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二重高阶对称破缺分歧点和它们的数值确定*

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摘　要

本文考虑带$Z_2\times Z_2$对称性的两参数非线性的二重高阶对称破缺分歧点。利用对称性，我们提出了相应的正则扩张系统来确定这类分歧点，同时指出存在两条平方音叉式分歧点的道路通过该点。

关键词　$Z_2\times Z_2$对称　二重高阶对称破缺分歧点　扩张系统

一、引　言

对称性是自然界中常见的一种性质，因而在自然科学的研究中考虑对称性是很自然的。求解具有对称性的非线性问题对我们来说是一个极具挑战性的任务。近年来，群论及其表示在处理这些涉及对称性的非线性问题时是极其有效的。利用这些工具，一个极其复杂的问题可以分解为一些简单的问题，而对这些简单问题已经知道如何求解或者很容易处理(见[5])。在数值逼近的范围内，系统地用群表示理论来计算带对称性的问题则是更为最近的事了。本文中我们考虑带$Z_2\times Z_2$对称非线性问题的二重高阶对称破缺分歧点的确定。

考虑带两个参数的非线性问题

$$f(x,\ \lambda,\ \mu)=0,\quad f:\ X\times R^2\to X \tag{1.1}$$

其中　X是Hilbert 空间，λ，μ是实参数，　f是$C^r(r\geqslant 3)$映照，它满足

$$\gamma f(x,\ \lambda,\ \mu)=f(\gamma x,\ \lambda,\ \mu)(\forall (x,\ \lambda,\ \mu)\in X\times R\times R,\ \gamma\in\Gamma) \tag{1.2}$$

这里Γ是$Z_2\times Z_2$在X上的表示，Γ的两个字母是α和β，$\Gamma=\{I,\ \alpha,\ \beta,\ \alpha\beta(=\beta\alpha)\}$。$\alpha$和$\beta$诱导出$X$和$X$的共轭空间$X'$的自然分解：

$$X=X_{ss}\oplus X_{sa}\oplus X_{as}\oplus X_{aa}, \tag{1.3a}$$

$$X'=X'_{ss}\oplus X'_{sa}\oplus X'_{as}\oplus X'_{aa} \tag{1.3b}$$

其中

$X_{ss}=\{x\in X:\ \alpha x=x,\ \beta x=x\}$，$X_{sa}=\{x\in X:\ \alpha x=x,\ \beta x=-x\}$，

$X_{as}=\{x\in X:\ \alpha x=-x,\ \beta x=x\}$，$X_{aa}=\{x\in X:\ \alpha x=-x,\ \beta x=-x\}$，

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$X'_{ss}=\{\psi\in X':\ \psi\alpha=\psi,\ \psi\beta=\psi\}$，$X'_{sa}=\{\psi\in X':\ \psi\alpha=\psi,\ \psi\beta=-\psi\}$，

$X'_{as}=\{\psi\in X':\ \psi\alpha=-\psi,\ \psi\beta=\psi\}$，$X'_{aa}=\{\psi\in X':\ \psi\alpha=-\psi,\ \psi\beta=-\psi\}$.

我们感兴趣的是(1.1)的奇异解，即此时$(x_0,\ \lambda_0,\ \mu_0)$满足

$$f(x_0,\ \lambda_0,\ \mu_0)=0, \tag{1.4a}$$

$$\dim N(f_x^0)=\operatorname{codim} R(f_x^0)\geqslant 1. \tag{1.4b}$$

这里 $N(f_x^0)$和$R(f_x^0)$分别表示$f_x(x_0,\ \lambda_0,\ \mu_0)$的零空间和域值.

众所周知，若$x_0\in X_{ss}$，那末f_x^0的一个零向量ϕ_0一定属于X_{ss}，X_{sa}，X_{as}，X_{aa}中的一个．当$\phi_0\in X_{ss}$时，这时通过奇异点的解分枝继续保持对称性，这种情况很容易处理．本文将考虑$\phi_0\notin X_{ss}$的情况，这时将发生对称破缺．

在上述假定下，很容易知道

$$\psi_0 x=0 \qquad (\forall x\in X_{ss}) \tag{1.5}$$

我们把λ看作分歧参数，μ看作辅助参数．下面先给出一些定义

定义1.1若

$$f(x_0,\ \lambda_0,\ \mu_0)=0 \qquad (x_0\in X_{ss}) \tag{1.6a}$$

$$N(f_x^0)=\operatorname{span}\{\phi_0\} \qquad (\phi_0\in X_i) \tag{1.6b}$$

$$R(f_x^0)=\{y\in X:\ \psi_0 y=0\} \qquad (\psi_0\in X'_i) \tag{1.6c}$$

$$b_\lambda:=\psi_0(f_{xx}^0 v_0+f_{x\lambda}^0)\phi_0\neq 0. \tag{1.6d}$$

其中v_0由

$$f_x^0 v_0+f_\lambda^0=0 \qquad (v_0\in X_{ss}) \tag{1.6e}$$

唯一确定，$X_i(i=1,\ 2,\ 3)$分别为子空间X_{sa}，X_{as}，X_{aa}，那末我们称$(x_0,\lambda_0,\ \mu_0)$是(1.1)关于λ的一个音叉式分歧点．

进一步，若

$$b_x:=\psi_0(f_{xxx}^0\phi_0\phi_0\phi_0+3f_{xx}^0\phi_0 v_x)\neq 0 \tag{1.7a}$$

其中v_x由

$$f_x^0 v_x+f_{xx}^0\phi_0\phi_0=0 \qquad (v_x\in X_{ss}) \tag{1.7b}$$

唯一确定，那末$(x_0,\ \lambda_0,\ \mu_0)$是(1.1)关于$\lambda$的一个平方音叉式分歧点.

