e_{15}^{(2)}}{\varepsilon_{11}^{(2)}}\left[A_3(s)e^{-\gamma^{(2)}h} + B_3(s)e^{\gamma^{(2)}h}\right] + \left[C_3(s)e^{-sh} + D_3(s)e^{sh}\right] - \frac{e_{15}^{(1)}}{\varepsilon_{11}^{(1)}}A_4(s)e^{-\gamma^{(1)}h} - C_4(s)e^{-sh} = 0. \tag{25}$$

应用边界条件(5)~(11),经过 Fourier 余弦变换,从方程(16)~(19)可以得到

$$\mu^{(1)}\gamma^{(1)}A_1(s)e^{-\gamma^{(1)}h} + e_{15}^{(1)}sC_1(s)e^{-sh} = \mu^{(2)}\gamma^{(2)}\left[A_2(s)e^{-\gamma^{(2)}h} - B_2(s)e^{\gamma^{(2)}h}\right] + e_{15}^{(2)}s\left[C_2(s)e^{-sh} - D_2(s)e^{sh}\right], \tag{26}$$

$$\mu^{(2)}\gamma^{(2)}\left[A_3(s)e^{-\gamma^{(2)}h} - B_3(s)e^{\gamma^{(2)}h}\right] + e_{15}^{(2)}s\left[C_3(s)e^{-sh} - D_3(s)e^{sh}\right] = \mu^{(1)}\gamma^{(1)}A_4(s)e^{-\gamma^{(1)}h} + e_{15}^{(1)}sC_4(s)e^{-sh}, \tag{27}$$

$$\varepsilon_{11}^{(1)}C_1(s)e^{-sh} = \varepsilon_{11}^{(2)}\left[C_2(s)e^{-sh} - D_2(s)e^{sh}\right], \tag{28a}$$

$$\varepsilon_{11}^{(2)}\left[C_3(s)e^{-sh} - D_3(s)e^{sh}\right] = \varepsilon_{11}^{(1)}C_4(s)e^{-sh}, \tag{28b}$$

$$A_2(s) + B_2(s) = A_3(s) + B_3(s),\quad A_2(s) - B_2(s) = -A_3(s) + B_3(s), \tag{29}$$

$$C_2(s) + D_2(s) = C_3(s) + D_3(s),\quad C_2(s) - D_2(s) = -C_3(s) + D_3(s). \tag{30}$$

解方程(22)~(30)可以得到 $A_k(s)$、$B_k(s)$、$C_k(s)$、$D_k(s)$ 的解,将结果代入方程(16)和(19)并利用边界条件(5)、(7)~(9),可以得到

$$\frac{2}{\pi}\int_0^\infty f_1(s)\cos(sx)\,ds = 0,\qquad x > a, \tag{31}$$

$$\frac{2}{\pi}\int_0^\infty f_2(s)\cos(sx)\,ds = 0,\qquad x > b, \tag{32}$$

$$\frac{2}{\pi}\int_0^\infty s\left[\alpha(s)f_1(s) + \beta(s)f_2(s)\right]\cos(sx)\,ds = -\tau_0,\qquad 0 \leqslant x \leqslant a, \tag{33}$$

$$\frac{2}{\pi}\int_0^\infty s\left[\beta(s)f_1(s) + \alpha(s)f_2(s)\right]\cos(sx)\,ds = -\tau_0 e^{-i\omega 2h/c_2},\qquad 0 \leqslant x \leqslant b, \tag{34}$$

其中 $\alpha(s)$ 和 $\beta(s)$ 是已知函数,见附录.

2 对偶积分方程的解

应用 Schmidt 方法,可以求解对偶积分方程(31)~(34). 裂纹表面位移差可以用下面的级数表示

$$f_1(x) = \sum_{n=1}^{\infty} a_n P_{2n-2}^{(1/2,1/2)}\left(\frac{x}{a}\right)\left(1 - \frac{x^2}{a^2}\right)^{1/2},\qquad 0 \leqslant x \leqslant a,\ y = h, \tag{35a}$$

$$f_2(x) = \sum_{n=1}^{\infty} b_n P_{2n-2}^{(1/2,1/2)}\left(\frac{x}{b}\right)\left(1 - \frac{x^2}{b^2}\right)^{1/2},\qquad 0 \leqslant x \leqslant b,\ y = -h, \tag{35b}$$

其中 a_n 和 b_n 是未知的待定参数, $P_n^{(1/2,1/2)}(x)$ 是 Jacobi 多项式(参见 Gradshteyn 和 Ryzhik[11]). 方程(35)的 Fourier 余弦变换为(参见 Erdelyi[12])

$$f_1(s) = \frac{1}{s}\sum_{n=1}^{\infty} a_n G_n J_{2n-1}(sa),\quad f_2(s) = \frac{1}{s}\sum_{n=1}^{\infty} b_n G_n J_{2n-1}(sb), \tag{36}$$

其中, $G_n = 2\sqrt{\pi}(-1)^{n-1}[\Gamma(2n-1/2)/(2n-2)!]$, $\Gamma(x)$ 和 $J_n(x)$ 是 Gamma 函数和 Bessel 函数.

