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扁柱面网壳的非线性动力学行为*

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(我刊编委叶开沅来稿)

摘要: 用连续化法建立了正三角形网格的三向单层扁柱面网壳的非线性动力学方程和协调方程. 在两对边简支条件下用分离变量函数法给出扁柱面网壳的横向位移. 由协调方程求出张力, 通过 Galerkin 作用得到了一个含二次、三次的非线性动力学微分方程. 通过求 Floquet 指数讨论平衡点邻域的稳定性, 用复变函数留数理论求出 Melnikov 函数, 可得到该动力学系统发生混沌运动的临界条件. 通过数值计算模拟和 Poincar 映射也证明了混沌运动存在.

关 键 词: 网壳; 连续法; 混沌; 临界条件
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引 言

兼有杆系结构和薄壳结构固有特性的曲面型网壳结构, 具有受力合理、重量轻、造价低等优点, 目前在大跨度覆盖建筑中得到广泛应用. 由于地震海啸频繁发生, 地球表面经常遭受狂风暴雨、暴雪袭击. 1963年罗马尼亚布加勒斯特国立经济展览馆、1978年美国康涅狄格州哈特福德城一座体育馆在大风雪中倒塌, 9415号台风引起温州机场网壳屋盖的破坏. 这种网壳结构动态稳定性问题自然提出来了. 国内外学者对这方面都有研究[1_8], 但关于网架、网壳结构的分岔和混沌的研究尚未查到, 我们在文献[9_11]中初步研究了网架网壳的分岔与混沌运动. 该文也是在网格结构研究分岔与混沌运动的继续.

1 物理方程和等效刚度

平面正三角形网格单元有效刚度[12]:

$$T_{xx} = T_{yy} = \frac{3\sqrt{3}}{4a_1}EA,\ T_{xy} = T_{yx} = \frac{\sqrt{3}}{4a_1}EA,\ B_{xx} = B_{yy} = \frac{3\sqrt{3}}{4a_1}EI,\ B_{xy} = B_{yx} = \frac{\sqrt{3}}{4a_1}EI,$$

其中 A 为杆横截面积, E 为弹性模量, I 为杆惯性矩, a_1 为杆长,

$$N_x = T_{xx}\varepsilon_x + T_{xy}\varepsilon_y = \frac{3\sqrt{3}}{4a_1}EA\left(\varepsilon_x + \frac{1}{3}\varepsilon_y\right),$$

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$$N_y = T_{yy}\varepsilon_y + T_{yx}\varepsilon_x = \frac{3\sqrt{3}EA}{4a_1}\left(\varepsilon_y + \frac{1}{3}\varepsilon_x\right),\quad N_{xy} = N_{yx} = T_{xy}\varepsilon_{xy} = \frac{\sqrt{3}EA}{4a_1}\varepsilon_{xy},$$

$$M_x = B_{xx}\chi_x + B_{xy}\chi_y = \frac{3\sqrt{3}EI}{4a_1}\left(\chi_x + \frac{1}{3}\chi_y\right),$$

$$M_y = B_{xy}\chi_x + B_{yy}\chi_y = \frac{3\sqrt{3}EI}{4a_1}\left(\chi_y + \frac{1}{3}\chi_x\right),\quad M_{xy} = B_{xy}\chi_{xy} = \frac{\sqrt{3}EI}{4a_1}\chi_{xy},$$

其中 $\chi_x = -\dfrac{\partial^2 W}{\partial x^2},\quad \chi_y = -\dfrac{\partial^2 W}{\partial y^2},\quad \chi_{xy} = -2\dfrac{\partial^2 W}{\partial x\partial y}.$

对图 1 所示扁柱壳,

$$\varepsilon_x = \frac{\partial u}{\partial x} + \frac{1}{2}\left(\frac{\partial W}{\partial x}\right)^2,\quad \varepsilon_y = \frac{\partial V}{\partial y} - \frac{1}{R}W + \frac{1}{2}\left(\frac{\partial W}{\partial y}\right)^2,$$

$$\varepsilon_{xy} = \frac{\partial U}{\partial y} + \frac{\partial V}{\partial x} + \frac{\partial W\partial W}{\partial y\partial x},$$

U、V、W 分别为 x、y 的方向上位移, R 为柱面网壳 y 方向的曲率半径.