定义1.2　若

$$f(x_0,\ \lambda_0,\ \mu_0)=0 \qquad (x_0\in X_{ss}) \tag{1.8a}$$

$$N(f_x^0)=\operatorname{span}\{\phi_1,\ \phi_2\} \qquad (\phi_1\in X_1,\ \phi_2\in X_2) \tag{1.8b}$$

$$R(f_x^0)=\{y\in X:\psi_1 y=0,\ \psi_2 y=0\} \qquad (\psi_1\in X'_1,\ \psi_2\in X'_2) \tag{1.8c}$$

$$\psi_1 x=0 \qquad (\forall x\in X_2);\ \psi_2 x=0 \qquad (\forall x\in X_1) \tag{1.8d}$$

$$A_i:=\psi_i(f_{xx}^0 v_0+f_{\lambda x}^0)\phi_i=0 \qquad (i=1,\ 2) \tag{1.8e}$$

$$\psi_i\phi_i\neq 0 \qquad (i=1,\ 2) \tag{1.8f}$$

则$(x_0,\ \lambda_0,\ \mu_0)$称为(1.1)关于$\lambda$的二重高阶对称破缺分歧点(DHSBP)．

带有对称性的非线性显示出极其丰富的分歧现象．Shearer[5]指出当辅助参数μ变化时，由扰动而产生的两次分歧中对称性所起的重要作用．另一方面，利用对称性，奇异性的余维数可大为减少，使问题得到简化．

下面引进两个在理论分析和实际计算中都十分重要的扩张系统

$$F^i(y,\mu)=\begin{pmatrix} f(x,\lambda,\mu) \\ f_x(x,\lambda,\mu)\phi \\ l_i\phi-1 \end{pmatrix}=0,\quad F^i: Y\times R\to Y:=X_{ss}\times X_i\times R \quad (i=1,2) \tag{1.9}$$

其中 $y=(x,\ \phi,\ \lambda)$，$y_0=(x_0,\ \phi_i,\ \lambda_0)$，$l_i\in X_i'$满足$l_i\phi_i=1$。

本文在第二节中对 DHSBP 附近的奇异点进行分歧分析，证明了二条平方音叉式分歧点通路通过该 DHSBP。在第三节中提出了计算 DHSBP 的正则扩张系统。最后在第四节中计算了几个数值例子。

二、分 歧 分 析

首先给出一个直接可从(1.2)推出的引理。

引理2.1 对于$x\in X_{ss}$，λ，$\mu\in R$，那末下列结论成立：

(a) f，f_λ，$f_{\lambda\lambda}\in X_{ss}$，

(b) X_{ss}，X_{sa}，X_{as}，X_{aa}是f_x，$f_{\lambda x}$，$f_{\mu x}$的不变子空间，

(c) $f_{xx}uv\in X_{ss}$ $(\forall u,\ v\in X_i)$

(d) $f_{xx}uv\in X_i$ $(\forall u\in X_{ss},\ v\in X_i)$

(f) $f_{xx}uv\in X_{aa}$ $(\forall u\in X_{sa},\ v\in X_{as})$；$f_{xx}uv\in X_{as}(\forall u\in X_{sa},\ v\in X_{aa})$；

$f_{xx}uv\in X_{sa}$ $(\forall u\in X_{as},\ v\in X_{aa})$

其中 $X_i(i=1,2,3)$在第一节中已定义。

引理2.2 假定$(x_0,\ \lambda_0,\ \mu_0)$是(1.1)关于$\lambda$的二重高阶对称破缺分歧点。

那末由(1.9)定义的映照$F^i(y,\mu)(i=1,2)$满足

$$N(F_y^0)=\mathrm{span}\{\Phi_i\},\quad R(F_y^0)=\{y\in Y:\Psi_i y=0\}$$

其中

$$F_y^0=F_y^i(y_0,\mu_0) \tag{2.1a}$$

$$\Phi_i=(v_0,w_i,1)^T,\quad \Psi_i=(\zeta_i,\psi_i,0) \tag{2.1b}$$

$$f_x^0 w_i+(f_{xx}^0 v_0+f_{x\lambda}^0)\phi_i=0,\quad w_i\in X_i,\quad l_i w_i=0 \tag{2.1c}$$

$$\zeta_i f_x^0+\psi_i f_{xx}^0\phi_i=0,\quad \zeta_i f_\lambda^0+\psi_i f_{x\lambda}^0\phi_i=0 \quad (\zeta_i\in X_{ss}') \tag{2.1d}$$

证明 考虑

$$F_y^i(y_0,\mu_0)w=0,\ w=(y_1,y_2,c_0)^T \quad (y_1\in X_{ss},y_2\in X_i,c_0\in R) \tag{2.2}$$

将(2.2)展开就是

$$f_x^0 y_1+c_0 f_\lambda^0=0 \tag{2.3a}$$

$$f_{xx}^0\phi_i y_1+f_x^0 y_2+c_0 f_{x\lambda}^0\phi_i=0 \tag{2.3b}$$

$$l_i y_2=0 \tag{2.3c}$$

从(2.3a)可知$y_1=c_0v_0$。将y_1代入(2.3b)，得到

$$c_0(f_{xx}^0\phi_i v_0+f_{x\lambda}^0\phi_i)+f_x^0 y_2=0 \tag{2.4}$$

由(2.4)和(2.3c)推断出 $y_2=c_0w_i$，其中w_i由(2.1c)给出。于是

$$N(F_y^i(y_0,\mu_0))=\mathrm{span}\{\Phi_i\},\quad \Phi_i=(v_0,w_i,1)^T.$$

展开

$$\xi F_y^i(y_0,\mu_0)=0,\ \xi=(\xi_1,\xi_2,c_1)\in X_{ss}'\times X_i'\times R \tag{2.5}$$

就是

$$\xi_1 f_x^0 + \xi_2 f_{xx}^0 \phi_i = 0 \tag{2.6a}$$

$$\xi_2 f_x^0 + c_1 l_i = 0 \tag{2.6b}$$

$$\xi_1 f_\lambda^0 + \xi_2 f_{x\lambda}^0 \phi_i = 0 \tag{2.6c}$$

将(2.6b)两边作用ϕ_i，推出$c_1=0$，于是$\xi_2=\beta_1\psi_i$。再将ξ_2代入(2.6a)和 (2.6c) 可直接得到$\xi_1=\beta_1\zeta_i$，其中$\zeta_i\in X'_{ss}$由(2.1d)给出，故引理的结论成立。