将方程(36)代入(31)~(34),方程(31)和(32)自动成立。对方程(33)和(34)在$[0, x]$(方程(33)中,$0< x < a$;方程(34)中,$0< x < b$)范围内做关于x的积分,方程(33)和(34)变成

$$\sum_{n=1}^{\infty} a_n G_n \int_0^{\infty} \frac{1}{s}\alpha(s) J_{2n-1}(sa)\sin(sx)\,ds + \sum_{n=1}^{\infty} b_n G_n \int_0^{\infty} \frac{1}{s}\beta(s) J_{2n-1}(sb)\sin(sx)\,ds = -\frac{\pi\tau_0}{2}x, \quad 0< x < a, \tag{37a}$$

$$\sum_{n=1}^{\infty} a_n G_n \int_0^{\infty} \frac{1}{s}\beta(s) J_{2n-1}(sa)\sin(sx)\,ds + \sum_{n=1}^{\infty} b_n G_n \int_0^{\infty} \frac{1}{s}\alpha(s) J_{2n-1}(sb)\sin(sx)\,ds = -\frac{\pi\tau_0}{2}x\,e^{-i\omega 2h/c_2}, \quad 0< x < b, \tag{37b}$$

方程(37)中的系数a_n和b_n可以用Schmidt方法得到。限于篇幅,这里简略,具体可见文献[7]和文献[13,14]。

3 强 度 因 子

当系数a_n和b_n获得后,整个应力场和电位移场就可以获得。从应力场和电位移的表达式可知,应力场和电位移的奇异部分可以表示为

$$\tau^{(1)} = -\frac{2\alpha_c}{\pi}\sum_{n=1}^{\infty} a_n G_n H_n(x), \quad D^{(1)} = -\frac{2\delta_c}{\pi}\sum_{n=1}^{\infty} a_n G_n H_n(x), \quad x > a, \tag{38a}$$

$$\tau^{(4)} = -\frac{2\alpha_c}{\pi}\sum_{n=1}^{\infty} b_n G_n L_n(x), \quad D^{(4)} = -\frac{2\delta_c}{\pi}\sum_{n=1}^{\infty} b_n G_n L_n(x), \quad x > b, \tag{38b}$$

其中

$$H_n(x) = \frac{(-1)^{n-1} a^{2n-1}}{\sqrt{x^2 - a^2}\,[x + \sqrt{x^2 - a^2}]^{2n-1}},$$

$$L_n(x) = \frac{(-1)^{n-1} b^{2n-1}}{\sqrt{x^2 - b^2}\,[x + \sqrt{x^2 - b^2}]^{2n-1}},$$

$\gamma(s)$、$\delta(s)$、α_c和δ_c见附录,α_c与δ_c为已知常数,与材料性质有关。

从而可以得到应力强度因子K_a和K_b

$$K_a = \lim_{x\to a^+}\sqrt{2\pi(x-a)}\,\tau^{(1)} = -\frac{4\alpha_c}{\sqrt{a}}\sum_{n=1}^{\infty} a_n \frac{\Gamma(2n-1/2)}{(2n-2)!}, \tag{39}$$

$$K_b = \lim_{x\to b^+}\sqrt{2\pi(x-b)}\,\tau^{(4)} = -\frac{4\alpha_c}{\sqrt{b}}\sum_{n=1}^{\infty} b_n \frac{\Gamma(2n-1/2)}{(2n-2)!}. \tag{40}$$

电位移强度因子D_a和D_b

$$D_a = \lim_{x\to a^+}\sqrt{2\pi(x-a)}\,D^{(1)} = -\frac{4\delta_c}{\sqrt{a}}\sum_{n=1}^{\infty} a_n \frac{\Gamma(2n-1/2)}{(2n-2)!} = \frac{\delta_c}{\alpha_c}K_a, \tag{41}$$

$$D_b = \lim_{x\to b^+}\sqrt{2\pi(x-b)}\,D^{(4)} = -\frac{4\delta_c}{\sqrt{b}}\sum_{n=1}^{\infty} b_n \frac{\Gamma(2n-1/2)}{(2n-2)!} = \frac{\delta_c}{\alpha_c}K_b. \tag{42}$$

4 数值计算及讨论

本文采用压电陶瓷作为压电材料进行了数值计算。图1结构中材料1和2分别为PZT_4和PZT_5H,材料常数见表1。如文献[9]和[10]中的讨论,可知取方程(37)中级数的前6项,Schmidt方法就可以满足有关精度要求。以下计算了无量纲的应力强度因子(K_a, K_b)和电位移强度因子(D_a, D_b)。计算结果见图2~图14,从图中可以看出:

表 1 压电陶瓷的材料常数

材料	c_{44}/(N/m²)	e_{15}/(C/m²)	ε_{11}/(C/V·m)	ρ/(kg/m³)
PZT_4	2.56×10^{10}	12.7	64.6×10^{10}	7 500
PZT_5H	2.3×10^{10}	17.0	150.4×10^{10}	7 500