2 基本方程和边界条件

由扁薄壳非线性动力学理论:

$$\frac{3\sqrt{3}EI}{4a}l_1(W) = q + l_2(W,\ \phi) + \frac{1}{R}\frac{\partial^2\phi}{\partial x^2} - c\frac{\partial W}{\partial t} - \gamma\frac{\partial^2 W}{\partial t^2}, \tag{1}$$

$$\frac{\sqrt{3}a}{2EA}l_1(\phi) = -\frac{1}{2}l_2(W,\ \phi) - \frac{1}{R}\frac{\partial^2 W}{\partial x^2}, \tag{2}$$

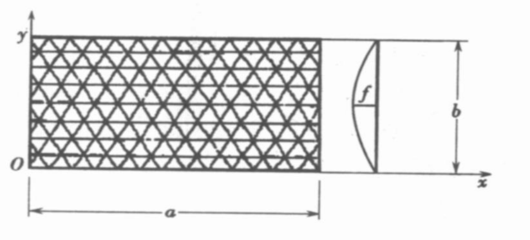

图 1 单层扁柱面网壳

这里

$$N_x = \frac{\partial^2\phi}{\partial y^2},\quad N_y = \frac{\partial^2\phi}{\partial x^2},\quad N_{xy} = -2\frac{\partial^2\phi}{\partial x\partial y},$$

$$l_1 = \frac{\partial^4}{\partial x^4} + 2\frac{\partial^4}{\partial x^2\partial y^2} + \frac{\partial^4}{\partial y^4},$$

$$l_2(W,\ \phi) = \frac{\partial^2 W}{\partial x^2}\frac{\partial^2\phi}{\partial y^2} + \frac{\partial^2 W}{\partial y^2}\frac{\partial^2\phi}{\partial x^2} - 2\frac{\partial^2\phi}{\partial x\partial y}\frac{\partial^2 W}{\partial x\partial y},$$

c 为阻尼系数, γ 为单元体的物体质量. 边界条件为两底对边简支:

当 $x = 0,\ x = a$ 时, 平均位移

$$e_x = 0, \tag{3}$$

当 $y = 0,\ y = b$ 时, 平均位移

$$e_y = 0, \tag{4}$$

e_x、e_y 分别为 x、y 方向上平均相对位移. 设扁柱壳位移为

$$W = f(t)\sin\alpha x\sin\beta y, \tag{5}$$

其中 $\alpha = \dfrac{\pi}{a},\quad \beta = \dfrac{\pi}{b}.$

将方程(5)代入方程(2)的右端, 解方程得

$$n_2\phi = \frac{f^2(t)}{32}\left[\left(\frac{a}{b}\right)^2\cos 2\alpha x + \left(\frac{b}{a}\right)^2\cos 2\beta y\right] +$$

$$\frac{1}{R\pi^2}f(t)\frac{a^2b^4}{a^2+b^2}\sin\alpha x\sin\alpha y-n_2\left(\frac{p_x y^2}{2}+\frac{p_y x^2}{2}\right). \tag{6}$$

将方程(5)、(6)代入方程(1)通过Galerkin作用可得

$$n_1\frac{\pi^6}{16}f(t)\left(\frac{1}{a^2}+\frac{1}{b^2}\right)+\frac{\pi^6}{256n_2}f^3(t)\left(\frac{1}{a^4}+\frac{1}{b^4}\right)+\frac{p_y}{R}-\frac{2\pi^2b^2}{3n_2R(a^2+b^2)^2}f^2(t)-$$
$$\frac{\pi^2}{24Rb^2n_2}f^2(t)+\frac{\pi^2b^4}{16R^2n_2(a^2+b^2)^2}f(t)-p_x\frac{\pi^4}{16a^2}f(t)-p_y\frac{\pi^4}{16b^2}f(t)+$$
$$\frac{\pi^2}{16}c\frac{\partial f(t)}{\partial t}+\frac{\pi^2}{16}\gamma\frac{\partial^2 f(t)}{\partial t^2}=q, \tag{7}$$

其中 $n_1=3\sqrt{3}EI/(4a)$, $n_2=\sqrt{3}a/(2EA)$, p_x、p_y 是边界侧张力平均值.