由引理2.2，可知$F_y^i(y_0,\ \mu_0)$限制在子空间$X_{ss}\times X_i\times R$上时有Φ_i张成的一维零空间．为了确保$(y_0,\ \mu_0)=(x_0,\ \phi_i,\ \lambda_0,\ \mu_0)$是$F^i(y,\mu)|_{X_{ss}\times X_i\times R^2}$关于$\mu$的简单折叠点，我们要假定下列条件满足

$$\Psi_i F_\mu^i(y_0,\mu_0)\neq 0 \qquad (i=1,2) \tag{2.7a}$$

它等价于

$$e_i:=\zeta_i f_\mu^0 + \psi_i f_{x\mu}^0 \phi_i \neq 0 \qquad (i=1,2) \tag{2.7b}$$

$$B_i:=\psi_i(f_{xx}^0 \phi_i u_0 + f_{x\mu}^0 \phi_i)\neq 0 \tag{2.7c}$$

其中u_0由

$$f_x^0 u_0 + f_\mu^0 = 0 \qquad (u_0\in X_{ss}) \tag{2.7d}$$

唯一确定

由引理2.1可知$f_{xx}^0 v_0^2+2f_{x\lambda}^0 v_0+f_{\lambda\lambda}^0\in X_{ss}$。于是存在唯一的$h_0\in X_{ss}$使得

$$f_x^0 h_0 + (f_{xx}^0 v_0^2 + 2f_{x\lambda}^0 v_0 + f_{\lambda\lambda}^0) = 0 \tag{2.8}$$

定理2.1 假定$(x_0,\ \lambda_0,\ \mu_0)$是(1.1)关于$\lambda$的二重高阶对称破缺分歧点，且(2.7)成立，那末当

$$E_i:=\psi_i(f_{xxx}^0 \phi_i v_0^2 + 2f_{xx}^0 w_i v_0 + 2f_{xx\lambda}^0 \phi_i v_0 + 2f_{x\lambda}^0 w_i + f_{x\lambda\lambda}^0 \phi_i + f_{xx}^0 \phi_i h_0)\neq 0 \tag{2.9}$$

时，$(y_0,\ \mu_0)=(x_0,\phi_i,\lambda_0,\mu_0)$是$F^i(y,\mu)|_{X_{ss}\times X_i\times R^2}=0$关于$\mu$的一个简单平方折叠点。

证明 令 $F_{yy}^0=F_{yy}^i(y_0,\mu_0)$，利用(2.8)和(2.1d)直接计算得：

$$\begin{aligned}\Psi_i F_{yy}^0(y_0,\mu_0)\Phi_i\Phi_i &= \zeta_i(f_{xx}^0 v_0^2 + 2f_{x\lambda}^0 + f_{\lambda\lambda}^0)\\ &\quad + \psi_i(f_{xxx}^0 \phi_i v_0^2 + 2f_{xx}^0 w_i v_0 + 2f_{xx\lambda}^0 \phi_i v_0 + 2f_{x\lambda}^0 w_i + f_{x\lambda\lambda}^0 \phi_i)\\ &= -\zeta_i f_x^0 h_0 + \psi_i(f_{xxx}^0 \phi_i v_0^2 + 2f_{xx}^0 w_i v_0 + 2f_{xx\lambda}^0 \phi_i v_0 + 2f_{x\lambda}^0 w_i + f_{x\lambda\lambda}^0 \phi_i)\\ &= \psi_i(f_{xxx}^0 \phi_i v_0^2 + 2f_{xx}^0 w_i v_0 + 2f_{xx\lambda}^0 \phi_i v_0 + 2f_{x\lambda}^0 w_i + f_{x\lambda\lambda}^0 \phi_i + f_{xx}^0 \phi_i h_0)\\ &= E_i \neq 0,\end{aligned}$$

由于假定(2.7)成立，可得定理的结论。

下面是定理2.1的一个明显推论。

推论2.1 假定$(x_0,\ \lambda_0,\ \mu_0)$是(1.1)关于$\lambda$的二重高阶对称破缺分歧点，同时(2.7)和(2.9)成立，那末$F^i(y,\mu)|_{X_{ss}\times X_i\times R^2}=0$仅存在一条解枝$L_i$通过$(y_0,\ \mu_0)$，且$L_i$在$(y_0,\mu_0)$处的切向量为$(\Phi_i,\ 0)$ $(i=1,2)$。

定理2.1在实际上和理论上都是重要的。它指出了通过计算(1.9)关于μ的简单二次折叠点枝来得到二重高阶对称破缺分歧点$(x_0,\ \lambda_0,\ \mu_0)$的过程。众所周知，再次应用扩张系统的方法，二次扩张系统

$$F_2^i(y,\Phi,\mu)=\begin{pmatrix} F^i(y,\mu)\\ F_y^i(y,\mu)\Phi\\ L_i\Phi-1\end{pmatrix}=0$$

在 DHSBP 处是正则的。

下面我们将假定

$$\Delta:=\det K\neq 0 \tag{2.10}$$

其中

$$K:=\begin{pmatrix} E_1 & e_1 \\ E_2 & e_2 \end{pmatrix}.$$

定理2.2　假定定理2.1的条件和(2.10)满足，且

$$D_i:=\psi_i(f^0_{xxx}\phi_i^3+3f^0_{xx}\phi_i z_i)\neq 0 \tag{2.11a}$$

其中z_i由

$$f^0_x z_i+f^0_{xx}\phi_i\phi_i=0 \quad (z_i\in X_{ss}) \tag{2.11b}$$

唯一决定。于是推论2.1中的解枝L_i除了(x_0, λ_0, μ_0)点外是原问题$f(x, \lambda, \mu)=0$关于λ的二次音叉式分歧点族。

证明　我们把证明分成两步：

(a). 首先证明在L_i上除了(x_0, λ_0, μ_0)外f_x的零特征值是单重的。

令　$(y(\varepsilon), \mu(\varepsilon))=(x(\varepsilon), \phi(\varepsilon), \lambda(\varepsilon), \mu(\varepsilon))\in l_i\subset X_{ss}\times X_i\times R^2$.

考虑如下扩张系统

$$M(m, \varepsilon)=\begin{pmatrix} f_x(\varepsilon)\theta(\varepsilon)-\beta(\varepsilon)\theta(\varepsilon) \\ l_i\theta(\varepsilon)-1 \end{pmatrix}=0,\quad M:X_j\times R^2\to X_j\times R \tag{2.12}$$

其中　$(i,j)=(1,2)$或者$(2,1)$，$f_x(\varepsilon)=f_x(x(\varepsilon), \lambda(\varepsilon), \mu(\varepsilon))$，$m=(\theta(\varepsilon), \beta(\varepsilon))\in X_j\times R$，$m_0=(\phi_j,0)$。$M(m_0,0)=0$，$M_m(m_0,0)$是正则的，由隐函数定理可知(2.12)存在唯一的解枝$m(\varepsilon)=(\theta(\varepsilon), \beta(\varepsilon))$使得$m(0)=m_0$。