(ⅰ) 当两个裂纹的长度相等, 简谐波的频率为零时, 本问题结果与文献[7]中的结果相等, 见图 2•

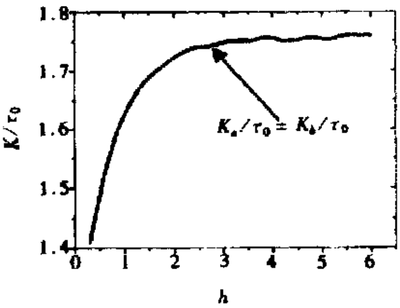

图 2 电位移强度因子随 h 的变化
($\omega/c_1 = 0$, $a = b = 1.0$, PZT_4/PZT_4/PZT_4)

(ⅱ) 从图 3~图 6 可以看出, K_a 随着裂纹 1 长度的增加而增加, 而 K_b 随着裂纹 1 长度的增加而减小• 特别的, 当 $a = 0.24$ 时, 图 3 中的两条曲线交在一起, 这表示即使 $a \neq b$ 在某种情况下 K_a 可以与 K_b 相等• 这是由于裂纹 1 和裂纹 2 的机械边界条件不同而引起的•

(ⅲ) 从图 7~图 10 可以看到, K_a 随着裂纹间距离的增加而增加, K_b 随着裂纹间距离的增加而减小• 这种现象叫裂纹屏蔽效应(参见 Ratwani[15])•

(ⅳ) 图 3~图 10 表示 K_a 随着裂纹 1 长度的增加而迅速增加• 然而, 应力强度因子 K_b 却随着平行裂纹间距离的增加而减小• 从这点可以看出, 裂纹 1 长度变化对 K_a 的影响要大于 K_b, 裂纹间距离的变化对 K_b 的影响要大于 K_a•

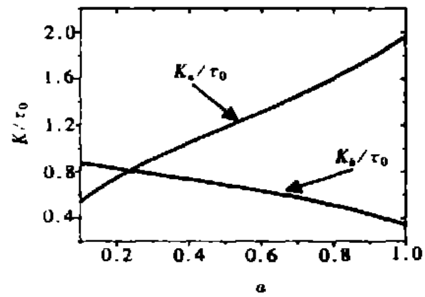

图 3 应力强度因子随 a 的变化
($h = 0.5$, $\omega/c_1 = 0.5$, $b = 1.0$, PZT_4/PZT_5H/PZT_4)

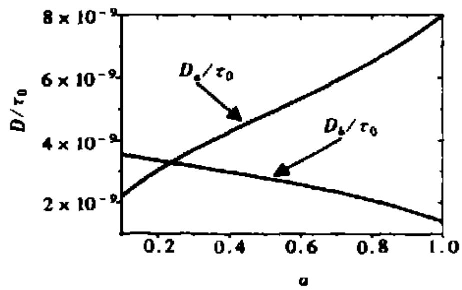

图 4 电位移强度因子随 a 的变化
($h = 0.5$, $\omega/c_1 = 0.5$, $b = 1.0$, PZT_4/PZT_5H/PZT_4)

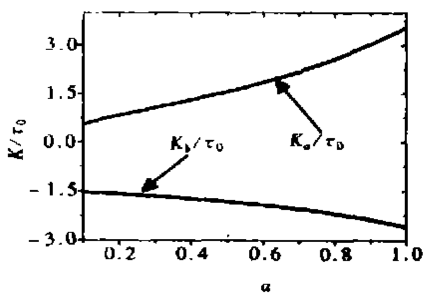

图 5 应力强度因子随 a 的变化
($h = 1.0$, $\omega/c_1 = 0.5$, $b = 1.0$, PZT_4/PZT_5H/PZT_4)

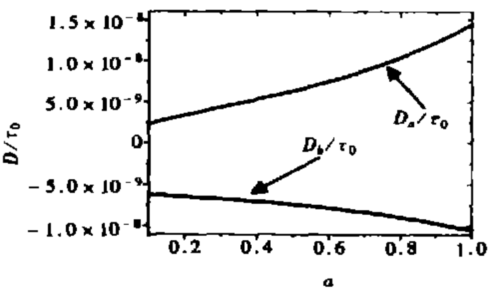

图 6 电位移强度因子随 a 的变化
($h = 1.0$, $\omega/c_1 = 0.5$, $b = 1.0$, PZT_4/PZT_5H/PZT_4)

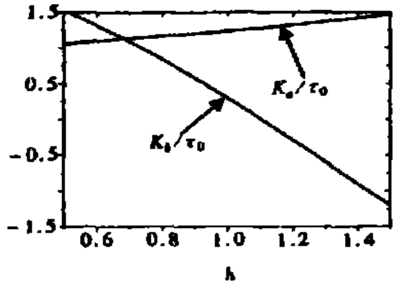

图 7 应力强度因子随 h 的变化
(a = 0.5, ω/c_1 = 0.5, b = 1.0,
PZT_4/ PZT_5H/ PZT_4)

