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圆形三向网架非线性动力稳定性分析*

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(我刊编委叶开沅来稿)

摘要: 用拟板法将网架简化为平板, 给出表层应变与中面位移的非线性关系• 根据薄板的非线性动力学理论, 建立了在直角坐标系中三向网架的非线性动力学方程, 又将此方程转化为极坐标系轴对称非线性动力学方程• 在周边固定条件下, 引入异于等厚度板的无量纲量, 对基本方程无量纲化• 利用 Galerkin 法得到一个三次非线性振动方程, 在无外激励情况下, 讨论了稳定性与分岔问题• 在外激励情况下, 用 Melnikov 方法研究了圆形三向网架可能发生的混沌运动• 通过数字仿真绘出了发生混沌的相平面图•

关　键　词: 三向网架; 拟板法; 分岔; 混沌
中图分类号: O343.5; O326　　文献标识码: A

引　　言

网架结构由于具有刚度大、重量轻、受力合理、造价低等优点, 在宇航、航海、机械、建筑、石油化工等工程中, 其大、中、小跨度都得到了广泛应用, 引起了不少学者的高度重视• 当前, 对网架结构的研究, 非线性弯曲较多[1~4], 非线性振动、振动稳定性较少[5,6]• 对于网架结构的分岔和混沌研究, 作者尚未见报导, 而在其它领域研究工作较多[7,8]• 本文主要是研究三向网架非线性动力学中的分岔与混沌问题, 其结果可供工程技术人员设计时参考•

1　物理方程的建立

1.1　板的表层应变与中面位移的关系

设直角坐标系在板的中面上, u、v、w 为中面上一点 x、y、z 方向的位移• 由薄板非线性理论, 上、下表层的应变写成矩阵形式:

$$\{\varepsilon^{u}\} = \{\varepsilon\} - h_1\{\chi\},\quad \{\varepsilon^{l}\} = \{\varepsilon\} + h_2\{\chi\},\tag{1}$$

$$\{\varepsilon\} = \left[\frac{\partial u}{\partial x} + \frac{1}{2}\left(\frac{\partial w}{\partial x}\right)^2 \quad \frac{\partial v}{\partial y} + \frac{1}{2}\left(\frac{\partial w}{\partial y}\right)^2 \quad \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} + \frac{\partial w}{\partial x}\frac{\partial w}{\partial y}\right]^{\mathrm{T}},\tag{2}$$

$$\{\chi\} = \left[-\frac{\partial^2 w}{\partial x^2} \quad -\frac{\partial^2 w}{\partial y^2} \quad -2\frac{\partial^2 w}{\partial x\partial y}\right]^{\mathrm{T}},\tag{3}$$

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其中, h_1、h_2 分别为 xy 平面到上、下层的距离, 板的厚度 $h = h_1 + h_2$.

1.2 板的中面内力(图 1)

板的上、下层内力分别为:

$$\{N^u\} = [D^u]\{\varepsilon\} - h_1[D^u]\{\chi\},\ \{N^l\} = [D^l]\{\varepsilon\} + h_2[D^l]\{\chi\}; \tag{4}$$

中面上内力为:

$$\{N\} = \{N^u\} + \{N^l\}; \tag{5}$$

中面内弯矩为:

$$\{M\} = [S]\{\varepsilon\} + [B]\{\chi\}; \tag{6}$$

其中, $[S] = -h_1[D^u] + h_2[D^l]$, $[B] = h_1^2[D^u] + h_2^2[D^l]$, $[D^u]$、$[D^l]$ 是板上、下表层的刚度系数矩阵[9].

1.3 三向网架物理方程

$$\varepsilon_x = E_1\left(N_x - \frac{1}{3}N_y\right),\ \varepsilon_y = E_1\left(N_y - \frac{1}{3}N_x\right),\ \varepsilon_{xy} = \frac{8}{3}E_1N_{xy}, \tag{7}$$

$$\begin{cases} M_x = -\dfrac{9EI}{8a}\left(\dfrac{\partial^2 w}{\partial x^2} - \dfrac{1}{3}\dfrac{\partial^2 w}{\partial y^2}\right), \\ M_y = -\dfrac{9EI}{8a}\left(\dfrac{\partial^2 w}{\partial y^2} - \dfrac{1}{3}\dfrac{\partial^2 w}{\partial x^2}\right), \\ M_{xy} = -\dfrac{3EI}{4a}\dfrac{\partial^2 w}{\partial x\partial y}, \end{cases} \tag{8}$$

其中, a 为弦杆长度, A_1、A_2 分别为上、下弦杆的横截面面积, E 为弦杆的弹性模量.