由推论2.1知道　　$\dot{x}(0)=v_0, \dot{\phi}(0)=w_i, \dot{\lambda}(0)=1, \dot{\mu}(0)=0$。　　将

$f_x(\varepsilon)\theta(\varepsilon)-\beta(\varepsilon)\theta(\varepsilon)=0$　　关于ε在$\varepsilon=0$处求导可得

$$(f^0_{xx}v_0+f^0_{x\lambda})\phi_j+f^0_x\dot{\theta}(0)-\dot{\beta}(0)\phi_j=0. \tag{2.13}$$

将ϕ_j作用于(2.13)，由$A_j=0$可知 $\dot{\beta}(0)=0$，$\dot{\theta}(0)=w_j$。

将ϕ_j作用于$f_x(\varepsilon)\theta(\varepsilon)-\beta(\varepsilon)\theta(\varepsilon)=0$关于$\varepsilon$在$\varepsilon=0$处的二阶导数可得

$$\begin{aligned}&\psi_j(f^0_{xxx}\phi_j v_0^2+2f^0_{jx}w_j v_0+2f^0_{xx\lambda}\phi_j v_0+2f^0_{x\lambda}w_j+f^0_{x\lambda\lambda}\phi_j)\\&\quad+\psi_j(f^0_{xx}\ddot{x}(0)+f^0_{x\lambda}\ddot{\lambda}(0)+f^0_{x\mu}\ddot{\mu}(0))\phi_j-\ddot{\beta}(0)\psi_j\phi_j=0\end{aligned} \tag{2.14}$$

由(2.1d)和(2.7c)可进一步得到

$$\begin{aligned}&\psi_j(f^0_{xxx}\phi_j v_0^2+2f^0_{xx}w_j v_0+2f^0_{xx\lambda}\phi_j v_0+2f^0_{x\lambda}w_j+f^0_{x\lambda\lambda}\phi_j)\\&\quad-\zeta_j(f^0_x\ddot{x}(0)+f^0_\lambda\ddot{\lambda}(0)+f^0_\mu\ddot{\mu}(0))\phi_j+e_j\ddot{\mu}(0)-\ddot{\beta}(0)\psi_j\phi_j=0\end{aligned} \tag{2.15}$$

另一方面，将$\phi(\varepsilon)f(\varepsilon)=0$关于$\varepsilon$在$\varepsilon=0$处求导得到

$$\dot{\psi}(0)f^0_x+\psi_i(f^0_{xx}v_0+f^0_{x\lambda})=0. \tag{2.16}$$

将$\phi(\varepsilon)f(\varepsilon)=0$关于$\varepsilon$在$\varepsilon=0$处的二阶导数作用于$\phi_i$可得

$$\begin{aligned}&2\dot{\psi}(0)(f^0_{xx}v_0+f^0_{x\lambda})\phi_i+\psi_i(f^0_{xxx}\phi_i v_0^2+2f^0_{xx\lambda}\phi_i v_0+f^0_{x\lambda\lambda}\phi_i)\\&\quad+\psi_i(f^0_{xx}\ddot{x}(0)+f^0_{x\lambda}\ddot{\lambda}(0)+f^0_{x\mu}\ddot{\mu}(0))\phi_i=0\end{aligned} \tag{2.17}$$

由(2.1d)，(2.7c)，(2.16)，(2.17)和w_i的定义进一步可得

$$\begin{aligned}&\psi_i(f^0_{xxx}\phi_i v_0^2+2f^0_{xx}w_i v_0+2f^0_{xx\lambda}\phi_i v_0+2f^0_{x\lambda}w_i+f^0_{x\lambda\lambda}\phi_i)\\&\quad-\zeta_i(f^0_x\ddot{x}(0)+f^0_\lambda\ddot{\lambda}(0)+f^0_\mu\ddot{\mu}(0))+e_i\ddot{\mu}(0)=0\end{aligned} \tag{2.18}$$

$f(x(\varepsilon), \lambda(\varepsilon), \mu(\varepsilon))=0$关于$\varepsilon$在$\varepsilon=0$处求二阶导数可得

$$f^0_{xx}v_0^2+2f^0_{x\lambda}v_0+f^0_{\lambda\lambda}+f^0_x\ddot{x}(0)+f^0_\lambda\ddot{\lambda}(0)+f^0_\mu\ddot{\mu}(0)=0 \tag{2.19}$$

由h_0的定义，将ζ_i作用于(2.19)，可推出

$$\zeta_i(f^0_{xx}v_0^2+2f^0_{x\lambda}+f^0_{\lambda\lambda})+\zeta_i(f^0_x\ddot{x}(0)+f^0_\lambda\ddot{\lambda}(0)+f^0_\mu\ddot{\mu}(0))=0 \tag{2.20}$$

$$\psi_i f^0_{xx}h_0\phi_i+\zeta_i(f^0_x\ddot{x}(0)+f^0_\lambda\ddot{\lambda}(0)+f^0_\mu\ddot{\mu}(0))=0 \tag{2.21}$$

将(2.18)和(2.19)相加，即得

$$E_i+e_i\ddot{\mu}(0)=0 \tag{2.22}$$

于是

$$\ddot{\mu}(0)=-\frac{E_i}{e_i} \tag{2.23}$$

将ζ_j作用于(2.19)可得

$$\psi_j f^0_{xx}h_0\phi_j+\zeta_j(f^0_x\ddot{x}(0)+f^0_\lambda\ddot{\lambda}(0)+f^0_\mu\ddot{\mu}(0))=0 \tag{2.24}$$

将(2.15)和(2.24)相加可得

$$E_j+e_j\ddot{\mu}(0)-\ddot{\beta}(0)\psi_j\phi_j=0 \tag{2.25}$$

将(2.23)代入上式推出

$$\ddot{\beta}(0)=-\frac{\det\begin{pmatrix}E_i & e_i\\ E_j & e_j\end{pmatrix}}{\psi_j\phi_j e_i}.$$

由假定，$\ddot{\beta}(0)\neq 0$，于是对于不等于0的小ε，$\beta(\varepsilon)\neq 0$，注意$f_x(0)$有$n-2$个非零特征值。由I的定义可知，$f_x(\varepsilon)$的零特征值是单重的。

(b)。令$\phi(\varepsilon)$，$\phi(\varepsilon)$分别是$f_x(\varepsilon)$在左，右特征向量，即

$$\psi(\varepsilon)f_x(\varepsilon)=0,\ \psi(\varepsilon)\in X_i',\ \psi(0)=\psi_i,$$

$$f_x(\varepsilon)\phi(\varepsilon)=0,\ \phi(\varepsilon)\in X_i,\ \phi(0)=\phi_i.$$