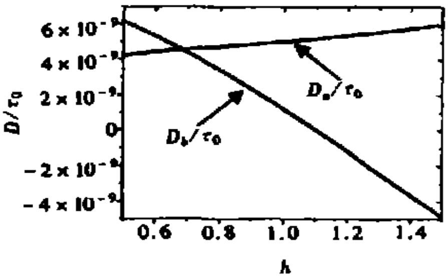

图 8 电位移强度因子随 h 的变化
(a = 0.5, ω/c_1 = 0.5, b = 1.0,
PZT_4/ PZT_5H/ PZT_4)

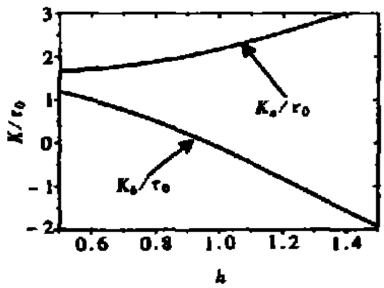

图 9 应力强度因子随 h 的变化
(a = 1.0, ω/c_1 = 0.5, b = 1.0,
PZT_4/ PZT_5H/ PZT_4)

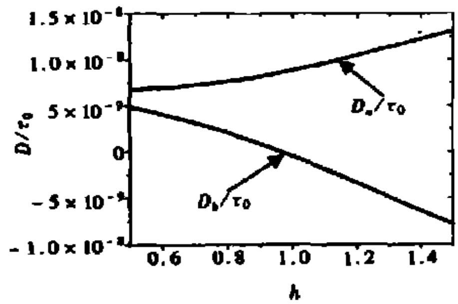

图 10 电位移强度因子随 h 的变化
(a = 1.0, ω/c_1 = 0.5, b = 1.0,
PZT_4/ PZT_5H/ PZT_4)

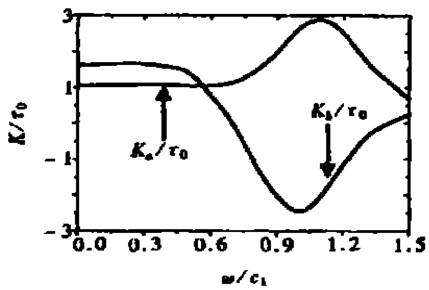

图 11 应力强度因子随 ω/c_1 的变化
(h = 0.5, a = 0.5, b = 1.0,
PZT_4/ PZT_5H/ PZT_4)

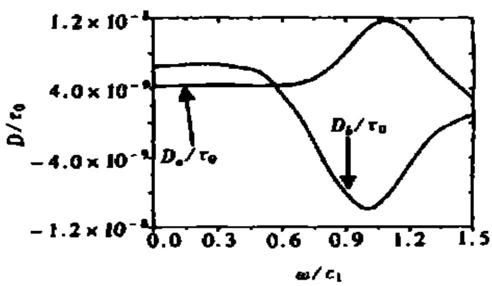

图 12 电位移强度因子随 ω/c_1 的变化
(h = 0.5, a = 0.5, b = 1.0,
PZT_4/ PZT_5H/ PZT_4)

(v) 从图 11~ 图 14 可以得到, 随着谐波频率的增加, 应力强度因子 K_b 变位负值• 这是由于裂纹 2 的边界条件引起的•

(vi) 从方程式(41)、(42) 可以得出, D_a 和 D_b 的变化趋势与 K_a 和 K_b 相同•

从以上结果中, 可以得出结论, 动态强度因子和电位移强度因子受到裂纹长度, 简谐波频率和裂纹间距离的影响•

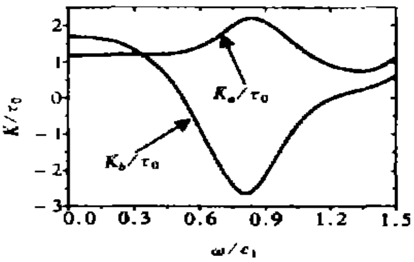

图 13　应力强度因子随 ω/c_1 的变化
($h=1.0$, $a=0.5$, $b=1.0$,
PZT_4/ PZT_5H/ PZT_4)

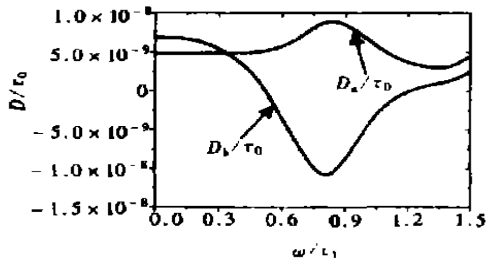

图 14　电位移强度因子随 ω/c_1 的变化
($h=1.0$, $a=0.5$, $b=1.0$,
PZT_4/ PZT_5H/ PZT_4)

