

扁锥面网壳非线性动力分岔与混沌运动^{*}

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摘要: 对曲面为正三角形网格的 3 向扁锥面单层网壳, 用拟壳法建立了轴对称非线性动力学方程。在几何非线性范围内给出了协调方程。网壳在周边固定条件下, 通过 Galerkin 作用得到一个含 2 次、3 次的非线性微分方程, 通过求 Floquet 指数讨论了分岔问题。为了研究混沌运动, 对一类非线性动力系统的自由振动方程进行了求解, 继之给出了单层扁锥面网壳非线性自由振动微分方程的准确解, 通过求 Melnikov 函数, 给出了发生混沌的临界条件, 通过数值仿真也证实了混沌运动的存在。

关键词: 网壳; 拟壳法; 分岔; 混沌

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引 言

空间网壳结构具有刚度大, 重量轻, 受力合理, 造价低等优点, 可用较短的杆件组成大跨度空间结构, 此结构也是衡量一个国家建筑技术水平的重要标志, 世界各国都很重视此领域的研究、应用和发展。以往工作对静态稳定研究的较多, 对动态稳定研究的较少^[1-6], 而只有文献 [5] 和文献 [6] 涉及到混沌问题。此文是在文献 [7] 和文献 [8] 工作的基础上继续在空间网格结构方面开展的研究, 这对网壳结构动力稳定性的研究有重要的意义。

1 物理方程

$$\begin{aligned} T_r = T_{00} = \frac{9}{8} \frac{EA}{b}, \quad T_{r0} = T_{0r} = \frac{3}{8} \frac{EA}{b}, \quad B_{rr} = B_{00} = \frac{9}{8} \frac{EI}{b}, \quad B_{r0} = B_{0r} = \frac{3}{8} \frac{EI}{b}, \\ N_r = T_{rr} \varepsilon + T_{r0} \varepsilon_0, \quad N_0 = T_{00} \varepsilon_0 + T_{0r} \varepsilon_r, \quad M_r = B_{rr} x_r + B_{r0} x_0, \quad M_0 = B_{00} x_0 + B_{0r} x_r, \\ x_r = -\frac{\partial^2 w}{\partial r^2}, \quad x_0 = -\frac{1}{r} \frac{\partial w}{\partial r}, \quad \varepsilon = \frac{\partial u}{\partial r} + \frac{H}{R} \frac{\partial w}{\partial r} + \frac{1}{2} \left(\frac{\partial w}{\partial r} \right)^2, \quad \varepsilon_0 = \frac{u}{r}, \end{aligned}$$

其中 E 为弹性模量, A 为杆的横截面积, I 为杆的惯性矩, l 为杆长, $b = (\sqrt{3}/2)l$, u 为径向位移, w 为横向位移, H 为锥壳拱高, R 为锥壳底半径。

2 基本方程和边界条件

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由薄壳非线性动力学理论

$$\frac{9RI}{8b}L(w) = q + \frac{1}{r} \frac{\partial}{\partial r} \left[rN_r \left(\frac{H}{R} + \frac{\partial w}{\partial r} \right) \right] - c \frac{\partial w}{\partial t} - \gamma \frac{\partial^2 w}{\partial t^2}, \quad (1)$$

$$\frac{b}{EA} r \frac{d}{dr} \frac{1}{r} \frac{d}{dr} (r^2 N_r) = - \frac{H}{R} \frac{\partial w}{\partial r} - \frac{1}{2} \left(\frac{\partial w}{\partial r} \right)^2. \quad (2)$$

边界条件为

$$\text{当 } r = R, w = \frac{\partial w}{\partial r} = 0, \frac{\partial}{\partial r} (rN_r) - \frac{1}{3} N_r = 0, \quad (3)$$

$$\text{当 } r = 0, w, \frac{\partial w}{\partial r}, N_r \text{ 有限.} \quad (4)$$

初始条件为

$$t = 0, w = w_0(r, 0), \frac{\partial w_0(r, 0)}{\partial t} = 0, \quad (5)$$

其中: $L = \frac{1}{r} \frac{\partial}{\partial r} r \frac{\partial}{\partial r} \frac{1}{r} \frac{\partial}{\partial r} r \frac{\partial}{\partial r}$, γ 为单位面积体质量, c 为阻尼系数.