为了证明$(x(\varepsilon),\ \lambda(\varepsilon),\ \mu(\varepsilon))\in L_i$是(1.1)关于$\lambda$的二次音叉式分歧点，只要验证对于小的$\varepsilon\neq 0$

$$b_\lambda(\varepsilon):=\psi(\varepsilon)(f_{xx}(\varepsilon)v_\lambda(\varepsilon)+f_{x\lambda}(\varepsilon))\phi(\varepsilon)\neq 0 \tag{2.26}$$

$$b_x(\varepsilon):=\psi(\varepsilon)(f_{xxx}(\varepsilon)\phi(\varepsilon)^3+3f_{xx}(\varepsilon)v_x(\varepsilon)\phi(\varepsilon))\neq 0 \tag{2.27}$$

其中$v_\lambda(\varepsilon)$，$v_x(\varepsilon)$由

$$f_x(\varepsilon)v_\lambda(\varepsilon)+f_\lambda(\varepsilon)=0,\ v_\lambda(\varepsilon)\in X_{ss},\ v_\lambda(0)=v_0 \tag{2.28a}$$

$$f_x(\varepsilon)v_x(\varepsilon)+f_{xx}(\varepsilon)\phi(\varepsilon)^2=0,\ v_x(\varepsilon)\in X_{ss},\ v_x(0)=z_i \tag{2.28b}$$

唯一决定。注意到$D_i\neq 0$，(2.27)显然满足，下面证明(2.26)。

由于$\dot{x}(0)=v_0$，$\dot{\phi}(0)=w_i$，$\dot{\lambda}(0)=1$，$\dot{\mu}(0)=0$，可知

$$\begin{aligned}b_\lambda(0)=\dot{\psi}(0)(f^0_{xx}\phi_i v_0+f^0_{x\lambda}\phi_i)+\psi_i(f^0_{xxx}\phi_i v_0^2+2f^0_{xx\lambda}\phi_i v_0\\+f^0_{x\lambda\lambda}\phi_i+f^0_{xx}w_i v_0+f^0_{xx}\phi_i\dot{v}_\lambda(0)+f^0_{x\lambda}w_i).\end{aligned} \tag{2.29}$$

$$\begin{aligned}b_\lambda(0)=-\dot{\psi}(0)f^0_x w_i+\psi_i(f^0_{xxx}\phi_i v_0^2+2f^0_{xx\lambda}\phi_i v_0+f^0_{x\lambda\lambda}\phi_i\\+f^0_{xx}w_i v_0+f^0_{xx}\phi_i\dot{v}_\lambda(0)+f^0_{x\lambda}w_i)\end{aligned} \tag{2.30}$$

将$\psi(\varepsilon)f(\varepsilon)=0$关于$\varepsilon$在$\varepsilon=0$处求导，可得到

$$\dot{\psi}(0)f^0_x+\psi_i(f^0_{xx}v_0+f^0_{x\lambda})=0 \tag{2.31}$$

于是

$$\begin{aligned}b_\lambda(0)=\psi_i(f^0_{xx}w_i v_0+f^0_{x\lambda}w_i)+\psi_i(f^0_{xxx}\phi_i v_0^2+2f^0_{xx\lambda}\phi_i v_0+f^0_{x\lambda\lambda}\phi_i\\+f^0_{xx}w_i v_0+f^0_{xx}\phi_i\dot{v}_\lambda(0)+f^0_{x\lambda}w_i)\end{aligned} \tag{2.32}$$

将(2.28a)式关于ε在$\varepsilon=0$处求导，可推出

$$f^0_{xx}v_0^2+2f^0_{x\lambda}v_0+f^0_{\lambda\lambda}+f^0_x\dot{v}_\lambda(0)=0 \tag{2.33}$$

从而$\dot{v}_\lambda(0)=h_0$，将它代入(2.32)即得$b_\lambda(0)=E_i\neq0$，再由$b_\lambda(0)=0$可知对不等于0的小ε，(2.26)成立。

三、正 则 扩 张 系 统

我们引进下列扩张系统，它们在 DHSBP 处的正则性保证了通常的牛顿方法可用来计算二重高阶对称破缺分歧点。假定(1.1)存在关于λ的二重高阶对称破缺分歧点(x_0, λ_0, μ_0)，其零空间由$\phi_1\in X_{sa}$，$\phi_2\in X_{as}$张成。

头二个扩张系统是由二次扩张系统$F_2^i=0(i=1, 2)$展开而成：

$$H_i(y)=\begin{pmatrix} f(x,\lambda,\mu) \\ f_x\phi \\ f_xv+f_\lambda \\ f_xw+f_{xx}\phi v+f_{x\lambda}\phi \\ l_i\phi-1 \\ l_iw \end{pmatrix}=0,\quad \begin{aligned}&H_i:Y\to Y:=X_{ss}\times X_i\times X_i\times R^2,\\ &y=(x,\phi,v,w,\lambda,\mu),\\ &y_0=(x_0,\phi_i,v_0,w_i,\lambda_0,\mu_0)\qquad (i=1,2)\end{aligned} \tag{3.1}$$

它们的正则性可由二次扩张系统$F=0$的正则性直接推出。于是我们有

定理3.1 假定(x_0, λ_0, μ_0)是(1.1)关于λ的二重高阶对称破缺分歧点，条件(2.7c)和(2.9)满足，那末$H_i(y)=0$有正则孤立解$(x_0, \phi_i, v_0, w_i, \lambda_0, \mu_0)$，$(i=1,2)$。

带二个开折参数α，β的第三个扩张系统能一下子计算出二个零向量ϕ_1，ϕ_2来：

$$H(y)=\begin{pmatrix} f(x,\lambda,\mu) \\ f_x\phi_{sa} \\ f_x\phi_{as}+\alpha\phi_{as} \\ f_xv+f_\lambda \\ f_xw_{sa}+f_{xx}\phi_{sa}v+f_{x\lambda}\phi_{sa} \\ f_xw_{as}+f_{xx}\phi_{as}v+f_{x\lambda}\phi_{as}+\beta\phi_{as} \\ l_1\phi_{sa}-1 \\ l_2\phi_{as}-1 \\ l_1w_{sa} \\ l_2w_{as} \end{pmatrix}=0,\quad \begin{aligned}&H:Y\to Y:=X_{ss}\times X_1\times X_2\times X_{ss}\\ &\qquad\times X_1\times X_2\times R^4,\\ &y=(x,\phi_{sa},\phi_{as},v,w_{sa},w_{as},\lambda,\mu,\alpha,\beta),\\ &y_0=(x_0,\phi_1,\phi_2,v_0,w_1,w_2,\lambda_0,\mu_0,0,0).\end{aligned} \tag{3.2}$$