$$E_1 = \frac{a}{E(A_1 + A_2)},\ I = \frac{A_1A_2}{A_1 + A_2}h^2.$$

2 控制方程

根据薄板非线性动力学理论, 其动力学方程为:

平衡方程:

$$\frac{\partial^2 M_x}{\partial x^2} + 2\frac{\partial^2 M_{xy}}{\partial x\partial y} + \frac{\partial^2 M_y}{\partial y^2} = -q + c\frac{\partial w}{\partial t} + \gamma\frac{\partial^2 w}{\partial t^2} - N_x\frac{\partial^2 w}{\partial x^2} - N_y\frac{\partial^2 w}{\partial y^2} - 2N_{xy}\frac{\partial^2 w}{\partial x\partial y}; \tag{9}$$

协调方程:

$$\frac{\partial^2 \varepsilon_x}{\partial y^2} + \frac{\partial^2 \varepsilon_y}{\partial x^2} - \frac{\partial^2 \varepsilon_{xy}}{\partial x\partial y} = \left(\frac{\partial^2 w}{\partial x\partial y}\right)^2 - \frac{\partial^2 w}{\partial x^2}\frac{\partial^2 w}{\partial y^2}; \tag{10}$$

这里 c 为阻尼系数, γ 为单位面积体的质量.

将方程(8)代入方程(9), 方程(7)代入方程(10), 转换为极坐标系下轴对称形式为:

$$\frac{9EI}{8a}\frac{1}{r}\frac{\partial}{\partial r}r\frac{\partial}{\partial r}\frac{1}{r}\frac{\partial}{\partial r}r\frac{\partial w}{\partial r} + c\frac{\partial w}{\partial t} + \gamma\frac{\partial^2 w}{\partial t^2} = q + \frac{1}{r}\frac{\partial}{\partial r}\left(rN_r\frac{\partial w}{\partial r}\right), \tag{11}$$

$$r\frac{\partial}{\partial r}\frac{1}{r}\frac{\partial}{\partial r}(r^2N_r) = -\frac{E(A_1 + A_2)}{2a}\left(\frac{\partial w}{\partial r}\right)^2. \tag{12}$$

将圆形三向网架(图 2)边缘近似看作圆形, 对于固定边界条件是:

$$r = R,\ w = \frac{\partial w}{\partial r} = 0,\ \frac{\partial}{\partial r}(rN_r) - \frac{1}{3}N_r = 0, \tag{13}$$

$$r = 0,\ w,\ \frac{\partial w}{\partial r},\ N_r\ \text{有限;} \tag{14}$$

初始条件取:

$$t = 0,\ w(r,0) = w_0,\ \frac{\partial w(r,0)}{\partial t} = 0. \tag{15}$$

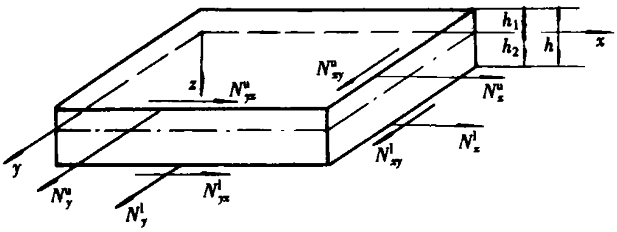

图 1　板的内力图

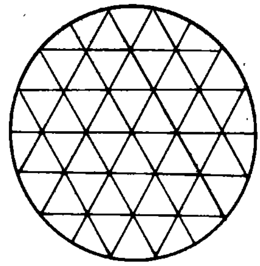

图 2　圆形三向网架

为了简化计算, 引入无量纲量:

$$\rho = \frac{r}{R},\ W = \frac{w}{\beta},\ S = \frac{8aR}{9EI}rN_r,\ Q = \frac{8aR^4q}{9E\beta^5},\ c_1 = \frac{8aR^4}{9EI}c,\ \gamma_1 = \frac{8aR^4}{9EI}\gamma,$$

则方程(11)、(12)分别化为:

$$\frac{1}{\rho}\frac{\partial}{\partial\rho}\rho\frac{\partial}{\partial\rho}\frac{1}{\rho}\frac{\partial}{\partial\rho}\rho\frac{\partial W}{\partial\rho} + c_1\frac{\partial W}{\partial t} + \gamma_1\frac{\partial^2 W}{\partial t^2} = Q + \frac{1}{\rho}\frac{\partial}{\partial\rho}\left(S\frac{\partial W}{\partial\rho}\right), \tag{16}$$

$$\rho\frac{\partial}{\partial\rho}\frac{1}{\rho}\frac{\partial}{\partial\rho}(\rho S) = -\beta_1\left(\frac{\partial W}{\partial\rho}\right)^2, \tag{17}$$