引入无量纲量

$$\xi(t)=\frac{f(t)}{\beta},\quad \lambda=\frac{a}{b},\quad k_y'=\frac{a^2}{R\beta},\quad P_x'=\frac{n_2p_xb^2}{\beta^2},\quad P_y'=\frac{n_2p_ya^2}{\beta^2},$$
$$Q=\frac{qn_2a^2b^2}{\beta^3},\quad n_3=\frac{q\beta^2}{8A},\quad c_1=\frac{c\pi^2a^2b^2n_2}{16\beta^2},\quad \gamma_1=\frac{n_2\pi^2a^2b^2\gamma}{16\beta^2}.$$

由方程(7)可得

$$\gamma_1\frac{\partial^2\xi(t)}{\partial t^2}+\left[n_3\frac{\pi^6}{16}\left(\frac{1}{\lambda}+\lambda\right)^2+k_y'^2\frac{\pi^2}{16}\left(\frac{1}{\lambda}+\lambda\right)^{-2}\right]\xi(t)+$$
$$c_1\frac{\partial\xi(t)}{\partial t}-\left(\frac{2\pi^2}{3}k_y'\frac{\lambda^2}{(1+\lambda^2)^2}+\frac{\pi^2}{24}k_y'\lambda^2\right)\xi^2(t)+$$
$$\frac{\pi^6}{256}\left(\frac{1}{\lambda^2}+\lambda^2\right)\xi^3(t)-\frac{\pi^2}{16}(p_x'+p_y')\xi(t)+k_y'p_y'=Q, \tag{8}$$

其中 $I=\beta^4$.

由边界条件(3)、(4)可得

$$p_x'=-\frac{\pi^2}{8}\left(\frac{1}{\lambda^2}+\mu\right)\frac{1}{1-\mu^2}\xi^2(t)+\mu k_y'\frac{4}{\pi^2(1-\mu^2)}\xi(t)-k_y'\frac{4\lambda^2}{\pi^2(1+\lambda^2)^2}\xi(t), \tag{9}$$
$$p_y'=-\frac{\pi^2}{8}\frac{(\lambda^2+\mu)}{1-\mu^2}\xi^2(t)+\lambda^2k_y'\frac{4}{\pi^2(1-\mu^2)}\xi(t)-k_y'\frac{4\lambda^2}{\pi^2(1+\lambda^2)^2}\xi(t). \tag{10}$$

取 $Q=Q_0\cos\Omega t$ 将方程(9)、(10)代入方程(8)得

$$\frac{\partial^2\xi(t)}{\partial t^2}+\omega^2\xi(t)+c_2\frac{\partial\xi(t)}{\partial t}-\alpha_1\xi^2(t)+\alpha_2\xi^3(t)=F_1\cos\Omega t, \tag{11}$$

其中

$$\omega^2=\frac{1}{\gamma_1}\left[n_3\frac{\pi^2}{16}\left(\frac{1}{\lambda}+\lambda\right)^2+\lambda^2\frac{\pi^2-64}{16\pi^2(\lambda^2+1)^2}k_y'^2+k_y'^2\lambda^2\frac{4}{\pi^2(1-\mu^2)}\right],$$
$$\alpha_1=\frac{1}{\gamma_1}\left[\frac{3\pi^2k_y'(\lambda^2+\mu)}{8(1-\mu^2)}+\frac{5\pi^2k_y'\lambda^2}{12(1+\lambda^2)^2}+\frac{\pi^2k_y'\lambda^2}{24}\right],$$
$$\alpha_2=\frac{1}{\gamma_1}\left[\frac{\pi^2(1+\lambda^4)}{256\lambda^2}+\frac{\pi^6(\lambda^4+2\lambda^2\mu+1)}{128(1-\mu^2)\lambda^2}\right],\quad c_2=\frac{c_1}{\gamma_1},\quad F_1=\frac{Q_0}{\gamma_1}.$$

取 $\tau=\omega t$, $\eta(t)=(\omega/\sqrt{\alpha_2})\xi(t)$, 则方程(11)变为标准方程

$$\frac{d^2\eta(\tau)}{d\tau^2}+\eta(\tau)-\beta_2\eta^2(\tau)+\eta^3(\tau)=F\cos\frac{\Omega}{\omega}\tau-\beta_0\frac{d\eta(\tau)}{d\tau}, \tag{12}$$