附　录

$(\gamma^{(1)})^2 = s^2 - \omega^2/(c_1)^2$, $(c_1)^2 = \mu^{(1)}/\rho^{(1)}$,

$\mu^{(1)} = c_{44}^{(1)} + (e_{15}^{(1)})^2/\varepsilon_{11}^{(1)}$, $(\gamma_N^{(1)})^2 = 1 - \omega^2/((c_1)^2 s^2)$;

$(\gamma^{(2)})^2 = s^2 - \omega^2/(c_2)^2$, $(c_2)^2 = \mu^{(2)}/\rho^{(2)}$,

$\mu^{(2)} = c_{44}^{(2)} + (e_{15}^{(2)})^2/\varepsilon_{11}^{(2)}$, $(\gamma_N^{(2)})^2 = 1 - \omega^2/((c_2)^2 s^2)$;

$H_1 = 1 + e^{-4h\gamma^{(2)}}$, $H_2 = 1 - e^{-4h\gamma^{(2)}}$, $H_3 = 1 + e^{-4hs}$, $H_4 = 1 - e^{-4hs}$;

$R_1 = 2H_1H_4(e_{15}^{(1)})^3 e_{15}^{(2)} \gamma_N^{(2)} \mu_2 \varepsilon_{11}^{(1)} (\varepsilon_{11}^{(2)})^3$, $R_2 = -2H_1H_4 e_{15}^{(1)} e_{15}^{(2)} \gamma_N^{(1)} \gamma_N^{(2)} \mu_1 \mu_2 (\varepsilon_{11}^{(1)})^2 (\varepsilon_{11}^{(2)})^3$,

$R_3 = -H_1H_4(e_{15}^{(1)})^2 \gamma_N^{(2)} \mu_2 (\varepsilon_{11}^{(2)})^2((e_{15}^{(2)})^2(\varepsilon_{11}^{(1)})^2 + (\varepsilon_{11}^{(2)})^2((e_{15}^{(1)})^2 - 2\gamma_N^{(1)}\mu_1\varepsilon_{11}^{(1)}))$,

$R_4 = H_1H_4 \gamma_N^{(1)} \gamma_N^{(2)} \mu_1 \mu_2 (\varepsilon_{11}^{(1)})^2 (\varepsilon_{11}^{(2)})^2 (2(e_{15}^{(2)})^2 \varepsilon_{11}^{(1)} - \gamma_N^{(1)} \mu_1 ((\varepsilon_{11}^{(1)})^2 (\varepsilon_{11}^{(2)})^2))$,

$R_5 = 2H_1H_3 \gamma_N^{(1)} \gamma_N^{(2)} \mu_1 \mu_2 (\varepsilon_{11}^{(1)})^2 \varepsilon_{11}^{(2)} ((e_{15}^{(2)})^2 (\varepsilon_{11}^{(1)})^2 - e_{15}^{(1)} e_{15}^{(2)} \varepsilon_{11}^{(1)} \varepsilon_{11}^{(2)} +$
$(\varepsilon_{11}^{(2)})^2((e_{15}^{(1)})^2 - \gamma_N^{(1)} \mu_1 \varepsilon_{11}^{(1)}))$,

$R_6 = -H_2H_4(e_{15}^{(2)})^4 \gamma_N^{(1)} \mu_1 (\varepsilon_{11}^{(1)})^4$, $R_7 = 2H_2H_4 e_{15}^{(1)} (e_{15}^{(2)})^3 \gamma_N^{(1)} \mu_1 (\varepsilon_{11}^{(1)})^3 \varepsilon_{11}^{(2)}$,

$R_8 = -H_2H_4(e_{15}^{(2)})^2 \gamma_N^{(1)} \mu_1 (\varepsilon_{11}^{(1)})^2 (\varepsilon_{11}^{(2)})^2((e_{15}^{(1)})^2 - \gamma_N^{(1)} \mu_1 \varepsilon_{11}^{(1)}))$,

$R_9 = H_2H_4(\gamma_N^{(2)})^2(\mu_2)^2 \varepsilon_{11}^{(1)} (\varepsilon_{11}^{(2)})^2(\gamma_N^{(1)}\mu_1\varepsilon_{11}^{(1)}((\varepsilon_{11}^{(1)})^2 + (\varepsilon_{11}^{(2)})^2) - (e_{15}^{(1)})^2(\varepsilon_{11}^{(2)})^2)$,

$R_{10} = H_2H_3(\varepsilon_{11}^{(1)})^2 \varepsilon_{11}^{(2)}((e_{15}^{(2)})^2(\gamma_N^{(1)})^2(\mu_1)^2(\varepsilon_{11}^{(1)})^2 + (\gamma_N^{(2)})^2(\mu_2)^2(\varepsilon_{11}^{(2)})^2((e_{15}^{(1)})^2 -$
$2\gamma_N^{(1)}\mu_1\varepsilon_{11}^{(1)}))$,