它的正则性由下列定理描述：

定理3.2 假定(x_0, λ_0, μ_0)是(1.1)关于λ的二重高阶对称破缺分歧点，条件(2.7c)和(2.9)满足，那末$H(y)=0$有正则孤立解$(x_0, \phi_1, \phi_2, v_0, w_1, w_2, \lambda_0, \mu_0, 0, 0)$。

证明 考虑

$$D_yH(y_0)w=z \tag{3.3}$$

其中 $D_yH(y_0)$表示$H(y)$在$y=y_0$处的 Jacobian 矩阵，

$z=(z_1,z_2,z_3,z_4,z_5,z_6,\theta_1,\theta_2,\theta_3,\theta_4)\in Y=X_{ss}\times X_{sa}\times X_{as}\times X_{ss}\times X_{sa}\times X_{as}\times R^4$，

$w=(y_1, y_2, y_3, y_4, y_5, y_6, \lambda, \mu, \alpha, \beta)$。

将(3.3)展开出来就是

$$f_x^0y_1+\lambda f_\lambda^0+\mu f_\mu^0=z_1 \tag{3.4a}$$

$$f_{xx}^{0}y_1\phi_1+f_x^0y_2+\lambda f_{x\lambda}^{0}\phi_1+\mu f_x^0\mu\phi_1=z_2 \tag{3.4b}$$

$$f_{xx}^{0}y_1\phi_2+f_x^0y_3+\lambda f_{x\lambda}^{0}\phi_2+\mu f_x^0\mu\phi_2+\alpha\phi_2=z_3 \tag{3.4c}$$

$$(f_{xx}^{0}v_0+f_{x\lambda}^{0})y_1+f_x^0y_4+\lambda(f_{x\lambda}^{0}v_0+f_{\lambda\lambda}^{0})+\mu(f_{x\mu}^{0}v_0+f_{\lambda\mu}^{0})=z_4 \tag{3.4d}$$

$$(f_{xx}^{0}w_1+f_{xxx}^{0}\phi_1v_0+f_{xx\lambda}^{0}\phi_1)y_1+(f_{xx}^{0}v_0+f_{x\lambda}^{0})y_2+f_{xx}^{0}\phi_1y_4+f_x^0y_5$$
$$+\lambda(f_{x\lambda}^{0}w_1+f_{xx\lambda}^{0}\phi_1v_0+f_{x\lambda\lambda}^{0}\phi_1)+\mu(f_{x\mu}^{0}w_1+f_{xx}^{0}\mu\phi_1v_0+f_{x\lambda\mu}^{0}\phi_1)=z_5 \tag{3.4e}$$

$$(f_{xx}^{0}w_2+f_{xxx}^{0}\phi_2v_0+f_{xx\lambda}^{0}\phi_2)y_1+(f_{xx}^{0}v_0+f_{x\lambda}^{0})y_3+f_{xx}^{0}\phi_2y_4+f_x^0y_6$$
$$+\lambda(f_{x\lambda}^{0}w_2+f_{xx\lambda}^{0}\phi_2v_0+f_{x\lambda\lambda}^{0}\phi_2)+\mu(f_{x\mu}^{0}w_2+f_{xx\mu}^{0}\phi_2v_0+f_{x\lambda\mu}^{0}\phi_2)+\beta\phi_2=z_6 \tag{3.4f}$$

$$l_1y_2=\theta_1,\quad l_2y_3=\theta_2,\quad l_1y_5=\theta_3,\quad l_2y_6=\theta_4 \tag{3.4g}$$

由(3.4a)，可解出$y_1=\lambda v_0+\mu u_0+\bar{y}_1$，$\bar{y}_1\in X_{ss}$由$f_x^0\bar{y}_1=z_1$唯一决定.

将y_1代入(3.4b)推出

$$\lambda(f_{xx}^{0}\phi_1v_0+f_{x\lambda}^{0}\phi_1)+\mu(f_{xx}^{0}\phi_1u_0+f_{x\mu}^{0}\phi_1)+f_{x\mu}^{0}\phi_1)+f_x^0y_2=\bar{z}_2 \tag{3.5}$$

其中$\bar{z}_2=z_2-f_{xx}^{0}\phi_1\bar{y}_1$.

将ϕ_1作用于(3.5)，由$A_1=0$，$B_1\neq0$可求出$\mu=\dfrac{\psi_1\bar{z}_2}{B_1}$，于是$y_2=B_1\phi_1+\lambda w_1+\bar{y}_2$，

其中 $\bar{y}_2$由$f_x^0\bar{y}_2=\bar{z}_2-\mu(f_{xx}^{0}\phi_1u_0+f_{x\mu}^{0}\phi_1)$，$l_1\bar{y}_2=0$唯一决定. 将$y_1$，$\mu$代入(3.4c)导出

$$f_x^0y_3+\lambda(f_{xx}^{0}\phi_2v_0+f_{x\lambda}^{0}\phi_1)+\alpha\phi_2=\bar{z}_3 \tag{3.6}$$

其中 $\bar{z}_3=z_3-f_{xx}^{0}\phi_2(\mu u_0+\bar{y}_1)-\mu f_{x\mu}^{0}\phi_2$.

将ϕ_2作用于(3.6)，推出$\alpha=\dfrac{\psi_2\bar{z}_3}{\psi_2\phi_2}$. 于是$y_3=\beta_2\phi_2+\lambda w_2+\bar{y}_3$，$\bar{y}_3$由$f_x^0\bar{y}_3=\bar{z}_3-\alpha\phi_2$，$l_2\bar{y}_3=0$

唯一决定. 将y_1，μ代入(3.4d)导出

$$f_x^0y_4+\lambda(f_{xx}^{0}v_0^2+2f_{x\lambda}^{0}v_0+f_{\lambda\lambda}^{0})=\bar{z}_4 \tag{3.7}$$

其中 $\bar{z}_4=z_4-(f_{xx}^{0}v_0+f_{x\lambda}^{0})(\mu u_0+\bar{y}_1)-\mu(f_{x\mu}^{0}v_0+f_{\lambda\mu}^{0})$.