引入无量纲量

$$\rho = \frac{r}{R}, W = \frac{w}{\beta}, N = \frac{8bR}{9EI} r N_r, Q = \frac{8bR^4 q}{9E\beta^5}, \gamma = \frac{8bR^4}{9EI} \gamma,$$

$$c = \frac{8bR^4}{9EI} c, K = \frac{H}{\beta}, I = \beta^4.$$

则方程(1)~ 方程(5)简化为

$$L_1(W) = Q + \frac{1}{\rho} \frac{\partial}{\partial \rho} \left[N \left(K + \frac{\partial W}{\partial \rho} \right) \right] - c \frac{\partial W}{\partial t} - \gamma \frac{\partial^2 W}{\partial t^2}, \quad (6)$$

$$L_2(N) = - \alpha \left(K + \frac{1}{2} \frac{\partial W}{\partial \rho} \right) \frac{\partial W}{\partial \rho}, \quad (7)$$

$$\text{当 } \rho = 0, W, \frac{\partial W}{\partial \rho}, N \text{ 有限,} \quad (8)$$

$$\text{当 } \rho = 1, W = \frac{\partial W}{\partial \rho} = 0, \frac{\partial N}{\partial \rho} - \frac{1}{3} N = 0, \quad (9)$$

$$\text{当 } t = 0, W = W_0(\rho, 0), \frac{\partial W}{\partial t} \neq 0, \quad (10)$$

这里,算子

$$L_1 = \frac{1}{\rho} \frac{\partial}{\partial \rho} \rho \frac{\partial}{\partial \rho} \frac{1}{\rho} \frac{\partial}{\partial \rho} \rho \frac{\partial}{\partial \rho}, L_2 = \rho \frac{\partial}{\partial \rho} \frac{1}{\rho} \frac{\partial}{\partial \rho}, \alpha = \frac{4A}{9\beta^2},$$

$$\text{取 } W = f(t) (1 - \rho^2)^2, \quad (11)$$

将(11)式代入(7)式,由边界条件得 N , 在将 N 和(11)式代入(6)式通过 Galerkin 作用,可得一个含 2 次、3 次项的非线性微分方程

$$\frac{d^2 f}{dt^2} + \omega^2 f - \alpha_1 f^2 + \alpha_2 f^3 = g \cos \Omega t - C_1 \frac{df}{dt}, \quad (12)$$

$$\text{其中 } \omega^2 = \frac{10}{\gamma} \left[\frac{32}{3} + 1.8761k^2 \right], C_1 = \frac{c}{\gamma}, \alpha_1 = \frac{35.546k}{\gamma}, \alpha_2 = \frac{14.86}{\gamma}, g \cos \Omega t = \frac{5Q}{3\gamma}.$$

取 $\tau = \omega t, f = (\omega / \sqrt{\alpha_2}) x$, 由(12)式可得

$$\frac{d^2 x}{d\tau^2} + x - \beta_2 x^2 + x^3 = F \cos \frac{\Omega}{\omega} \tau - \beta_0 \frac{dx}{d\tau}, \quad (13)$$

其中, $\beta_0 = C_1 / \omega, \beta_2 = \alpha_1 / (\omega \sqrt{\alpha_2}), F = g \sqrt{\alpha_2} / \omega^3$. 方程(13)式不考虑外激励,其等价系统

为

$$\dot{x}_1 = x_2, \quad \dot{x}_2 = -x_1 + \beta_2 x_1^2 - x_1^3 - \beta_0 x_2,$$

此系统3个平衡点为 $(0, 0)$, $(\beta_2/2 \pm \sqrt{\beta_2^2/4 - 1}, 0)$ 。

1) 在平衡点 $(0, 0)$ 处, Floquet 指数为 $\lambda_{1,2} = -\beta_0/2 \pm \sqrt{\beta_0^2/4 - 1}$, 当 $\beta_0 > 2$ 时, 则此平衡点为稳定的结点; 当 $0 < \beta_0 < 2$ 时, λ 离开数轴在复平面上, 有稳定的结点; 当 $\beta_0 = 2$ 时, λ 为相等的负数, 平衡点为临界结点; 当 $\beta_0 = 0$ 时, λ 是纯虚数, 解的曲线是封闭的, 即极限环。这时发生 Hopf 分岔。