$R_{11} = -8e^{-2h(\gamma^{(2)}+s)} e_{15}^{(2)} \gamma_N^{(1)} \gamma_N^{(2)} \mu_1 \mu_2 (\varepsilon_{11}^{(1)})^3 \varepsilon_{11}^{(2)} (e_{15}^{(2)} \varepsilon_{11}^{(1)} - e_{15}^{(1)} \varepsilon_{11}^{(2)})$,

$R = R_1 + R_2 + R_3 + R_4 + R_5 + R_6 + R_7 + R_8 + R_9 + R_{10} + R_{11}$;

$S_1 = 2e^{-2h\gamma^{(2)}} e^{-4hs} \gamma_N^{(2)} \mu_2 (\varepsilon_{11}^{(1)})^2(-e_{15}^{(1)} e_{15}^{(2)} \varepsilon_{11}^{(1)} + \gamma_N^{(1)} \mu_1 \varepsilon_{11}^{(1)} (\varepsilon_{11}^{(1)} - \varepsilon_{11}^{(2)}) + (e_{15}^{(1)})^2 (\varepsilon_{11}^{(2)})^2)$,

$S_2 = -2e^{-4h\gamma^{(2)}} e^{-2hs} (\varepsilon_{11}^{(1)})^2 \varepsilon_{11}^{(2)} (e_{15}^{(2)} \gamma_N^{(1)} \mu_1 \varepsilon_{11}^{(1)} - e_{15}^{(1)} \gamma_N^{(2)} \mu_2 \varepsilon_{11}^{(2)})^2$,

$S_3 = 2e^{-2hs} (\varepsilon_{11}^{(1)})^2 \varepsilon_{11}^{(2)} (e_{15}^{(2)} \gamma_N^{(1)} \mu_1 \varepsilon_{11}^{(1)} + e_{15}^{(1)} \gamma_N^{(2)} \mu_2 \varepsilon_{11}^{(2)})^2$,

$S_4 = -2e^{-2h\gamma^{(2)}} \gamma_N^{(2)} \mu_2 (\varepsilon_{11}^{(2)})^2 (e_{15}^{(1)} e_{15}^{(2)} \varepsilon_{11}^{(1)} - (e_{15}^{(1)})^2 \varepsilon_{11}^{(2)} + \gamma_N^{(1)} \mu_1 \varepsilon_{11}^{(1)} (\varepsilon_{11}^{(1)} + \varepsilon_{11}^{(2)}))^2$,

$S = S_1 + S_2 + S_3 + S_4$;

$T_1 = H_2H_4(e_{15}^{(1)})^3 \gamma_N^{(1)} \mu_1 (\varepsilon_{11}^{(1)})^4 \varepsilon_{11}^{(2)}$, $T_2 = H_1H_4(e_{15}^{(1)})^3 \gamma_N^{(2)} \mu_2 \varepsilon_{11}^{(1)} (\varepsilon_{11}^{(2)})^4$,

$T_3 = -H_2 e_{15}^{(1)} (\gamma_N^{(2)})^2 (\mu_2)^2 (\varepsilon_{11}^{(1)})^2 (\varepsilon_{11}^{(2)})^3 (\varepsilon_{11}^{(1)} H_3 - \varepsilon_{11}^{(2)} H_4)$,

$T_4 = -2H_1H_4(e_{15}^{(1)})^2 e_{15}^{(2)} \gamma_N^{(2)} \mu_2 (\varepsilon_{11}^{(1)})^2 (\varepsilon_{11}^{(2)})^3$,

$T_5 = -H_2 e_{15}^{(1)} (\gamma_N^{(1)})^2 (\mu_1)^2 (\varepsilon_{11}^{(1)})^3 (\varepsilon_{11}^{(2)})^2 (\varepsilon_{11}^{(1)} H_3 - \varepsilon_{11}^{(2)} H_4)$,

$T_6 = H_2H_4(e_{15}^{(1)})^2 e_{15}^{(2)} \gamma_N^{(1)} \mu_1 (\varepsilon_{11}^{(1)})^2 (\varepsilon_{11}^{(2)})^3$,

$T_7 = -(H_3H_1 - 4e^{-2h\gamma^{(2)}} e^{-2hs}) e_{15}^{(2)} \gamma_N^{(1)} \gamma_N^{(2)} \mu_1 \mu_2 (\varepsilon_{11}^{(1)})^4 (\varepsilon_{11}^{(2)})^2$,

$T_8 = - H_1H_4e_{15}^{(2)}\gamma_N^{(1)}\gamma_N^{(2)}\mu_1\mu_2(\varepsilon_{11}^{(1)})^3(\varepsilon_{11}^{(2)})^3,$

$T_9 = - 2H_2H_4e_{15}^{(1)}(e_{15}^{(2)})^2\gamma_N^{(1)}\mu_1(\varepsilon_{11}^{(1)})^3(\varepsilon_{11}^{(2)})^2,\ T_{10} = H_1H_4e_{15}^{(1)}(e_{15}^{(2)})^2\gamma_N^{(2)}\mu_2(\varepsilon_{11}^{(1)})^3(\varepsilon_{11}^{(2)})^2,$