由(3.7)可解出$y_4=\lambda h_0+\bar{y}_4$，$\bar{y}_4$由$f_x^0\bar{y}_4=\bar{z}_4$，$\bar{y}_4\in X_{ss}$唯一决定. 将$y_1$，$y_2$，$y_4$和$\mu$代入(3.4e)推出

$$\lambda(f_{xxx}^{0}\phi_1v_0^2+2f_{xx\lambda}^{0}\phi_1v_0+2f_{xx}^{0}w_1v_0+2f_{x\lambda}^{0}w_1+f_{x\lambda\lambda}^{0}\phi_1+f_{xx}^{0}\phi_1h_0)$$
$$+\beta_1(f_{xx}^{0}v_0+f_{x\lambda}^{0})\phi_1+f_x^0y_5=\bar{z}_5 \tag{3.8}$$

其中

$$\bar{z}_5=z_5-(f_{xx}^{0}w_1+f_{xxx}^{0}\phi_1v_0+f_{xx\lambda}^{0}\phi_1)(\mu u_0+\bar{y}_1)-(f_{xx}^{0}v_0+f_{x\lambda}^{0})\bar{y}_2$$
$$-f_{xx}^{0}\phi_1\bar{y}_4-\mu(f_{x\mu}^{0}w_1+f_{xx\mu}^{0}\phi_1v_0+f_{x\lambda\mu}^{0}\phi_1).$$

将ϕ_1作用于(3.8)，可求得$\lambda=\dfrac{\psi_1\bar{z}_5}{E_1}$，于是$y_5=\beta_3\phi_1+\beta_1w_1+\bar{y}_5$由$f_x^0\bar{y}_5=\bar{z}_5-\lambda(f_{xxx}^{0}\phi_1v_0^2$

$+2f_{xx\lambda}^{0}\phi_1v_0+2f_{xx}^{0}w_1v_0+2f_{x\lambda}^{0}w_1+f_{x\lambda\lambda}^{0}\phi_1+f_{xx}^{0}\phi_1h_0)$，$l_1\bar{y}_5=0$唯一决定.

由(3.4g)可知$\beta_1=\theta_1$，$\beta_2=\theta_2$，$\beta_3=\theta_3$，从而已经完全解出了y_1，y_2，y_3，y_4，y_5，λ，μ，α. 剩下的是要解出y_6和β. 将y_1，y_3，y_4，λ，μ代入(3.4f)，可知y_6和β满足

$$f_x^0y_6+\beta\phi_2=\bar{z}_6 \tag{3.9}$$

其中

$$\bar{z}_6=z_6-(f_{xx}^{0}w_2+f_{xxx}^{0}\phi_2v_0+f_{xx\lambda}^{0}\phi_2)y_1-(f_{xx}^{0}v_0+f_{x\lambda}^{0})y_3+f_{xx}^{0}\phi_2y_4$$
$$-\lambda(f_{x\lambda}^{0}w_2+f_{xx\lambda}^{0}\phi_2v_0+f_{x\lambda\lambda}^{0}\phi_2)-\mu(f_{x\mu}^{0}w_2+f_{xx\mu}^{0}\phi_2v_0+f_{x\lambda\mu}^{0}\phi_2).$$

将ϕ_2作用于(3.9)可解得$\beta=\frac{\psi_2 z_6}{\psi_2\phi_2}$，于是$y_6=\beta_4\phi_2+\bar{y}_6$，$\bar{y}_6$由$f_x^0\bar{y}_6=z_6-\beta\phi_2$，$l_2\bar{y}_6=0$唯一决定。由$l_2y_6=\theta_4$进一步可得$\beta_4=\theta_4$。至此，对给定的$z$，我们可以唯一求出(3.3)的解$w$。同时容易验证当$z=0$时，(3.3)的解$w=0$。应用开映照定理，可知$H(y)=0$存在孤立解。

四、数 值 例 子

本文给出二个数值例子，表明用扩张系统(3.2)来计算二重高阶对称破缺分歧点是十分有效的。

例4.1 考虑在R^4中的非线性方程组

$$f(x,\lambda,\mu)=\begin{pmatrix}2x_1-x_1\exp[x_1]+(-\lambda^2+\lambda)x_3+\mu x_2+2x_4\\2x_2-x_2\exp[x_2]+(-\lambda^2+\lambda)x_4+\mu x_1+2x_3\\2x_3-x_3\exp[x_3]+(-\lambda^2+\lambda)x_1+\mu x_4+2x_2\\2x_4-x_4\exp[x_4]+(-\lambda^2+\lambda)x_2+\mu x_3+2x_1\end{pmatrix}=0 \qquad (4.1)$$

其中 $x=(x_1,\ x_2,\ x_3,\ x_4)$。容易验证$f(x,\ \lambda,\ \mu)$满足$Z_2\times Z_2$对称性条件，其母元

$$\alpha=\begin{pmatrix}0&1&0&0\\1&0&0&0\\0&0&0&1\\0&0&1&0\end{pmatrix},\quad \beta=\begin{pmatrix}0&0&0&1\\0&0&1&0\\0&1&0&0\\1&0&0&0\end{pmatrix}.$$

于是

$$X_{ss}=\{(v,v,v,v):v\in R\},\ X_{sa}=\{(v,v-v,-v):v\in R\},$$
$$X_{as}=\{(v,-v,-v,v):v\in R\},X_{aa}=\{(v,-v,v,-v):v\in R\}.$$

令

$$x=(x_1,x_1,x_1,x_1)\in X_{ss},\ \ v=(v_1,v_1,v_1,v_1)\in X_{ss},$$
$$\phi_1=(s_1,s_1,-s_1,-s_1)\in X_{sa},\phi_2=(s_2,-s_2,-s_2,s_2)\in X_{as},$$
$$w_1=(u_1,u_1,-u_1,-u_1)\in X_{sa},\ \ w_2=(u_2,-u_2,\ -u_2,\ u_2)\in X_{as}.$$