其中 $\beta_0=c_2/\omega$, $\beta_2=\alpha_1/(\omega\sqrt{\alpha_2})$, $F=F_1/\omega^3$. 方程(2)的自由振动方程解为[10_11]

$$\eta_{1,2}=\frac{2}{a\pm b\sin(\tau+c)}, \tag{13}$$

这里

$$a = 2\beta_2/3,\ b = \sqrt{a^2 - 2}.$$

若初始速度为 0, 位移不为 0, 取 $c = n\pi + \pi/2\ (n = 0, 1, 2, 3, \cdots)$, 则

$$\eta_{1,2} = \frac{2}{a \pm b\cos\tau}. \tag{14}$$

若初始速度不为 0, 位移不为 0 可取 $c = 0$ 则可得

$$\eta_{1,2} = \frac{2}{a \pm b\sin\tau}. \tag{15}$$

3 Melnikov 函数[12]

为了书写方便, 方程(12) 中以 t 代 τ, 以 ω 代替 Ω/ω. 取(14) 式中

$$\eta = \frac{2}{a - b\cos\tau} \tag{16}$$

$$\frac{d\eta}{dt} \frac{-2b\sin t}{(a - b\cos t)^2}, \tag{17}$$

$$M(t_0) = \int_{-\infty}^{+\infty}\left[-\beta_0\left(\frac{d\eta}{dt}\right)^2 + F\frac{d\eta}{dt}\cos\omega(t + t_0)\right]dt. \tag{18}$$

通过留数计算可得

当 $\omega = 1$ 时,

$$M(t_0) = -\frac{\sqrt{2}}{3}\beta_0\beta_2\left(\frac{2}{3}\beta_2 + \sqrt{2}\right)\left(\frac{2}{3}\beta_2 - \sqrt{2}\right)\pi - \frac{2\sqrt{2}F\pi\sqrt{2\beta_2/3 - \sqrt{2}}}{\sqrt{2\beta_2/3 + \sqrt{2}}}\cos t_0; \tag{19}$$

当 $\omega = 3$ 时,

$$M(t_0) = -\frac{2}{3}\beta_0\beta_2\left(\frac{2}{3}\beta_2 + \sqrt{2}\right)\left(\frac{2}{3}\beta_2 - \sqrt{2}\right)\pi - \frac{6\sqrt{2}F(2\beta_2/3 - \sqrt{2})^2\pi}{(2\beta_2/3 + \sqrt{2})\sqrt{4\beta_2^2/9 - 2}}\cos 3t_0. \tag{20}$$

当

$$\omega = 1,\ F > \frac{\beta_0\beta_2}{6}\left(\frac{2}{3}\beta_2 + \sqrt{2}\right)\sqrt{\frac{4}{9}\beta_2^2 - 2}, \tag{21}$$

$$\omega = 3,\ F > \frac{\beta_0\beta_2}{18}\frac{(2\beta_2/3 + \sqrt{2})^2\sqrt{4\beta_2^2/9 - 2}}{\sqrt{2} - 2\beta_2/3} \tag{22}$$

时就存在同宿点, 系统可能产生混沌运动.

从下面的时程曲线图(图 2、3), Poincar 映射图(图 4、5), 相轨曲线图(图 6、7) 可看出该动力系统在一定条件下产生混沌运动.

4 讨 论

1) 由式(21)、(22) 可知阻尼越小外激励越大, 越容易产生混沌运动.

2) 由式(14) 的两个解, 无论用哪个解求出的 Melnikov 函数是一样的.

3) 由固有频率 ω^2 的式子可以看出, 扁柱网壳结构动力学特性和结构特征尺寸及材料性质有关.

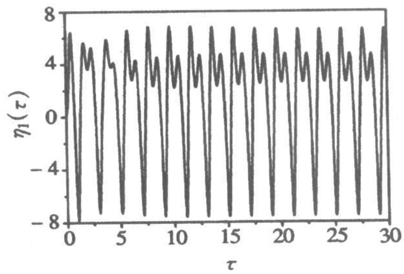

图 2 时程曲线图 (β_0 = 0.5, F = 100)

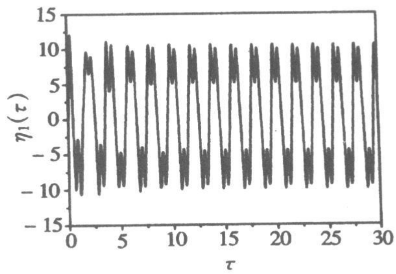

图 3 时程曲线图 (β_0 = 0.5, F = 500)