2) 在平衡点 $(\beta_2/2 + \sqrt{\beta_2^2/4 - 1}, 0)$ 处, Floquet 指数为

$$\lambda_{1,2} = -\frac{\beta_0}{2} \pm \sqrt{\left(\frac{\beta_0}{2}\right)^2 + 2 - \frac{1}{2}\beta_2^2 - \beta_2 \sqrt{\left(\frac{\beta_0}{2}\right)^2 - 1}} \quad (\beta_2 > 2);$$

当 $\beta_0 > \sqrt{-8 + 2\beta_2^2 + 4\beta_2 \sqrt{(\beta_2/2)^2 - 1}}$, 此平衡点为稳定的结点或鞍点;

当 $0 < \beta_0 < \sqrt{-8 + 2\beta_2^2 + 4\beta_2 \sqrt{(\beta_2/2)^2 - 1}}$, 此平衡点有稳定的焦点;

当 $\beta_0 = \sqrt{-8 + 2\beta_2^2 + 4\beta_2 \sqrt{(\beta_2/2)^2 - 1}}$, 平衡点为临界结点;

当 $\beta_0 = 0$, 在此平衡点发生 Hopf 分岔。

3) 在平衡点 $(\beta_2/2 - \sqrt{\beta_2^2/4 - 1}, 0)$ 处, Floquet 指数为

$$\lambda_{1,2} = -\frac{\beta_0}{2} \pm \sqrt{\left(\frac{\beta_0}{2}\right)^2 + 2 - \frac{1}{2}\beta_2^2 + \beta_2 \sqrt{\left(\frac{\beta_0}{2}\right)^2 - 1}}, \quad (\beta_2 > 2);$$

当 $\beta_0 > \sqrt{-8 + 2\beta_2^2 - 4\beta_2 \sqrt{(\beta_2/2)^2 - 1}}$, 此平衡点为稳定的结点或鞍点;

当 $0 < \beta_0 < \sqrt{-8 + 2\beta_2^2 - 4\beta_2 \sqrt{(\beta_2/2)^2 - 1}}$, 此平衡点有稳定的焦点;

当 $\beta_0 = \sqrt{-8 + 2\beta_2^2 - 4\beta_2 \sqrt{(\beta_2/2)^2 - 1}}$, 平衡点为临界结点;

当 $\beta_0 = 0$, 在此平衡点发生 Hopf 分岔。

3 一类非线性动力系统自由振动方程的解

考虑方程

$$x'' + \beta_1 x - \beta_2 x^2 + \beta_3 x^3 = 0 \quad (\beta_1, \beta_2, \beta_3 > 0), \quad (14)$$

通过求此方程同宿轨道可得解为

$$x = \frac{2\beta_1}{(2\beta_2/3) \pm \sqrt{(2\beta_2/3)^2 - 2\beta_1\beta_3 \sin \sqrt{\beta_1}(\tau + c)}}, \quad (15)$$

由方程(13)式知 $\beta_1 = \beta_3 = 1$, 则得(13)式的自由振动方程的解为

$$x_{1,2} = 2/(a \pm b \sin(\tau + c)),$$

其中 $a = 2\beta_2/3$, $b = \sqrt{a^2 - 2}$ 。取 $c = 0$, 初始条件得到满足(对其它初始条件在另文讨论)。

则

$$x_{1,2} = 2/(a \pm b \sin \tau). \quad (16)$$

4 Melnikov 函数

为了书写方便, 方程(13)式中, 以 t 代 τ , 以 ω 代 Ω/ω 。由方程(16)式可取 $x(t) = 2/(a -$

$bsint)$, 则 $dx/dt = y(t) = 2b \cos t / (a - bsint)^2$.

由定义的 Melnikov 函数^[10]

$$M(t_0) = \int_{-\infty}^{\infty} [-\beta_0 y^2(t) + Fy(t) \cos \omega(t + t_0)] dt = -4\beta_0 b^2 \int_{-\infty}^{\infty} \frac{\cos^2 t}{(a - bsint)^4} dt + 2bF \int_{-\infty}^{\infty} \frac{\cos t \cos \omega(t + t_0)}{(a - bsint)^2} dt, \quad (17)$$

经过留数计算可得

$$\text{当 } \omega = 1 \text{ 时, } M(t_0) = \frac{\sqrt{2}}{3} \left[\frac{4}{9} \beta_2^2 - 2 \right] \beta_0 \beta_2 \pi - \frac{4F\pi(2\beta_2/3 + 1) \cos t_0}{\sqrt{4\beta_2^2/9 - 2}},$$