相应的扩张系统(3.2)是

$$\left.\begin{aligned}&4x_1-x_1\exp[x_1]+(-\lambda^2+\lambda)x_1+\mu x_1=0\\&(-\exp[x_1]-x_1\exp[x_1]+\mu+\lambda^2-2\lambda)s_1=0\\&(4-\exp[x_1]-x_1\exp[x_1]-\mu+\lambda^2-2\lambda)s_2+\alpha s_2=0\\&(4-\exp[x_1]-x_1\exp[x_1]+\mu-\lambda^2+2\lambda)v_1-1+\alpha v_1+(-2\lambda+2)x_1=0\\&(-\exp[x_1]-x_1\exp[x_1]+\mu+\lambda^2-2\lambda)u_1+(-2\exp[x_1]\\&\quad-x_1\exp[x_1])v_1+2\lambda-2=0\\&(4-\exp[x_1]-x_1\exp[x_1]-\mu+\lambda^2-2\lambda)u_2+(-2\exp[x_1]\\&\quad-x_1\exp[x_1])v_1+2\lambda-2+\beta s_2=0\\&s_1=1,\ \ s_2=1,u_1=0,\ \ u_2=0\end{aligned}\right\}$$

用牛顿方法，我们找到该问题的一个二重高阶对称破缺分歧点$(x_0,\ \lambda_0,\ \mu_0)=(0,\ 0,\ 0,\ 0,\ 1.0,\ 2.0)$。数值结果由表1所示：

表 1

代选次数	x_1	λ	μ	α	β	$\|\delta y\|$
0	−0.5	0.5	2.5	−0.5	−0.5	
1	−0.055427	−0.787731	1.745467	−0.509064	0.000000	0.754533
2	0.003173	0.992562	2.043348	0.086700	0.000000	0.595764
3	0.000021	1.000009	2.000079	0.000161	0.000000	0.865390E−01
4	0.000000	1.000000	1.999999	−0.000000	0.000000	0.953674E−06
5	0.000000	1.000000	2.000000	0.000000	0.000000	0.000000E+00

例4.2 考虑下列两阶常微分方程组

$$\left.\begin{aligned}&\frac{d^2x_1}{dt^2}+(\lambda^2+\mu)x_1+x_1^2+\mu(x_2-2x_3+x_4)=0\\&\frac{d^2x_2}{dt^2}+(\lambda^2+\mu)x_2+x_2^2+\mu(x_1-2x_4+x_3)=0\\&\frac{d^2x_3}{dt^2}+(\lambda^2+\mu)x_3+x_3^2+\mu(x_4-2x_1+x_2)=0\\&\frac{d^2x_4}{dt^2}+(\lambda^2+\mu)x_4+x_4^2+\mu(x_3-2x_2+x_1)=0\end{aligned}\right\}\tag{4.2a}$$

边界条件为

$$x_i(0)=x_i(1)=0\qquad(i=1,2,3,4)\tag{4.2b}$$

令

$$X=\{(x_1(t),x_2(t),x_3(t),x_4(t))\,|\,x_i(t)\in C^2(0,1),x_i(0)=x_i(1)=0\}$$

容易验证(4.2)满足$Z_2\times Z_2$对称条件(1.2)，其母元α，β为

$$\alpha x(t):=\alpha(x_1(t),x_2(t),x_3(t),x_4(t))=(x_2(t),x_1(t),x_4(t),x_3(t)),$$
$$\beta x(t):=\beta(x_1(t),x_2(t),x_3(t),x_4(t))=(x_4(t),x_3(t),x_2(t),x_1(t)).$$

于是

$$X_{ss}=\{(x_1(t),x_1(t),x_1(t),x_1(t))\},$$
$$X_{sa}=\{(x_1(t),x_1(t),-x_1(t),-x_1(t))\},$$
$$X_{as}=\{(x_1(t),-x_1(t),-x_1(t),x_1(t))\},$$
$$X_{aa}=\{(x_1(t),\ -x_1(t),x_1(t),-x_1(t))\}.$$

再令

$$x(t)=(x_1(t),x_1(t),x_1(t),x_1(t))\in X_{ss},$$
$$\Phi_1(t)=(\phi_1(t),\phi_1(t),-\phi_1(t),-\phi_1(t))\in X_{sa},$$
$$\Phi_2(t)=(\phi_2(t),\ -\phi_2(t),\ -\phi_2(t),\phi_2(t))\in X_{as},$$
$$v(t)=(v_1(t),v_1(t),v_1(t),v_1(t))\in X_{ss},$$
$$W_1(t)=(w_1(t),\ w_1(t),-w_1(t),-w_1(t))\in X_{sa},$$
$$W_2(t)=(w_2(t),-w_2(t),-w_2(t),w_2(t))\in X_{as}.$$

于是相应的扩张系统为

$$\left.\begin{aligned}
&\frac{d^2x_1}{dt^2}+(\lambda^2+\mu+x_1)x_1=0\\
&\frac{d^2\phi_1}{dt^2}+(\lambda^2+3\mu+2x_1)\phi_1=0\\
&\frac{d^2\phi_2}{dt^2}+(\lambda^2+3\mu+2x_1)\phi_2+\alpha\phi_2=0\\
&\frac{d^2v_1}{dt^2}+(\lambda^2+\mu+2x_1)v_1+2\lambda x_1=0\\
&\frac{d^2w_1}{dt^2}+(\lambda^2+3\mu+2x_1)w_1+2\lambda\phi_1+2\phi_1v_1=0\\
&\frac{d^2w_2}{dt^2}+(\lambda^2+3\mu+2x_1)w_2+2\lambda\phi_2+2\phi_2v_1+\beta\phi_2=0\\
&l_1\phi_1-1=0,\quad l_2\phi_2-1=0\\
&l_1w_1=0,\quad l_2w_2=0
\end{aligned}\right\}\tag{4.3a}$$

边界条件为

$$x_1(0)=x_1(1)=\phi_i(0)=\phi_i(1)=v_1(0)=v_1(1)=w_i(0)=w_i(1)=0\tag{4.3b}$$

对(4.3)进行中心差分逼近，步长h取为1/20，用牛顿方法我们可以找到该问题的二重高阶对称破缺分歧点$(x_0,\lambda_0,\mu_0)=(0,\ 0,\ 3.283109)$。

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Double High Order S-Breaking Bifurcation Points and Their Numerical Determination

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Abstract

We consider double high order S-breaking bifurcation points of two-parameter dependent nonlinear problems with $Z_2 \times Z_2$-symmetry. Because of the underlying symmetry, we could propose some regular extended systems to determine double high order S-breaking bifurcation points, and we could also show that there exist two quadratic pitchfork bifurcation point paths passing through the point being considered.

Key words $Z_2 \times Z_2$-symmetry, double high order S-breaking bifurcation point, extended system