其中

$$I = \beta^4,\ \beta_1 = \frac{4(A_1 + A_2)}{9\beta^2}.$$

则边界条件(13)、(14)化为:

$$\rho = 1,\ W = \frac{\partial W}{\partial\rho} = 0,\ \frac{\partial S}{\partial\rho} - \frac{1}{3}S = 0, \tag{18}$$

$$\rho = 0,\ W,\ \frac{\partial W}{\partial\rho},\ S\ \text{有限;} \tag{19}$$

初始条件为:

$$t = 0,\ W = W_0,\ \frac{\partial W}{\partial t} = 0. \tag{20}$$

取

$$W = f(t)(1-\rho^2)^2, \tag{21}$$

由(17)和边界条件可得:

$$S = \frac{1}{3}\beta_1 f^2(t)(6\rho - 6\rho^3 + 4\rho^5 - \rho^7). \tag{22}$$

将(21)、(22)代入(16)用 Galerkin 法可得:

$$\frac{d^2 f(t)}{dt^2} + \omega^2 f(t) + c_1\frac{df(t)}{dt} + \alpha f^3(t) = g\cos(\Omega t), \tag{23}$$

其中

$$\omega^2 = \frac{320}{3\gamma_1},\ g = \frac{5G}{3\gamma_1},\ Q = G\cos(\Omega t),\ \alpha = \frac{200\beta_1}{21\gamma_1}.$$

取

$$\tau = \omega t,\ f(t) = \frac{\omega}{\sqrt{\alpha}}\eta(t),\ \alpha_1 = \frac{c_1}{\omega},\ g_1 = \frac{\sqrt{\alpha}}{\omega^3},$$

则方程(23)化为:

$$\frac{d^2\eta}{d\tau^2} + \eta + \alpha_1\frac{d\eta}{d\tau} + \eta^3 = g_1\cos\left(\frac{\Omega}{\omega}\tau\right)\cdot \tag{24}$$

3 稳定性分析

取 $g_1 = 0$, 则方程(24)等价系统为:

$$\eta_1' = \eta_2,\ \eta_2' = -\eta_1 - \eta_1^3 - \alpha_1\eta_2\cdot \tag{25}$$

此方程组平衡点为(0,0), Floquet 指数

$$\lambda = -\frac{\alpha_1}{2} \pm \sqrt{\left(\frac{\alpha_1}{2}\right)^2 - 1}\cdot$$

当 $\alpha_1 = 2$ 时, $\lambda_{1,2}$ 为相等的负数, 则平衡点为临界结点;

当 $\alpha_1 > 2$ 时, $\lambda_{1,2}$ 为不等的负数, 则平衡点为稳定的结点;

当 $0 < \alpha_1 < 2$ 时, λ 离开数轴在复平面上, 有稳定的焦点;

当 $\alpha_1 = 0$ 时, λ 为纯虚数, 解的曲线是极限环, 这时就发生 Hopf 分岔;

当 $\alpha_1 < 0$ 时, 失去物理意义, 这里不讨论·

方程(24)自由振动的解为:

$$\eta_1 = \sqrt{\frac{2b^2}{1-2b^2}}\,\mathrm{cn}\left(\frac{\tau}{\sqrt{1-2b^2}}\right), \tag{26}$$

$$\eta_2 = -\frac{\sqrt{2b^2}}{1-2b^2}\mathrm{sn}\left(\frac{\tau}{\sqrt{1-2b^2}}\right)\mathrm{dn}\left(\frac{\tau}{\sqrt{1-2b^2}}\right); \tag{27}$$

Hamilton 函数为:

$$H(\eta_1, \eta_2) = \frac{1}{2}(\eta_1^2 + \eta_2^2) + \frac{1}{4}\eta_1^4 = H, \tag{28}$$

b 满足:

$$H = \frac{2b^2}{1-2b^2} + \frac{2b^4}{(1-2b^2)^2}, \qquad b \in \left(0, \frac{\sqrt{2}}{2}\right);$$

Melnikov 函数为:

$$M(\tau) = \int_{-\infty}^{+\infty}\eta_2(\tau-\tau_0)\left[g_1\cos\left(\frac{\Omega}{\omega}\tau\right) - \alpha_1\eta_2(\tau-\tau_0)\right]d\tau =$$

$$-\alpha_1 I_\delta(m,n) + g_1 I_\lambda(m,n)\sin\left(\frac{\Omega}{\omega}\tau\right); \tag{29}$$