$T_{11} = -(H_3H_1 + 4e^{-2h\gamma^{(2)}}e^{-2hs})e_{15}^{(1)}\gamma_N^{(1)}\gamma_N^{(2)}\mu_1\mu_2(\varepsilon_{11}^{(1)})^3(\varepsilon_{11}^{(2)})^3,$

$T_{12} = - H_1H_4e_{15}^{(1)}\gamma_N^{(1)}\gamma_N^{(2)}\mu_1\mu_2(\varepsilon_{11}^{(1)})^2(\varepsilon_{11}^{(2)})^4,$

$T = T_1 + T_2 + T_3 + T_4 + T_5 + T_6 + T_7 + T_8 + T_9 + T_{10} + T_{11} + T_{12};$

$U_1 = 2e^{-2h\gamma^{(2)}}e_{15}^{(1)}(e_{15}^{(2)})^2\gamma_N^{(2)}\mu_2(\varepsilon_{11}^{(1)})^3(\varepsilon_{11}^{(2)})^3,$

$U_2 = 2e^{-4h\gamma^{(2)}}e^{-2hs}(\gamma_N^{(1)}\mu_1 - \gamma_N^{(2)}\mu_2)(\varepsilon_{11}^{(1)})^3(\varepsilon_{11}^{(2)})^2(e_{15}^{(2)}\gamma_N^{(1)}\mu_1\varepsilon_{11}^{(1)} - e_{15}^{(1)}\gamma_N^{(2)}\mu_2\varepsilon_{11}^{(2)}),$

$U_3 = - 2e^{-2hs}(\gamma_N^{(1)}\mu_1 + \gamma_N^{(2)}\mu_2)(\varepsilon_{11}^{(1)})^3(\varepsilon_{11}^{(2)})^2(e_{15}^{(2)}\gamma_N^{(1)}\mu_1\varepsilon_{11}^{(1)} + e_{15}^{(1)}\gamma_N^{(2)}\mu_2\varepsilon_{11}^{(2)}),$

$U_4 = - 2e^{-2h\gamma^{(2)}}e^{-4hs}\gamma_N^{(2)}\mu_2\varepsilon_{11}^{(1)}(\varepsilon_{11}^{(2)})^2(e_{15}^{(2)}\varepsilon_{11}^{(1)} + e_{15}^{(1)}\varepsilon_{11}^{(2)})(e_{15}^{(1)}e_{15}^{(2)}s\varepsilon_{11}^{(1)} - (e_{15}^{(1)})^2s\varepsilon_{11}^{(2)}),$

$U_5 = 2e^{-2h\gamma^{(2)}}e_{15}^{(1)}\gamma_N^{(2)}\mu_2\varepsilon_{11}^{(1)}(\varepsilon_{11}^{(2)})^3((e_{15}^{(1)})^2\varepsilon_{11}^{(2)} - \gamma_N^{(1)}\mu_1\varepsilon_{11}^{(1)}(\varepsilon_{11}^{(1)} + \varepsilon_{11}^{(2)})),$

$U_6 = 2e^{-2h\gamma^{(2)}}e_{15}^{(2)}\gamma_N^{(2)}\mu_2(\varepsilon_{11}^{(1)})^2(\varepsilon_{11}^{(2)})^2(- 2(e_{15}^{(1)})^2\varepsilon_{11}^{(2)} + \gamma_N^{(1)}\mu_1\varepsilon_{11}^{(1)}(\varepsilon_{11}^{(1)} + \varepsilon_{11}^{(2)})),$

$U_7 = - 2e^{-2h\gamma^{(2)}}e^{-4hs}\gamma_N^{(1)}\gamma_N^{(2)}\mu_1\mu_2(\varepsilon_{11}^{(1)})^2(\varepsilon_{11}^{(2)})^2(e_{15}^{(2)}\varepsilon_{11}^{(1)} + e_{15}^{(1)}\varepsilon_{11}^{(2)})(\varepsilon_{11}^{(2)} - \varepsilon_{11}^{(1)}),$

$U = U_1 + U_2 + U_3 + U_4 + U_5 + U_6 + U_7;$

$V_1 = H_2H_4(e_{15}^{(2)})^2(\varepsilon_{11}^{(1)})^3(e_{15}^{(2)}\varepsilon_{11}^{(1)} - 4e_{15}^{(1)}\varepsilon_{11}^{(2)}),$

$V_2 = 2H_2H_4(e_{15}^{(2)})^2(\varepsilon_{11}^{(1)})^2(\varepsilon_{11}^{(2)})^2(3(e_{15}^{(1)})^2 - \gamma_N^{(1)}\mu_1\varepsilon_{11}^{(1)}),$

$V_3 = 4H_2H_4e_{15}^{(1)}e_{15}^{(2)}\varepsilon_{11}^{(1)}(\varepsilon_{11}^{(2)})^3(-(e_{15}^{(1)})^2 + \gamma_N^{(1)}\mu_1\varepsilon_{11}^{(1)}),$