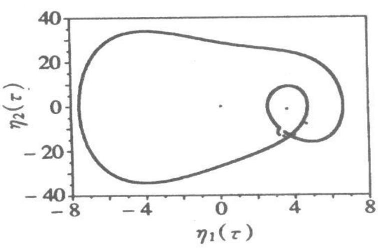

图 4 Poincar 映射图 (β_0 = 0.5, F = 100)

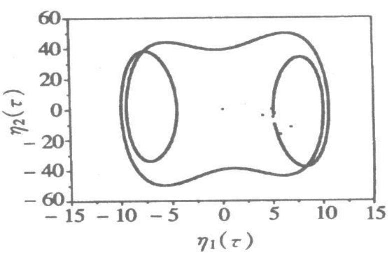

图 5 Poincar 映射图 (β_0 = 0.5, F = 500)

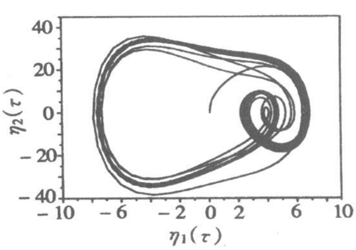

图 6 相轨曲线图 (β_0 = 0.5, F = 100)

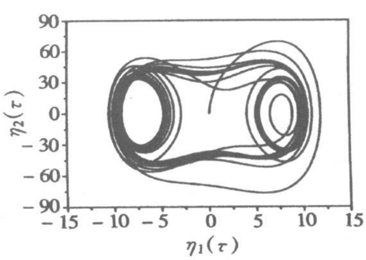

图 7 相轨曲线图 (β_0 = 0.5, F = 500)

[参 考 文 献]

[1] 孙建恒, 夏亨熹. 网壳结构非线性动力稳定性分析[J]. 空间结构, 1994, **1**(1): 25_31.

[2] 周岱, 刘红玉, 李春祥. 拉索_网壳结构的动力特性和非线性动力反应[J]. 振动与冲击, 2002, **21**(1): 7_11.

[3] 桂国庆, 林智斌. 单层网壳的动力稳定性分析[J]. 南昌大学学报, 2003, **25**(1): 43_47.

[4] 郭海山, 沈世钊. 单层网壳结构动力稳定性分析法[J]. 建筑结构学报, 2003, **24**(3): 1_9.

[5] Afraimovich V S, Glebsky I Ya, Nekorkin V I. Stability of stationary states and spatial choas in multidimensional lattice dynamical systems[J]. Random Computer Dynamic, 1994, **2**: 287_303.

[6] Bunimovich L A, CaHen E A. On the problem of stability dynamical systems[J]. Journal of Differential Equations, 1995, **123**: 213_229.

[7] 丁雪兴, 何世权, 王新志. 静载荷下圆形三向网架的非线性振动[J]. 甘肃工业学报, 2003, **29**(1):

102_105.

[8] 王 策. 在强地震作用下球面网壳的动力稳定性[J]. 清华大学学报(自然科学版), 2000, **40**(11): 57_60.

[9] 王新志, 王钢, 赵艳影, 等. 圆形三向网架非线性动力稳定性分析[J]. 应用数学和力学, 2004, **25**(4): 331_336.

[10] 王新志, 梁从兴, 栗蕾, 等. 扁锥面单层网壳的非线性动力学特性[J]. 动力学与控制学报, 2004, **2**(3): 14_17.

[11] 王新志, 梁从兴, 丁雪兴, 等. 单层扁锥面网壳的非线性动力稳定性分析[J]. 工程力学, 2005, **22**: 172_176.

[12] 陈士华, 陆君安. 混沌动力学初步[M]. 武汉: 武汉水利电力大学出版社, 1998.

Nonlinear Dynamical Behavior of Shallow Cylindrical Reticulated Shells

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Abstract: By using the method of quasi_shells, the nonlinear dynamic equations of three_dimensional single_layer shallow cylindrical reticulated shells with equilateral triangle cell were founded. By using the method of the separating variable function, the transverse displacement of the shallow cylindrical reticulated shells were given under the conditions of two edges simple support. The tensile force was solving from the compatible equations, a nonlinear dynamic differential equation containing the second and third order is derived by using the method of Galerkin. The stability near the equilibrium point was discussed by solving the Floquet exponent and the critical condition is obtained by using Melnikov function. Existence of the chaotic motion of the single_layer shallow cylindrical reticulated shell is approved by using the digital simulation method and Poincar mapped.

Key words: reticulated shells; method of quasi_shells; chaotic motion; critical condition