其中

$$I_\delta(m,n) = \frac{8n}{3\sqrt{1-2b^2}}\left[\frac{1-b^2}{1-2b^2}K(b) - E(b)\right],$$

$K(b)$、$E(b)$ 分别为第一、二类完全椭圆函数·

$$I_\lambda(m,n) = \begin{cases} 0, & n > 0 \text{ 或 } m \text{ 为偶数}, \\ \dfrac{2\sqrt{2}\,\omega_0\pi}{\mathrm{ch}(\omega_0 K'\sqrt{1-2b^2})}, & n = 1 \text{ 或 } m \text{ 为奇数}, \end{cases}$$

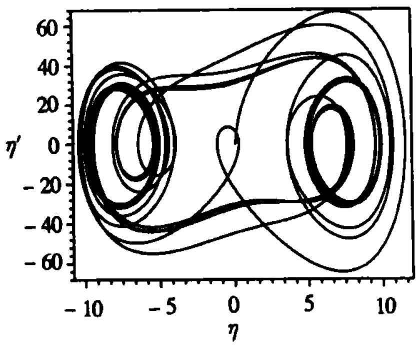

图 3　相图 $\alpha_1 = 1$, $g_1 = 500$

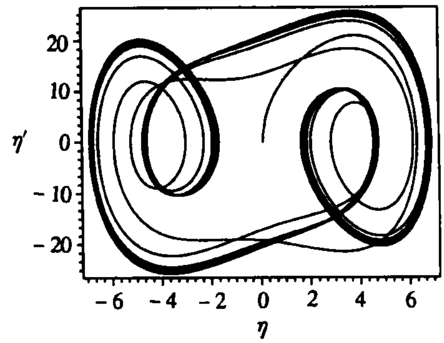

图 4　相图 $\alpha_1 = 1$, $g_1 = 100$

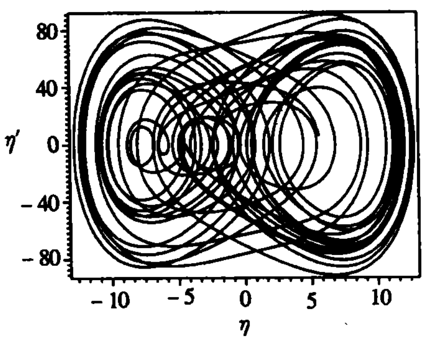

图 5　相图 $\alpha_1 = 0.01$, $g_1 = 500$

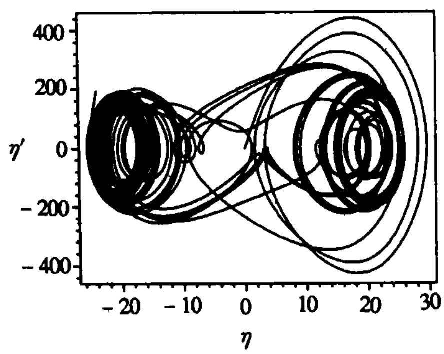

图 6　相图 $\alpha_1 = 1$, $g_1 = 10\,000$

其中

$$K' = \sqrt{1-b^2},\quad \omega_0 = \frac{\Omega}{\omega}.$$

当 $g_1 I_\lambda > \alpha_1 I_\delta$ 时, 存在同宿点, 可能发生混沌现象. 从数字仿真图 3 至图 6 可看出, 阻尼系数相当敏感, 阻尼越小, 外激励越大, 越容易发生混沌振动.

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Nonlinear Dynamical Stability Analysis of the Circular Three_Dimensional Frame

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Abstract: The three_dimensional frame is simplified into flat plate by the method of quasi_plate. The nonlinear relationships between the surface strain and the midst plane displacement are established. According to the thin plate nonlinear dynamical theory, the nonlinear dynamical equations of three-dimensional frame in the orthogonal coordinates system are obtained. Then the equations are translated into the axial symmetry nonlinear dynamical equations in the polar coordinates system. Some dimensionless quantities different from the plate of uniform thickness are introduced under the boundary conditions of fixed edges, then these fundamental equations are simplified with these dimensionless quantities. A cubic nonlinear vibration equation is obtained with the method of Galerkin. The stability and bifurcation of the circular three_dimensional frame are studied under the condition of without outer motivation. The contingent chaotic vibration of the three_dimensional frame is studied with the method of Melnikov. Some phase figures of contingent chaotic vibration are plotted with digital artificial method.

Key words: three_dimensional frame; quasi_plate method; bifurcation; chaos