$V_4 = H_2H_4(\varepsilon_{11}^{(2)})^2((e_{15}^{(1)})^4(\varepsilon_{11}^{(2)})^2 - 2(e_{15}^{(1)})^2\gamma_N^{(1)}\mu_1\varepsilon_{11}^{(1)}(\varepsilon_{11}^{(2)})^2 +$
$(\gamma_N^{(1)})^2(\mu_1)^2(\varepsilon_{11}^{(1)})^2((\varepsilon_{11}^{(1)})^2 + (\varepsilon_{11}^{(2)})^2)),$

$V_5 = H_2H_4(\gamma_N^{(2)})^2(\mu_2)^2(\varepsilon_{11}^{(1)})^2(\varepsilon_{11}^{(2)})^2((\varepsilon_{11}^{(1)})^2 + (\varepsilon_{11}^{(2)})^2),$

$V_6 = 8e^{-2h\gamma^{(2)}}e^{-2hs}\gamma_N^{(2)}\mu_2(\varepsilon_{11}^{(1)})^2\varepsilon_{11}^{(2)}(e_{15}^{(2)}\varepsilon_{11}^{(1)} + e_{15}^{(1)}\varepsilon_{11}^{(2)})^2,$

$V_7 = - 2H_2H_3e_{15}^{(2)}\gamma_N^{(1)}\mu_1(\varepsilon_{11}^{(1)})^3\varepsilon_{11}^{(2)}(e_{15}^{(2)}\varepsilon_{11}^{(1)} - 2e_{15}^{(1)}\varepsilon_{11}^{(2)}),$

$V_8 = 2H_2H_3(\varepsilon_{11}^{(1)})^2(\varepsilon_{11}^{(2)})^3(-(e_{15}^{(1)})^2\gamma_N^{(1)}\mu_1 + (\gamma_N^{(1)})^2(\mu_1)^2\varepsilon_{11}^{(1)} + (\gamma_N^{(2)})^2(\mu_2)^2\varepsilon_{11}^{(1)}),$

$V_9 = - 2H_1H_4e_{15}^{(2)}\gamma_N^{(2)}\mu_2(\varepsilon_{11}^{(1)})^2(\varepsilon_{11}^{(2)})^2(e_{15}^{(2)}\varepsilon_{11}^{(1)} - 2e_{15}^{(1)}\varepsilon_{11}^{(2)}),$

$V_{10} = 2H_1H_4\gamma_N^{(2)}\mu_2\varepsilon_{11}^{(1)}(\varepsilon_{11}^{(2)})^2(-(e_{15}^{(1)})^2(\varepsilon_{11}^{(2)})^2 + \gamma_N^{(1)}\mu_1\varepsilon_{11}^{(1)}((\varepsilon_{11}^{(1)})^2 + (\varepsilon_{11}^{(2)})^2)),$

$V_{11} = - 2H_1H_3e_{15}^{(2)}\gamma_N^{(2)}\mu_2(\varepsilon_{11}^{(1)})^3\varepsilon_{11}^{(2)}(e_{15}^{(2)}\varepsilon_{11}^{(1)} - 2e_{15}^{(1)}\varepsilon_{11}^{(2)}),$

$V_{12} = - 2H_1H_3\gamma_N^{(2)}\mu_2(\varepsilon_{11}^{(1)})^2(\varepsilon_{11}^{(2)})^3((e_{15}^{(1)})^2 - 2\gamma_N^{(1)}\mu_1\varepsilon_{11}^{(1)}),$

$V = V_1 + V_2 + V_3 + V_4 + V_5 + V_6 + V_7 + V_8 + V_9 + V_{10} + V_{11} + V_{12};$

$\alpha(s) = R/V,\ \beta(s) = S/V,\ \delta(s) = T/V,\ \gamma(s) = U/V,$

$\lim_{s\to\infty}\alpha(s) = \alpha_c,\ \lim_{s\to\infty}\beta(s) = \beta_c,\ \lim_{s\to\infty}\delta(s) = \delta_c.$

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Dynamic Behavior of Two Unequal Parallel Permeable Interface Cracks in a Piezoelectric Layer Bonded to Two Half Piezoelectric Materials Planes

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Abstract: The dynamic behavior of two unequal parallel permeable interface cracks in a piezoelectric layer bonded to two half_piezoelectric material planes subjected to harmonic anti_plane shear waves is investigated. By using the Fourier transform, the problem can be solved with the help of two pairs of dual integral equations in which the unknown variables were the jumps of the displacements across the crack surfaces. Numerical results are presented graphically to show the effects of the geometric parameters, the frequency of the incident wave on the dynamic stress intensity factors and the electric displacement intensity factors. Especially, the present problem can be returned to static problem of two parallel permeable interface cracks. Compared with the solutions of impermeable crack surface condition, it is found that the electric displacement intensity factors for the permeable crack surface conditions are much smaller.

Key words: interface crack; elastic wave; intensity factor; piezoelectric material; Schmidt method