则得

$$4F \left[\frac{2}{3} \beta_2 + 1 \right] > \frac{\sqrt{2}}{3} \left[\frac{4}{9} \beta_2^2 - 2 \right]^{3/2} \beta_2 \beta_0, \quad (18)$$

$$\text{当 } \omega = 2 \text{ 时, } M(t_0) = \frac{\sqrt{2}}{3} \left[\frac{4}{9} \beta_2^2 - 2 \right] \beta_0 \beta_2 \pi - \frac{2F\pi(4\sqrt{2}\beta_2 + 24\beta_2 + 18\sqrt{2}) \sin t_0}{2\beta_2^2 - 9},$$

则得

$$2F(4\sqrt{2}\beta_2 + 24\beta_2 + 18\sqrt{2}) > \frac{3\sqrt{2}}{3} \left[\frac{4}{9} \beta_2^2 - 2 \right]^2 \beta_2 \beta_0, \quad (19)$$

由(18)式、(19)式可知存在同宿点, 系统可能发生混沌运动.

从下面相平面图上也可以看出产生的混沌运动.

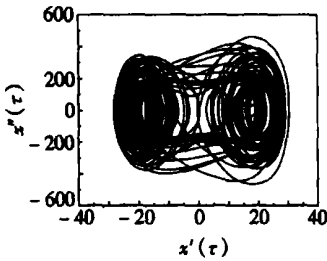


图1 相轨图 ($\beta_0 = 0.5, F = 10\,000$)

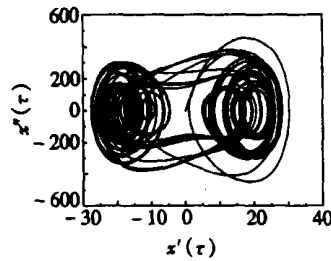


图2 相轨图 ($\beta_0 = 1, F = 10\,000$)

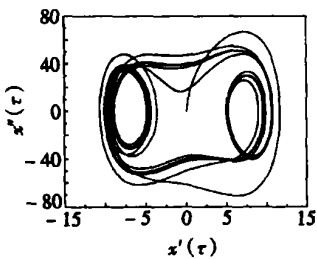


图3 相轨图 ($\beta_0 = 1, F = 500$)

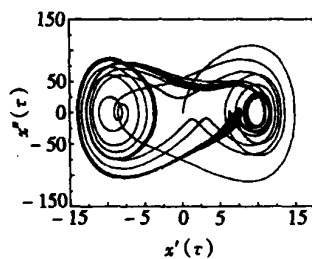


图4 相轨图 ($\beta_0 = 1, F = 1\,000$)

5 讨 论

1) 从方程(18)式、(19)式可以明显看出, 阻尼越小, 激励越大, 此动力系统越容易发生混沌运动.

2) 该文求出的自由振动方程(14)式的准确解, 使作者很容易求出 Melnikov 函数, 对此类

非线性动力学系统在研究混沌现象可推广应用。

3) 该文只求出 $\omega = 1$, $\omega = 2$ 时的 Melnikov 函数, 若工程需要也可对 $\omega = 3$ 求出 Melnikov 函数进行讨论。

4) (14) 式方程的两个解, 取其中任一个计算结果是一样的。

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Nonlinear Dynamical Bifurcation and Chaotic Motion of a Shallow Conical Lattice Shell

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Abstract: The nonlinear dynamical equations of axle symmetry were established by using the method of quasi_shells for three_dimensional shallow conical single_layer lattice shells. The compatible equations were given in geometrical nonlinear range. A nonlinear differential equation containing the second and the third order nonlinear items was derived under the boundary conditions of fixed and clamped edges by using the method of Galerkin. The problem of bifurcation is discussed by solving the Floquet exponent. In order to study chaotic motion, the equations of free oscillation to a kind of nonlinear dynamics system were solved. Then an exact solution to nonlinear free oscillation of the single_layer shallow conic lattice shell was found as well. The critical conditions of chaotic motion were obtained by solving Melnikov functions, some phase planes were drawn by using digital simulation and proved the existence of chaotic motion.

Key words: lattice shell; the method of quasi_shell; bifurcation; chaotic motion